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NASA Conference Publication 10088  
ICOMP-92-02; CMOTT-92-02

# Workshop on Engineering Turbulence Modeling

*Proceedings of the Workshop on  
Engineering Turbulence Modeling  
sponsored by the Institute for  
Computational Mechanics in Propulsion  
and Center for Modeling of Turbulence  
and Transition  
NASA Lewis Research Center  
Cleveland, Ohio  
August 21-22, 1991*



**NASA**

(NASA-CP-10088) WORKSHOP ON ENGINEERING  
TURBULENCE MODELING (NASA) 510 p CSCI 200

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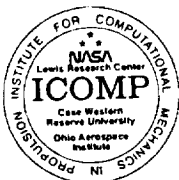
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1992







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# WORKSHOP ON ENGINEERING TURBULENCE MODELING

Center for Modeling of Turbulence and Transition (CMOTT)  
ICOMP, NASA Lewis Research Center  
Room 175, Sverdrup Building, Cleveland, Ohio

August 21-22, 1991

## 1. OBJECTIVES

The purpose of this meeting is to discuss the **present status** and the **future direction** of various levels of engineering turbulence modeling related to CFD computations for propulsion. For each level of complication, there are a few turbulence models which represent the state of the art for that level. However, it is important to know their capabilities as well as their deficiencies in CFD computations in order to help engineers select and implement the appropriate models in their real world engineering calculations. This will also help turbulence modelers perceive the future directions for improving turbulence models.

The focus of this meeting will be one-point closure models (i.e. from algebraic models to higher order moment closure schemes and pdf methods) which can be applied to CFD computations. However, other schemes helpful in developing one-point closure models, such as RNG, DIA, LES and DNS, will be also discussed to some extent.

## 2. FORMAT

This meeting will consist of three sessions and will last about one and half days.

Each session will have three or five position presentations. In the first two sessions each position presentation (40 minutes) will be followed by a comment presentation (10 minutes) and a discussion. In session III, there are five position presentations (30 minutes) and one discussion. The presentations will be made by invited speakers and the discussions will be led by the session chairman (see outline of the workshop for details).

The viewgraphs of the presentations will be collected to be distributed later.

## 3. ORGANIZING COMMITTEE

T.-H. Shih (Chairman)  
J. L. Lumley (Honorary Chairman)  
P. Moin  
M. Goldstein  
L. A. Povinelli  
E. Reshotko  
J. M. Barton

#### **4. OUTLINE OF THE WORKSHOP**

**August 21, 1991 (Wednesday)**

08:00-08:15 am Registration

08:15-08:30 am Welcome by L. Povinelli

##### **Session I: Turbulence Modeling in CFD and Algebraic Closure models.**

**Chairman: E. Reshotko**

08:30-09:10 am B.E. Launder, "The current status of turbulence modeling in CFD and its future prospects."

09:10-09:20 am D.M. Bushnell, "Comment paper."

09:20-09:40 am Discussion

09:40-10:20 am D. Wilcox, "The present state and the future direction of eddy viscosity models."

10:20-10:30 am P. Spalart, "Comment paper."

10:30-10:40 am T. Coakley, "Comment paper."

10:40-11:00 am Discussion

11:00-11:40 am D. Taulbee, "The present state and future direction of algebraic Reynolds stress models."

11:40-11:50 am A.O. Demuren, "Comment paper."

11:50-12:00 am Discussion

12:00-01:30 pm Lunch Break

##### **Session II: Second Order Closure and PDF Method.**

**Chairman: J.L. Lumley**

1:30-2:10 pm T.-H. Shih, "The present state and the future direction of second order closure models for incompressible flows."

2:10-2:20 pm J.R. Ristorcelli, Jr., "Comment paper."

2:20-2:30 pm C.G. Speziale, "Comment paper."

2:30-2:50 pm Discussion

2:50-3:30 pm T.B. Gatski, "The present state and the future direction of second order closure models for compressible flows."

3:30-3:40 pm J. Viegas, "Comment paper."

3:40-3:50 pm G. Huang, "Comment paper."

3:50-4:10 pm Discussion

4:10-4:50 pm S. Pope, "The present state and the future direction of pdf methods."

4:50-5:00 pm E.E. O'Brien, "Comment paper."

5:00-5:10 pm J.Y. Chen, "Comment paper."

5:10-5:20 pm Discussion

6:30-9:00 pm Banquet (Pierre Radisson Inn, Great Northern Blvd.)



*Center for Modeling of Turbulence and Transition  
Workshop on Engineering Turbulence Modeling - 1991*

## **Session I**

# **Turbulence Modeling in CFD and Algebraic Closure Models**



August 22, 1991 (Thursday)

**Session III: Unconventional Turbulence Modeling.**

**Chairman: J.H. Ferziger**

08:30-09:00 am A. Yoshizawa, "The present state of DIA models and their impact on one point closures."

09:00-09:30 am J. Weinstock, "The present state of two-point closure schemes and their impact on one point closures."

09:30-10:00 am S. Orszag, "The present state of RNG and its impact on one point closure."

10:00-10:30 am R.R. Mankbadi, "The present state of application of RDT to unsteady turbulent flows."

10:30-11:20 am W.K. George, J.H. Ferziger "The role of experiments and DNS & LES in supporting turbulence modeling efforts."

11:20-11:40 am Discussion

11:40-12:00 Concluding Remarks.

**5. INVITED PARTICIPANTS**

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A. Hsu	K.H. Kao	S. Kim
S.W. Kim	T. Le	M.F. Liou
M.S. Liou	W.W. Liou	C.J. Marek
R. Mankbadi	E.J. Mularz	L.A. Povinelli
D.R. Reddy	L. Reid	D. Rigby
B. Rubinstein	J. Schwab	J. Scott
A. Shabbir	T.H. Shih	J.S. Shuen
F. Simon	C. Steffen	W.M. To
C. Towne	T. VanOverbeke	C. Wang
Z. Yang	S.T. Yu	

LeRC	8/91	Title	1
<p>Turbulence modelling in CFD: Present status, future prospects</p> <p>by</p> <p>Brian Launder UMIST, Manchester, UK</p>			

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p-10

LeRC	8/91	Aim and Scope	2
<ul style="list-style-type: none"> <li>• To provide a (personal) view of the status of turbulence modelling for use with the <u>fully averaged</u> equations of motion, energy, etc.</li> <li>• To give concrete examples of what types of problem can/should be tackled with different levels of closure model</li> <li>• Particular emphasis on applications in turbomachinery and near-wall treatments</li> <li>• Models considered: <ul style="list-style-type: none"> <li>* (Isotropic) Eddy Viscosity Models (EVM)</li> <li>* Algebraic Second-Moment Closures (ASM)</li> <li>* Differential Second-Moment Closures (DSM)</li> </ul> </li> <li>• No space to consider numerical strategies needed for non-EVM treatments</li> </ul>			

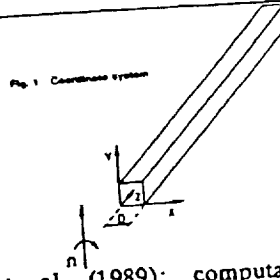
LeRC	8/91	Eddy viscosity models - I: Physical Basis	3
<ul style="list-style-type: none"> <li>• Can be interpreted as an implication of the turbulent kinetic energy (<math>k</math>) equation for a <u>simple shear flow</u> (<math>U_1 \neq 0, U_2 = U_3 = 0</math>) when production and dissipation of <math>k</math> are effectively in balance (<math>P_k = \epsilon</math>)</li> <li>• Main industrial interest is in applying turbulence models in conditions where these conditions are <u>not</u> satisfied!</li> <li>• Seem to perform worst in 2D curved flows and where body forces act in direction of primary velocity gradient</li> <li>• Compressibility effects on turbulence not adequately accounted for with an eddy-viscosity stress-strain relation</li> <li>• Nevertheless models of this type are relatively easy to use and will be replaced only where demonstrably superior alternatives are available</li> </ul>			

LeRC	8/91	Eddy viscosity models - II: Choice	4
<ul style="list-style-type: none"> <li>Available in versions requiring solution of 0-4 turbulent transport equations, of which one is (usually) that for <math>k</math></li> <li>No extensive testing beyond 2-equation level</li> <li>Generalized statement of 2-equation model <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <math display="block">\nu_t = c_\mu k^2 / \epsilon</math> <math display="block">\frac{Dk}{Dt} = d_k + P_k - \epsilon</math> <math display="block">\frac{Dz}{Dt} = d_z + c_{z1} \frac{P_k z}{k} - c_{z2} \frac{zk}{\nu_t} + S_z \quad z = k a b</math> </div> </li> <li>Most popular strategy takes <math>\epsilon = k^3 / 2l</math> as second dependent variable mainly because <math>S_\epsilon</math> can be taken as zero in many simple flows</li> <li>Need for <u>non-zero</u> <math>S_\epsilon</math> becomes evident in separated and impinging flows to prevent excessive near-wall length scales developing</li> </ul>			

LeRC	8/91	Eddy viscosity models - III: Near-Wall Strategy	5
<ul style="list-style-type: none"> <li>EVM's rarely give satisfactory levels of <math>\overline{u_i u_j}</math> away from wall vicinity: if Reynolds stresses are important there, second-moment closure is needed</li> <li>More difficult to devise suitable ASM's/DSM's for near-wall sublayer</li> <li>Hence most current research on EVM's concerned with treatment of this "low-Reynolds-number" region</li> <li>Log laws are generally inadequate .....</li> <li>..... even a mixing-length scheme is better</li> <li>One-equation models for sublayer currently seem a good compromise, especially if used with a "floating" <math>\kappa</math> (the length-scale gradient)</li> </ul>			

LeRC	8/91	Eddy viscosity models - IV: Low-Re $k$ - $\epsilon$ models	6
<ul style="list-style-type: none"> <li>Devised by reference to <u>2-dimensional</u> flows <u>parallel</u> to <u>plane</u> walls</li> <li>Fairly satisfactory in predicting laminarization and diffusion-controlled transition</li> <li>Return results of uncertain accuracy when used in 3D, separated or impinging flows or on curved walls</li> <li>Need for about 40 nodes across sublayer means that computations at this level for 3D flows are only just feasible</li> <li>Further development work still required, guided by DNS data banks (Rodi, Mansour)</li> <li>At present it is often better to use a one-equation EVM across sublayer blended to a two-equation model in fully turbulent region</li> </ul>			

LeRC	8/91	Eddy viscosity models - V: Application	7
<ul style="list-style-type: none"> <li>• <u>Problem</u>: Flow through square duct rotating in orthogonal mode</li> <li>• <u>Relevant to</u>: Internal cooling of turbine blades</li> <li>• Importance of both Coriolis and buoyancy forces</li> <li>• Experimental data of Wagner et al (1989); computations Bo, Iacovides, Launder (1991)</li> <li>• 3D parabolic code with <math>35 \times 67 \times 200</math> grid covering half cross section and 20 hydraulic diameters</li> <li>• Standard <math>k-\epsilon</math> model in core matched to one-equation low-Re treatment across sublayer; <math>\sigma_\theta = 0.9</math> in both regions</li> </ul>			

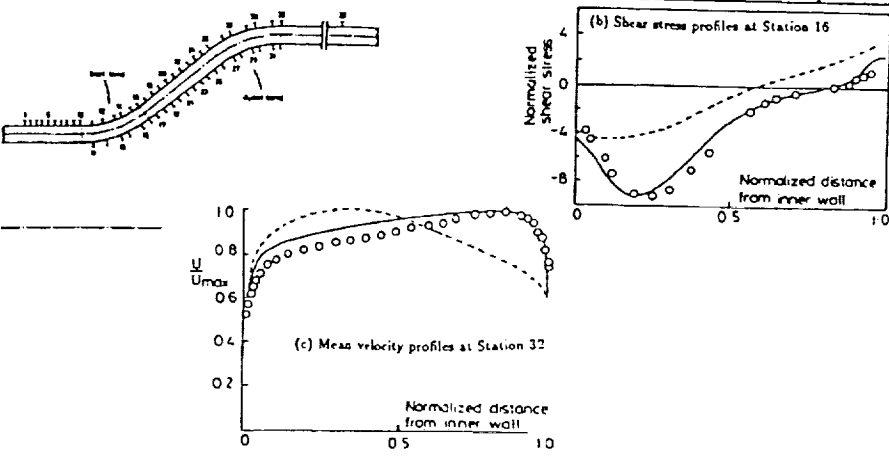


LeRC	8/91	Eddy viscosity models - VI: Results	8
<p>The left graph shows <math>\frac{Nu}{Nu_0}</math> vs <math>x/D</math> for <math>Ro = 0.12</math>. The y-axis ranges from 0.5 to 2.0, and the x-axis from 0 to 35. Two curves are shown: 'pressure' (upper) and 'suction' (lower). Both curves start at 1.0 and rise to approximately 1.8 and 0.8 respectively by <math>x/D = 35</math>.</p> <p>The right graph shows <math>Nu</math> vs <math>x/D</math> for <math>Ro = 0.24</math>. The y-axis ranges from 0.0 to 2.0, and the x-axis from 0 to 35. Two curves are shown: 'pressure' (upper) and 'suction' (lower). The pressure curve starts at 1.0 and rises to approximately 2.0, while the suction curve starts at 1.0 and drops to approximately 0.5.</p>			
<ul style="list-style-type: none"> <li>• Satisfactory results for this very complex flow due to weak influence of force fields in <u>turbulence</u> equations and to predominant importance of sublayer region</li> <li>• Standard two-equation low-Re model gives far worse results than one-equation model</li> </ul>			

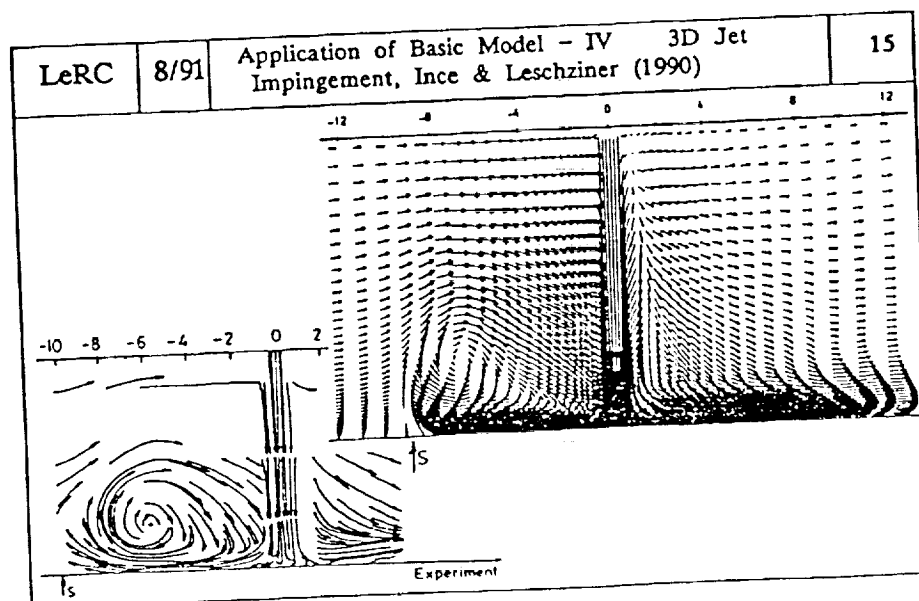
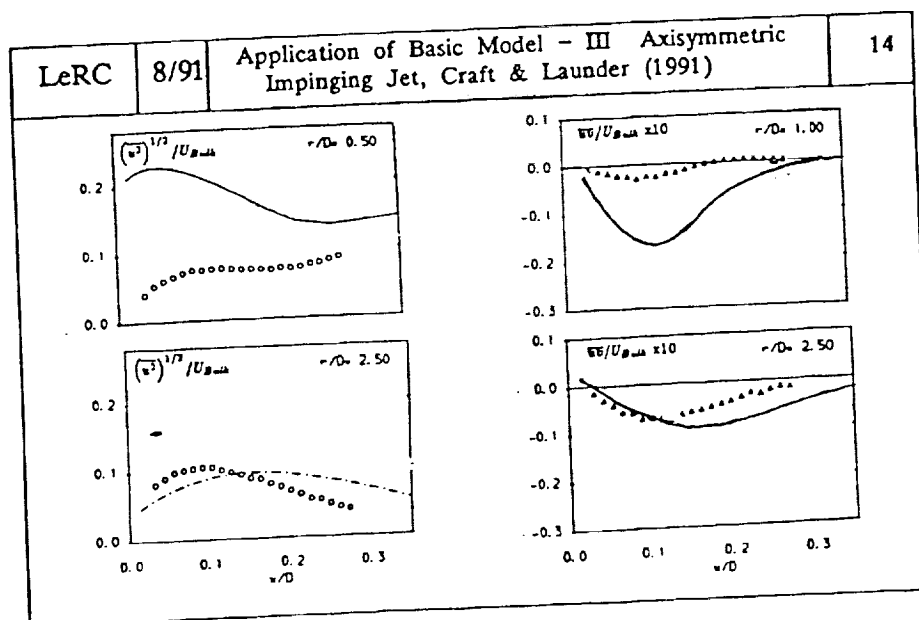
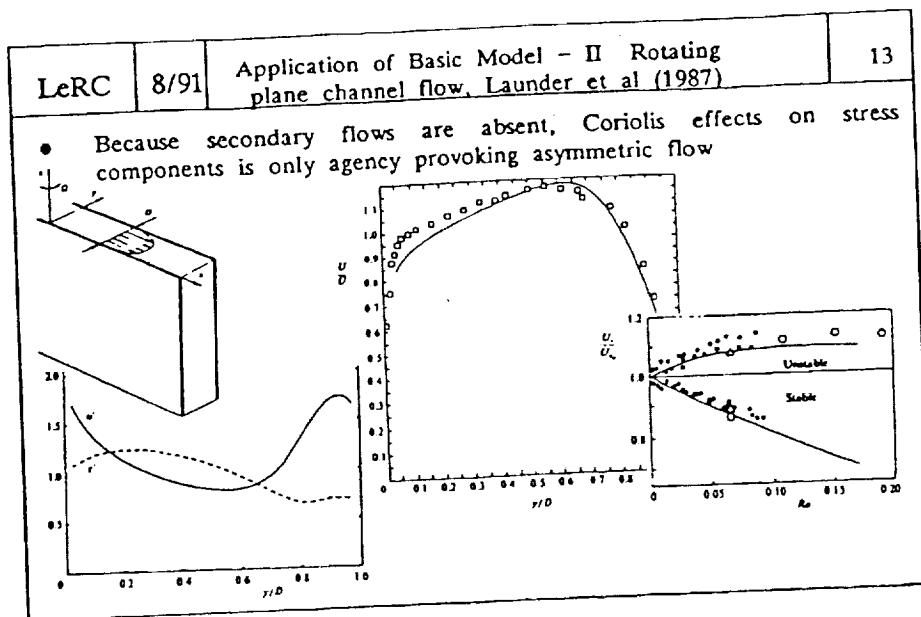
LeRC	8/91	Second-Moment Closure (SMC)	9
<ul style="list-style-type: none"> <li>• Modelling level based on approximated set of rate equations for Reynolds stresses and any other influential second moments (e.g. heat fluxes)</li> <li>• Convective transport of <math>\overline{u_i u_j}</math> together with stress generation due to shear, buoyancy, Coriolis forces etc. <u>all handled exactly</u> at this level</li> <li>• A modelling level intrinsically better able to cope with complex flows than EVM's</li> <li>• Approximations needed for             <ul style="list-style-type: none"> <li>* Pressure-strain correlation, <math>\phi_{ij}</math></li> <li>* Dissipation, <math>\epsilon_{ij}</math> (and hence <math>\epsilon</math>)</li> <li>* Diffusion, <math>d_{ij}</math></li> </ul> </li> </ul>			

LeRC	8/91	Status of SMC - I The Basic Model	10
<ul style="list-style-type: none"> <li>• A simple closure based on: <ul style="list-style-type: none"> <li>• Rotta's linear return-to-isotropy concept for non-linear part of <math>\varphi_{ij}</math>.</li> <li>• "Isotropization-of-production" concept for linear ("rapid") parts of <math>\varphi_{ij}</math>.</li> <li>• Daly-Harlow generalized gradient diffusion hypothesis for <math>d_{ij}</math>.</li> <li>• Local isotropy for <math>\epsilon_{ij}</math>.</li> <li>• <math>\epsilon</math> equation used in <math>k-\epsilon</math> model</li> </ul> </li> </ul> <p>has been extensively applied in 2-D and 3-D subsonic flows</p> <ul style="list-style-type: none"> <li>• Performance nearly always superior to <math>k-\epsilon</math> EVM - often markedly so</li> <li>• Scheme now becoming available in many commercial codes (FLUENT, FLOW3D, PHOENICS, etc.)</li> </ul>			

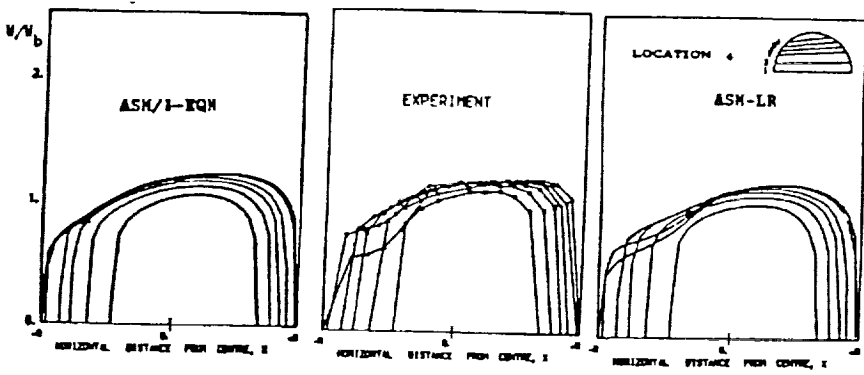
LeRC	8/91	Status of SMC - II The Basic Model (cont'd)	11
<ul style="list-style-type: none"> <li>• Empirical extension of model to low-Re sublayer has been extensively applied by Shima (1989) and colleagues to laminarizing flows</li> <li>• This model apparently does not do well in high M boundary layers, however (Huang, personal communication)</li> <li>• "Wall echo" part of <math>\varphi_{ij}</math> performs quite incorrectly in impinging flows</li> <li>• Performs rather poorly in free flows (round/plane jet "anomaly"; strong/weak shear flow "paradox")</li> </ul>			

LeRC	8/91	Application of Basic Model - I The Faired Annular Diffuser (Jones & Manners, 1989)	12
 <p>The figure contains three parts: a schematic of a faired annular diffuser at the top left, and two plots below it. Plot (b) shows 'Normalized shear stress' vs 'Normalized distance from inner wall' at Station 16, with data points and a dashed curve. Plot (c) shows 'Mean velocity profiles' at Station 32, with data points and a dashed curve. The y-axis for (c) is labeled 'U/Umax' and ranges from 0 to 1.0. The x-axis for both (b) and (c) ranges from 0 to 1.0.</p>			





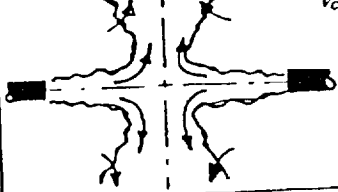
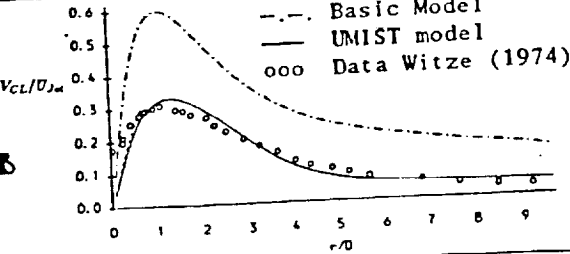
LeRC	8/91	Algebraic Stress Transport Hypothesis (ASTH)	16
<ul style="list-style-type: none"> <li>By suitably approximating transport (convection and diffusion) of <math>u_i u_j</math> in terms of <math>k</math> transport, the closure becomes one where: <ul style="list-style-type: none"> <li>* algebraic equations are solved for <math>\overline{u_i u_j}</math>'s</li> <li>* a differential equation is solved for <math>k</math></li> </ul> </li> <li>This is what we mean by an <u>ASM</u> closure</li> <li>Technique is most powerful where transport terms are small ... i.e. in wall flows</li> <li>Most widely used ASTH's not coordinate frame invariant</li> <li>Properly invariant versions have been proposed (Ahmadi, ICASE) but do not so far seem to have been extensively tested in crucial flows</li> </ul>			

LeRC	8/91	Application of ASM: Circular sectioned S-bend, Abou Haidar et al (1991)	17
			

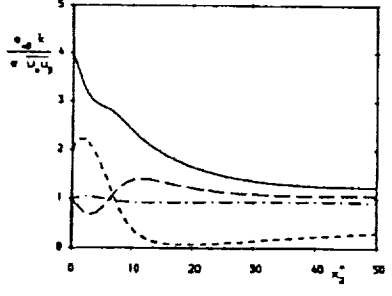
LeRC	8/91	Status of ASM's	18
<ul style="list-style-type: none"> <li>When transport is small, useful reduction in computational effort achieved while retaining virtually same results as DSM</li> <li>Nature of ASTH is then unimportant</li> <li>System of equations is stiffer than when DSM is used: convergence is often more difficult</li> <li>ASM's are on their way out; not worth developing new software for this level of model</li> </ul>			

LeRC	8/91	"New" generation SMC's	19
<ul style="list-style-type: none"> <li>Approximation of <math>\varphi_{ij}</math> (and other processes) designed to <u>comply with extreme states of turbulence</u>: e.g. isotropic turbulence, 2-component turbulence, ....</li> <li>Proper frame indifference</li> <li>Extensive use made of stress anisotropy invariants: <math display="block">A_2 = a_{ij}a_{ij} \quad ; \quad A_3 = a_{ik}a_{kj}a_{ji}</math> <math display="block">a_{ij} = (\overline{u_i u_j} - \frac{1}{3}\delta_{ij} \overline{u_k u_k})/k</math> </li> <li>Extensive use made of results of direct numerical simulations</li> <li>Algebraically far more complex than basic model but greater numerical stability can lead to a <u>reduction</u> in overall computer time</li> </ul>			

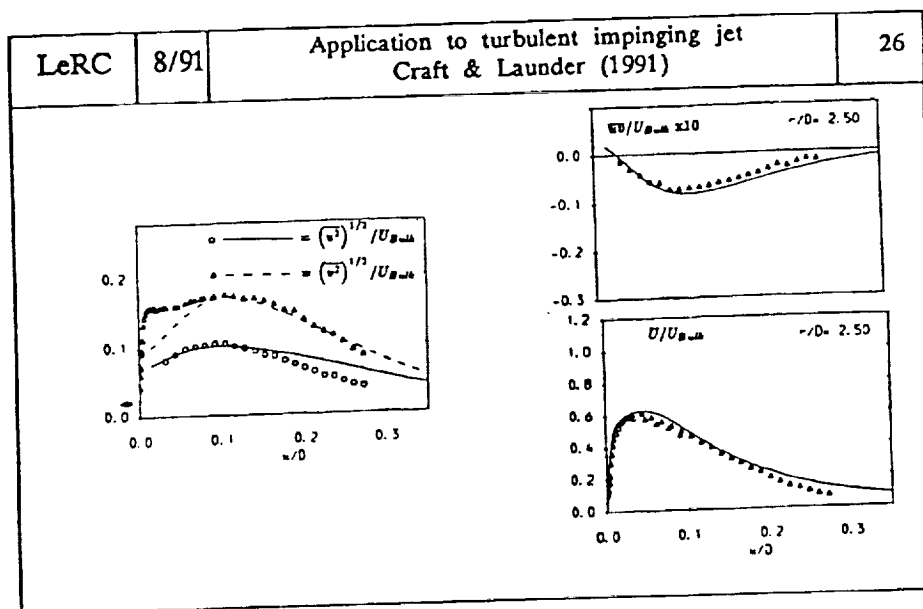
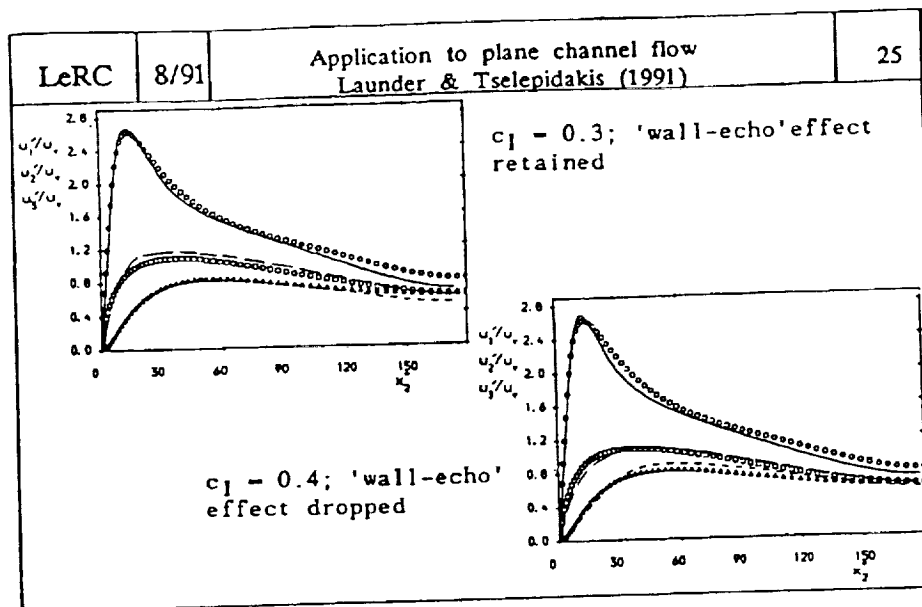
LeRC	8/91	Status of new generation models: I Free flows	20
<ul style="list-style-type: none"> <li>Extensive testing in homogeneous flows (ICASE, Stanford, Cornell, LeRC, UMIST)</li> <li>Moderate testing in 2D free shear flows</li> <li>UMIST model tested in 2D recirculating and colliding flows</li> <li>Far greater width of applicability demonstrated than with basic model</li> <li>Only known test for swirling recirculating flows not fully successful</li> </ul>			

LeRC	8/91	Application of UMIST model to free shear flows	21																								
<ul style="list-style-type: none"> <li>Dimensionless rates of spread of equilibrium free shear flows <table border="1"> <thead> <tr> <th>Flow</th><th>Expt</th><th>Basic Model</th><th>New Model</th></tr> </thead> <tbody> <tr> <td>Plane jet</td><td>0.105-0.110</td><td>0.100</td><td>0.110</td></tr> <tr> <td>Round jet</td><td>0.095</td><td>0.105</td><td>0.098</td></tr> <tr> <td>Plane wake</td><td>0.098</td><td>0.078</td><td>0.098</td></tr> <tr> <td>Plane plume</td><td>0.12</td><td>0.078</td><td>0.118</td></tr> <tr> <td>Mixing layer</td><td>0.16-0.20</td><td>0.16</td><td>0.176</td></tr> </tbody> </table> </li> <li>Colliding Round Jets  </li> </ul>				Flow	Expt	Basic Model	New Model	Plane jet	0.105-0.110	0.100	0.110	Round jet	0.095	0.105	0.098	Plane wake	0.098	0.078	0.098	Plane plume	0.12	0.078	0.118	Mixing layer	0.16-0.20	0.16	0.176
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LeRC	8/91	Key elements in UMIST model	22
<ul style="list-style-type: none"> <li>• More intricate interconnection among stress and strain components in mean-strain (rapid) part of <math>\varphi_{ij}</math></li> <li>• Weakening of return-to-isotropy coefficient as stress field becomes more isotropic and also as it approaches 2-component limit</li> <li>• Diminished effect of mean strain on evolution of <math>\epsilon</math></li> </ul> $\frac{D\epsilon}{Dt} = d\epsilon + \left\{ \frac{c_{\epsilon 1}}{2} \left[ \frac{P_k}{\epsilon} + \frac{\nu_t}{\epsilon} \left( \frac{\partial U_i}{\partial x_j} \right)^2 \right] - c_{\epsilon 2} \right\} \frac{\epsilon^2}{k}$ <p>where <math>c_{\epsilon 1} = 0.7</math> (versus 1.44)</p> <p><math>c_{\epsilon 2} = 1.92 / (1 + 1.65 A_2^{1/2} A)</math> (versus 1.92)</p> <p><math>A = 1 - \frac{9}{8} (A_2 - A_3)</math></p>			

LeRC	8/91	Issues in modelling near-wall turbulence	23
<ul style="list-style-type: none"> <li>• DNS data bank suggest <math>\epsilon_{ij}</math> far less isotropic than usually presumed; behaviour of <math>\epsilon_{12}</math> especially strange</li> <li>• Peak level of <math>\epsilon</math> at wall</li> <li>• Bradshaw et al (1987) have shown <u>inhomogeneous</u> effects on <math>\varphi_{ij}</math>; very important in buffer layer</li> <li>• "Wall-reflection" models of <math>\varphi_{ij}</math> need to consider different constraints imposed by parallel and impinging wall flows and free-surface flows</li> </ul> 			

LeRC	8/91	Some features of UMIST's approach to near-wall turbulence	24
<ul style="list-style-type: none"> <li>• <math>\epsilon_{ij}</math> modelled to satisfy exact component ratios at wall</li> <li>• Inhomogeneous effects on mean-strain part of <math>\varphi_{ij}</math> accommodated through use of <u>effective</u> velocity gradient</li> </ul> $\frac{\partial U_\ell}{\partial x_m} + c_1 \ell_n \frac{\partial \ell_n}{\partial x_k} \frac{\partial^2 U_\ell}{\partial x_k \partial x_m}, \text{ Launder \& Tselepidakis (1991)}$ <ul style="list-style-type: none"> <li>• New wall-reflection model designed to handle impinging and parallel shear flows (but, alas, <u>not</u> free surface effects), Launder and Craft (1991)</li> <li>• Best way of placing <math>\epsilon</math> maximum at wall is to solve transport equation for</li> </ul> $\bar{\epsilon} = \epsilon - 2\nu \left( \frac{\partial k^{1/2}}{\partial x_j} \right)^2 \text{ Kawamura (1991)}$			



LeRC	8/91	Present status of new generation DSM's	27
<ul style="list-style-type: none"> <li>• Substantial advances demonstrated over earlier models for free shear flows</li> <li>• Significant unresolved (or incompletely resolved) problems remain in modelling near-wall turbulence that limit range of applicability of available models</li> <li>• Much remains to be done in high-speed flows: present suggestions for modelling extra terms and/or physical processes seem generally to be "quick fixes"</li> <li>• Many improvements foreseen over next 3-5 years</li> <li>• New-generation closures now being incorporated into general purpose 3D solvers</li> </ul>			

LeRC	8/91	Prospects	28
<ul style="list-style-type: none"> <li>• EVM's, ASM's and DSM's will remain in use though with steady decline in importance of EVM's and ASM's in favour of DSM's</li> <li>• Improved versions of low-Re two-equation EVM's should lead to more reliable predictions of separated flows than at present</li> <li>• New-generation DSM closures will soon (2-3 years) replace basic model even in commercial codes</li> <li>• Further refinement of sub-models in second moment closures can be expected throughout this decade</li> <li>• Increasing attention to interfacing SMC with higher order approaches such as LES</li> <li>• Increasing use of two-time-scale schemes providing distinct time scales for large and (fairly) small eddies</li> </ul>			

LeRC	8/91	A Reminder	29
<ul style="list-style-type: none"> <li>• Extensive collaborative testing/assessment of turbulence models currently underway - coordinated/organized by Professor P. Bradshaw (Stanford) (with a little help from JLL and BEL)</li> <li>• Outcome of that exercise will offer a more complete and objective view of state of turbulence modelling than is currently available</li> </ul>			


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p. 20

*Center for Modeling of Turbulence and Transition  
Workshop on Engineering Turbulence Modeling - 1991*

**Comment on:  
The current status of turbulence modeling in CFD and its future prospects**

**by**

**D. M. Bushnell**

**NASA Langley Research Center**

*(D.M. Bushnell Comments on Launder Paper)*

## FUNDAMENTAL PROBLEM WITH CONVENTIONAL TURBULENCE MODELING

- Phase/Spectral information lost ("averaged out")  
—implicit assumption of Similar Spectra

However (Kline, 1981)

"Contrary to earlier ideas, turbulence is not a single or even a simple set of states; it is a very complex and variable set of states that react in sometimes unanticipated ways to a great variety of circumstances"

"The Quasi-Coherent parts of the turbulent flows, what we call the large or medium eddy structures, are not the same in different kinds of turbulent flows"

- zonal modeling is an attempt to include this dynamic richness



# FUNDAMENTAL PROBLEMS WITH THE "STATE OF THE ART" TURBULENCE MODELING APPROACH OF "EDDY VISCOSITY"

- Mixing length, Cebeci-Smith, Baldwin-Lomax, K-E etc.  
—relate turbulence stress directly to mean field, however, turbulence field cannot track rapid, inviscidly-induced, mean flow alterations  
—lack innate capability to generate turbulence-induced secondary flows, handle curvature/rotation effects
- Various "Fixes" have been proffered including Johnson-King and ASM.

## THE MAJOR PROBLEMS WITH RSE CLOSURES

- Numerical/computer efficiency
  - Numerics/computers barely capable of good resolution for 2-D separated flows, a major reason why RSE closures are relatively unevaluated/unoptimized, esp. at high speeds ("stiffness" problem)
- Length Scale Equations
  - usual dissipation rate eq. is mainly Adhoc, anisotropic and/or split spectral equations may be required
- Wall Region Treatments
  - wall functions not suitable for separated/3-D flows, must compute to wall

- Pressure-strain and pressure-dilatation modeling
  - High-Speed flows may be different due to hyperbolic nature of pressure field

## RSE CHALLENGES PROBABLY ADDRESSABLE WITHIN THE CONTEXT OF REYNOLDS AVERAGING

- Compressibility
  - simplex mean density/temperature variation
  - baroclinic torque/mean dilatation effects
  - alteration of eddy-eddy interaction at high  $M^1$ ,  $M_c$
  - energy radiation via acoustic waves ( $M \gg 1$ ?)
  - bulk viscosity ( $M \gg 1$ ?)
- Curvature
  - longitudinal and in-plane streamline curvature
  - induced by pressure gradients and organized vorticity as well as geometry
  - stationary curvature-induced Gortler vortices probably computable with good numerics
  - included to first order in RSE approaches

- Low Reynolds Number
  - transitional flow artifacts require  $O(100\delta)$  for relaxation, both free and bound flows, especially important at high Mach  $N^\circ$
  - wall regions, especially for unsteady, separated 3-D, flows and complex wall boundary conditions

## RSE CHALLENGES WHICH MAY REQUIRE THEORETICAL/NUMERICAL SPECTRAL ADJUNCTS FOR SATISFACTORY SOLUTIONS

- Shock Interaction
  - Both direct and indirect amplification of incident disturbance fields
  - Direct amplification accompanied by intermodal energy transfer
  - Shock motions/oscillations convert mean flow energy into fluctuation field
  - Often accompanied by pressure gradient/bulk dilatation and streamline curvature which further amplify turbulence levels
  - Produces spectral alteration, with production of large scale structures

## SHOCK OSCILLATIONS

- IDEAL GAS SHOCK WAVES ARE, BY THEMSELVES, STABLE (BUT SHOCK PERTURBATIONS CAN DECAY SLOWLY AND SHOCK WAVES IN REACTING FLOWS CAN BE UNSTABLE)
- SHOCK OSCILLATIONS PRODUCED BY
  - INNATELY UNSTABLE FLOW FIELDS (E.G., SEPARATION (SHEAR LAYER, BUBBLE MODES), TRANSONIC EDGE TONES/HELMHOLTZ RESONATOR, HARTMANN TUBE)
  - BIFURCATION BETWEEN STABLE SHOCK PATTERNS (E.G., "BORDERLINE OPERATION" BETWEEN EDNEY PATTERNS, REGULAR VS. MACH REFLECTION ETC.)
  - INTERACTION WITH/REFLECTION FROM/PRODUCTION BY TURBULENT FIELDS
  - POSSIBILITY OF SHOCK INSTABILITY IN REACTING FLOWS

—Overall amplifications of factors of 2 to 3 not uncommon



## RSE CHALLENGES WHICH MAY REQUIRE THEORETICAL NUMERICAL SPECTRAL ADJUNCTS FOR SATISFACTORY SOLUTIONS

- Intermittency
  - Transitional Flow Regions
  - Shear Flow “free boundary” regions
  - Chemically reacting flows
  - Function of pressure gradient, Mach number, roughness,
- Discrete Spectral Entities — initial/boundary conditions and embedded instabilities
  - example of discrete B.C.’s — stator/rotor wake interaction in rotating machinery
  - examples of imbedded instabilities (in turbulent flows)
    - a. Gortler (dynamic)
    - b. Karman shedding
    - c. Intersection region horseshoe vortex interactions.

## TOOLS AVAILABLE TO AID RSE MODELING

- Experimental
  - Quantitative Flow Visualization, direct (3 space and time!) measurements of terms/influences (e.g. Holographic velocimeter)
  - Mean and Second order (average and spectra) data to check validity of predictions, develop post-dictions, determine presence of “new physics” (from discrepancies, discrete spectral peaks), provide initial boundary conditions.
  - Structure/visualization/physics experiments.
- Numerical Simulations
  - Only source of detailed/complete data
  - Historically limited to simple geometry/low  $ReN^0$ , but both boundaries are expanding

- Theory
  - Rapid distortion theory
  - 3<sup>rd</sup> order and 2-pt. eqs
- Restrictions/Requirements
  - Realizability
  - Symmetry
  - Frame Indifference
  - Behavior at end conditions such as infinite  $ReN^0$ , 2-D Turb., homogenous shear
  - 2<sup>nd</sup> law of thermodynamics
- Dimensional Analysis

## CONCLUDING REMARKS

- Currently there are only 2 ways to “make numbers” in Turbulent Engineering Flows
  - Numerical Simulations
  - Modeling

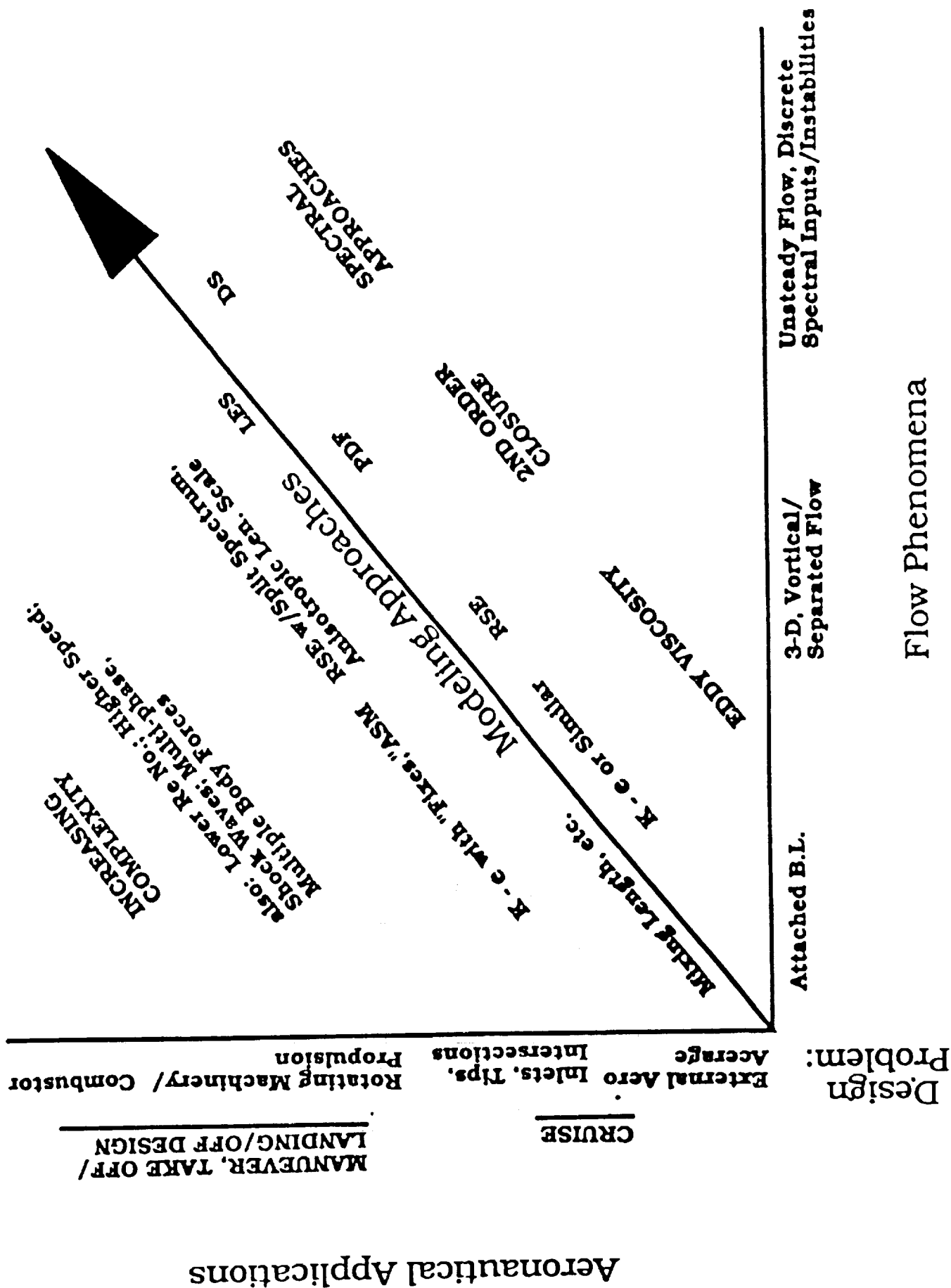
and for the usual complex flows simulation is not feasible; therefore we must model

- Simplex (Eddy viscosity) approaches such as mixing length/K-E etc. Provide reasonable estimates for simplex (quasi-parallel, near-equilibrium etc.) flows. Complex flows require second order (RSE) closures.
- 18 years of experience with K-E has resulted in numerous “fixes” which (tenuously) can extend its range of applicability

- RSE closures are in their infancy, due (historically) primarily to numerical difficulties and computer limitations. WE DO NOT YET KNOW HOW GOOD (OR BAD) THE RSE CLOSURES ARE OR CAN BE.
- Launder's group(s) have pioneered the low speed engineering applications of RSE closure, with Lumley's group providing many of the advanced concepts.
- RSE research frontiers include
  - Numerical efficiency improvements for the stiff eq sets
  - Length scale equation(s) (Anisotropic, Split Spectrum)
  - Pressure strain modeling
  - Wall region treatments
  - Compressibility

- Several applications may require theoretical/numerical spectral adjuncts
  - Shock interaction/oscillation
  - Discrete dynamical flow elements as initial/boundary conditions
  - Discrete dynamic instabilities of turbulent flows
  - Combustion/intermittency

# SIMPLEX VIEW OF STATUS - TURBULENCE MODELING



## COMPLICATING FEATURES — TURBULENT FLOWS

- 3-D:
  - Boundary Layers
  - Separated Flows
  - Vortical Flows
  - Free Mixing Layers
- Possibly Influenced By:
  - Complex 3-D Geometry (Corners/Intersections, roughness, etc)
  - Pressure Gradients, Curvature(s)
  - Compressibility
  - Low Reynolds Number (transition, walls, edges)
  - Discrete Organized Vorticity (instability, B. C.)



- Shock Wave Interaction(s)
- Combustion/Chemistry
- Multiple Phase/Phase Change
- Buoyancy/Stratified Media
- Plasma/Relativistic/Radiation Effects
- Non-Newtonian Fluids



## DISCUSSION

J. Bardina (to B.E. Launder)

I just have a question for Prof. Launder. You dismissed the algebraic Reynolds stress models real fast. To me a natural way to go from two equation models to higher models is to go through algebraic relations first. Do you think that the poor predictions are only due to the poor models and not related to the numerical instability issues.

B.E. Launder (reply)

It's a matter of taste and depends on the problem you are looking at. It's true that if one is thinking of it in that relation, the idea of using just the same two equation  $k - \epsilon$  or  $k - \omega$  or more complicated stress-strain relation got some appeal. But nobody would suggest ASM is an improvement of physics over  $k - \epsilon$ . The problem you encounter in stiffer equations make it an unattractive level to fall to. It's beginning to pass. It could be that for a particular discrete set of problems it would make a lot of sense. On the overall if you can not use an eddy viscosity model, you should just bite the bullet and use the Reynolds stress model.

A.K. Singhal

I would like to make a comment about the use of nomenclature. You can notice that even the names used for the Reynolds stress models by the invited speakers were different. It would be very helpful for industry if modelers could use the same nomenclature.

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# THE PRESENT STATE AND THE FUTURE DIRECTION OF EDDY VISCOSITY MODELS

by  
David C. Wilcox

Workshop on Engineering Turbulence Modeling  
August 21, 1991

DCW Industries, Inc.

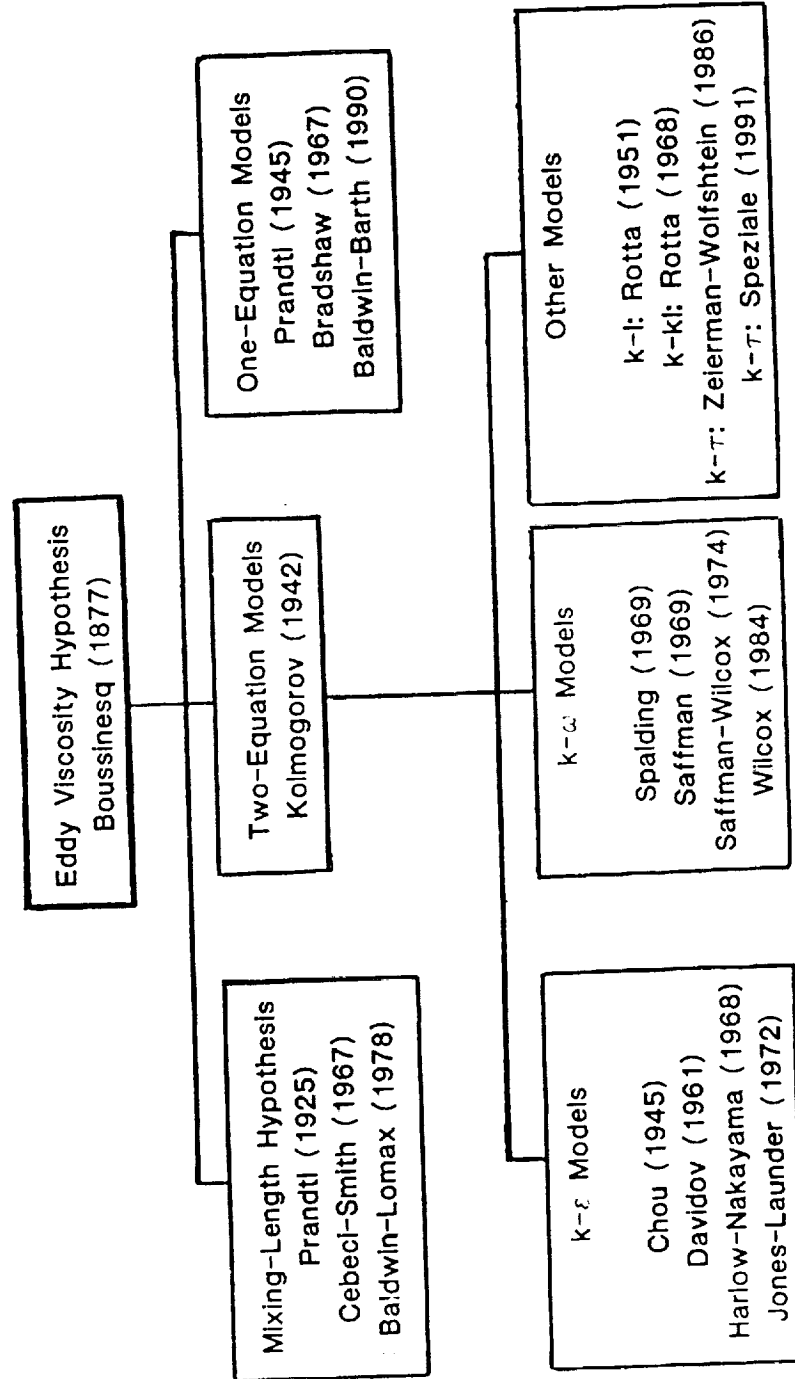
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# TALK OUTLINE

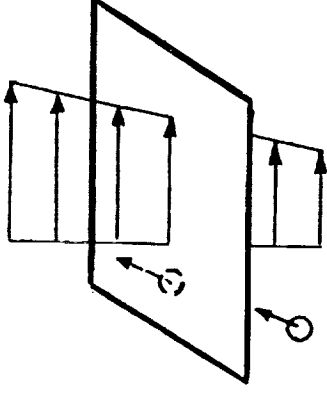
- Evolution of eddy-viscosity models
- The eddy-viscosity dilemma
- Two-equation models
- Free shear flows
- Attached wall-bounded flows
- Separated flows
- The past (where we've been)
- The future (where we should be going)

# EVOLUTION OF EDDY VISCOSITY MODELS



# THE EDDY VISCOSITY DILEMMA

- The Boussinesq approximation is based upon making an analogy with molecular transport of momentum



- Hence, we say

$$-\overline{u'\nabla^2} = \nu_{\text{mix}} \left[ \mathcal{L}_{\text{mix}} \frac{\partial u}{\partial y} + \frac{1}{2} \mathcal{L}_{\text{mix}}^2 \frac{\partial^2 u}{\partial y^2} + \dots \right] \approx \nu_T \frac{\partial u}{\partial y}$$

$$\nu_T = \nu_{\text{mix}} \mathcal{L}_{\text{mix}}$$

For this to work, we require

$$\text{Kn} = \mathcal{L}_{\text{mix}} / L \quad \text{where} \quad L \equiv \frac{|\partial u / \partial y|}{|\partial^2 u / \partial y^2|} \quad \text{and} \quad \mathcal{L}_{\text{mix}} / \nu_{\text{mix}} < < |\partial u / \partial y|^{-1}$$

- Neither condition is satisfied in a typical turbulent shear flow
- "SOLVE NOW, WORRY LATER" (J. Cole)



# TWO EQUATION MODELS

- Models that fail to provide sufficient information to determine both a turbulence velocity scale and a turbulence length scale are **incomplete**
- A two-equation model is the simplest **complete** model of turbulence
- Virtually all two-equation models begin with the turbulence kinetic energy equation – the differences appear in the choice of a second dependent variable...

$\varepsilon$  – Dissipation Rate

$\omega$  – Specific Dissipation Rate

$\tau$  – Dissipation Time Scale

$\lambda$  – Dissipation Length Scale

# EQUATIONS OF MOTION

- Two-equation models use the Boussinesq approximation so that

$$\tau_{ij} = -\overline{u_i u_j} = 2 \nu_T S_{ij} - \frac{1}{3} k \delta_{ij}$$

- The turbulence kinetic energy equation is

$$\frac{dk}{dt} = \tau_{ij} \frac{\partial u_i}{\partial x_j} - \epsilon + \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T / \sigma_k \right) \frac{\partial k}{\partial x_j} \right]$$

- For the k-ε model...

$$\frac{d\epsilon}{dt} = C_{\epsilon 1} \frac{\epsilon}{k} \tau_{ij} \frac{\partial u_i}{\partial x_j} - C_{\epsilon 2} \frac{\epsilon^2}{k} + \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T / \sigma_\epsilon \right) \frac{\partial \epsilon}{\partial x_j} \right]$$

$$\nu_T = C_\mu f_\mu k^2 / \epsilon$$

- For the k-ω model...

$$\frac{d\omega}{dt} = \alpha \frac{\omega}{k} \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T / \sigma_\omega \right) \frac{\partial \omega}{\partial x_j} \right]$$

$$\nu_T = k / \omega \quad \text{and} \quad \epsilon = C_\mu \omega k$$

# RELATIONSHIP BETWEEN $\varepsilon$ AND $\omega$

- If we use  $\varepsilon = C_\mu \omega k$  as the definition of  $\omega$ , transforming the  $\varepsilon$  equation (with  $\sigma_k = \sigma_w$  for simplicity) yields

$$\frac{d\omega}{dt} = \alpha \frac{\omega}{k} \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[ (\nu + \nu_T / \sigma_w) \frac{\partial \omega}{\partial x_j} \right] + 2 \underbrace{\frac{(\nu + \nu_T / \sigma_k)}{k} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}}_{\text{Cross Diffusion}}$$

- The cross-diffusion term does the following things:
  1. Yields an excellent prediction for the plane jet
  2. Disrupts defect-layer response to pressure gradient
  3. Makes k- $\varepsilon$  model very stiff in the sublayer and dictates the need for viscous damping terms

# Free Shear Flows

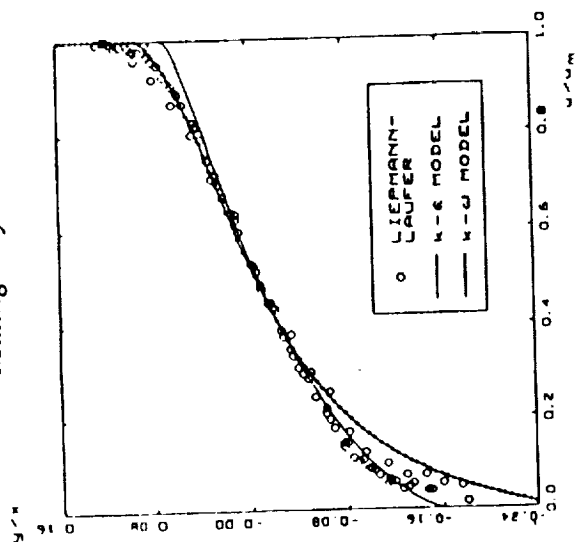
- $k-\omega$  models are sensitive to the freestream value of  $\omega$  when applied to free shear flows

Flow	Wilcox	$k-\varepsilon$	Speziale	Measured
Mixing Layer	.100-.141	.099	.115	.116
Plane Jet	.090-.133	.108	.130	.100-.110
Round Jet	.115-.175	.120	.155	.086
Radial Jet	.088-.186	.095	.112	.096-.110
Plane Wake	.221-.498	.257	.285	.365

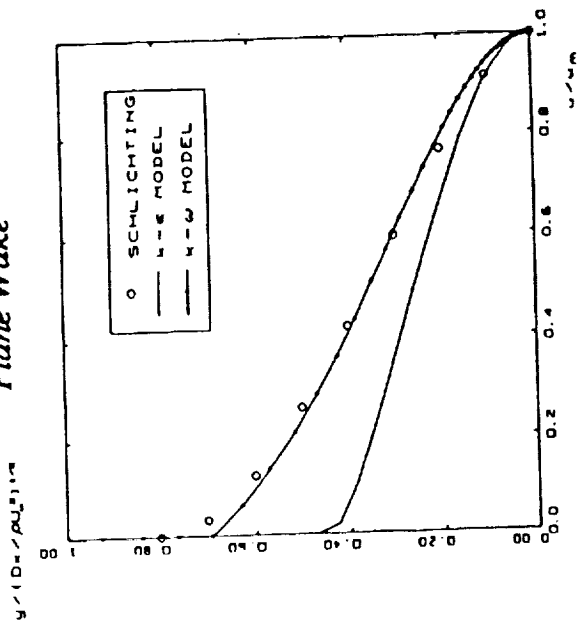
- Like the  $k-\varepsilon$  model, the  $k-\omega$  model suffers from the so-called round-jet/plane-jet anomaly

# INCOMPRESSIBLE FREE SHEAR FLOWS

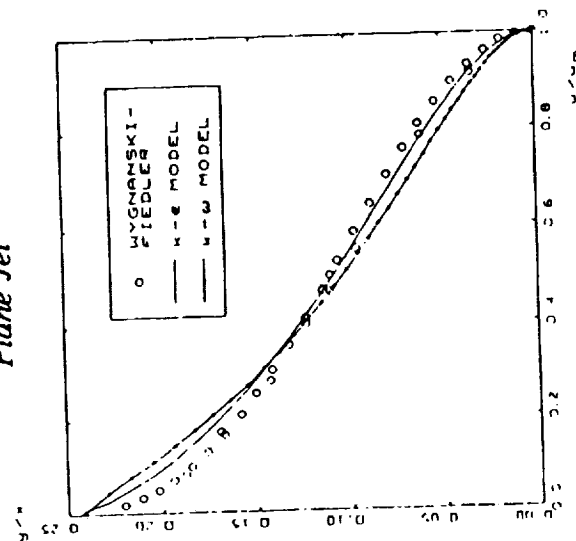
Mixing Layer



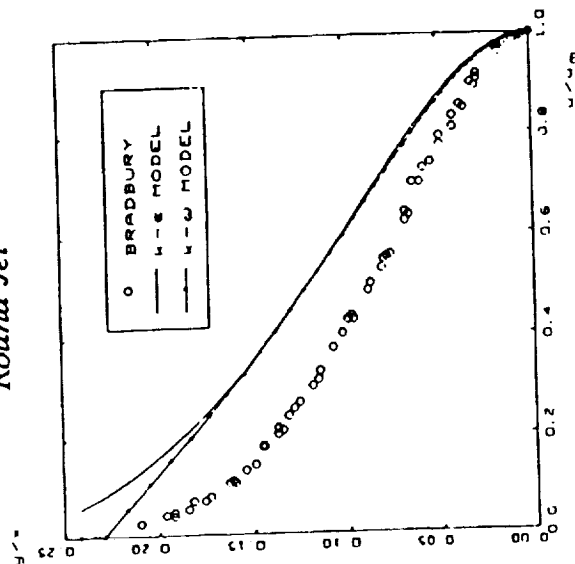
Plane Wake



Plane Jet

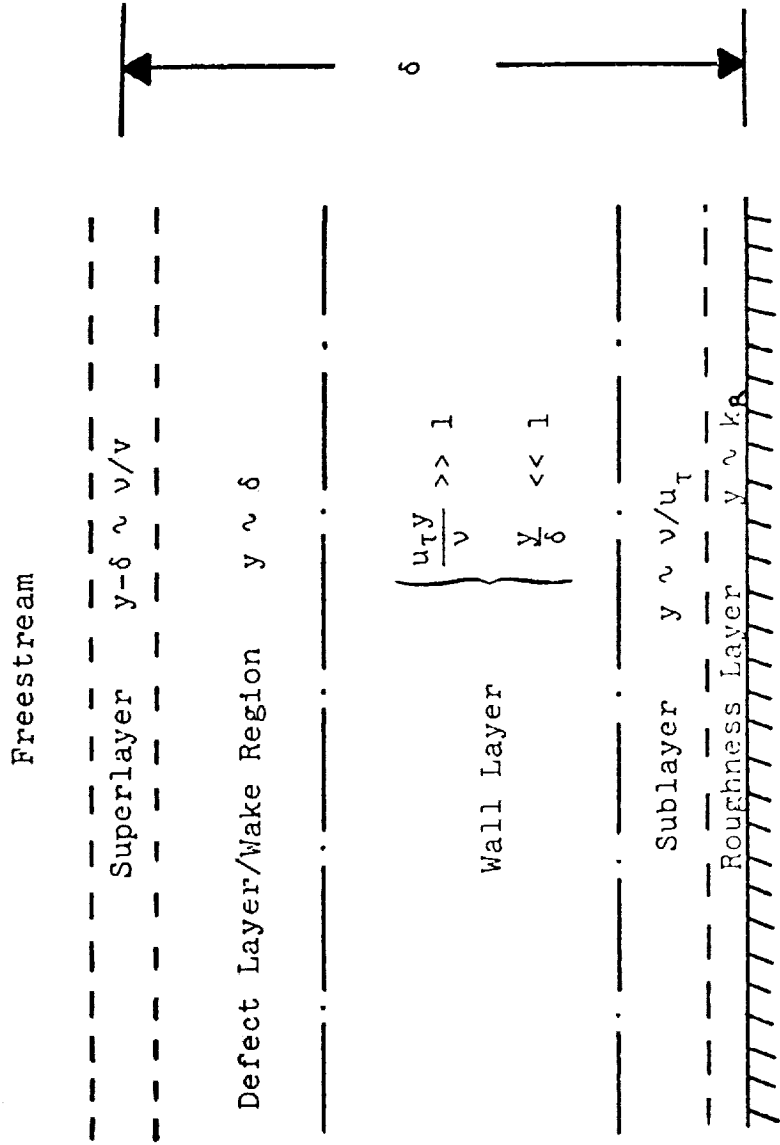


Round Jet



# MODEL-PREDICTED BOUNDARY LAYER STRUCTURE

- Turbulence transport equations rarely have closed-form solutions
- Perturbation methods are invaluable for analyzing turbulence models
- Five distinct layers have been identified for the  $k-\omega$  model...



# DEFECT-LAYER ANALYSIS

- Using perturbation methods, can simplify equations to similarity form

$$\left\{ \begin{array}{l} (U_e - u)/u_\tau = f(\eta) \\ \eta = y/\Delta \\ \Delta = U_e \delta^*/u_\tau \end{array} \right.$$

- Pressure gradient comes in through the equilibrium parameter

$$\beta_T = \frac{\delta^*}{\tau_w} \frac{dp}{dx} \quad (\text{Bradshaw } A = -0.255 \Rightarrow \beta_T \approx 5)$$

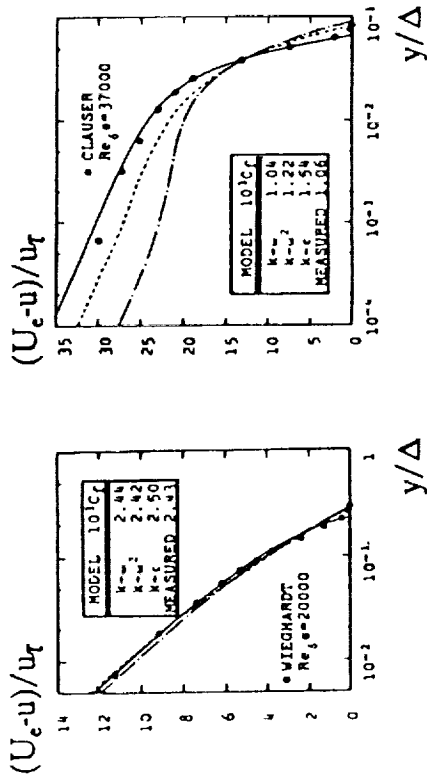
- The analysis shows that

$$(U_e - u)/u_\tau \sim -\kappa^{-1} \log(y/\Delta) + A - C \beta_T (y/\Delta) \log(y/\Delta) \quad \text{as } y/\Delta \rightarrow 0$$

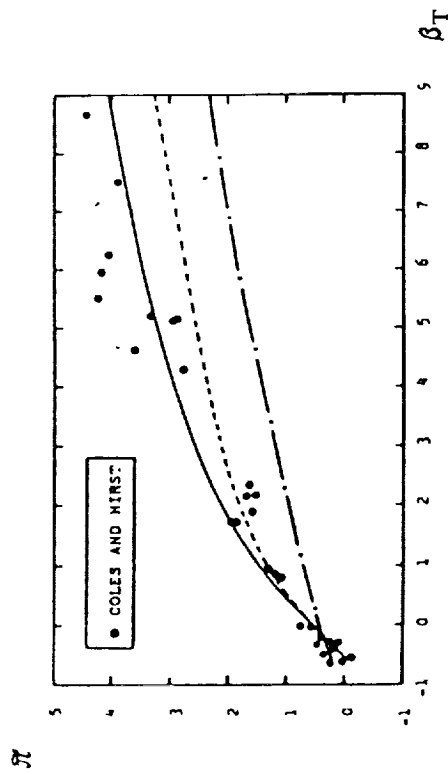
Model	Type	A	C
Wilcox	k- $\omega$	13.1	1.18
Wilcox-Rubesin	k- $\omega^2$	9.8	2.60
Kolmogorov	k- $\omega$	10.0	1.14
Lauder	k- $\epsilon$	5.2	6.09
Speziale	k- $\omega$	5.1	4.96

# Effects of Pressure Gradient

- Perturbation analysis results show that Kolmogorov did remarkably well without the aid of a computer!



(a) Velocity profiles,  $\beta_T = 0$  (b) Velocity profiles,  $\beta_T = 8.7$



(c) Variation of wake strength with pressure gradient

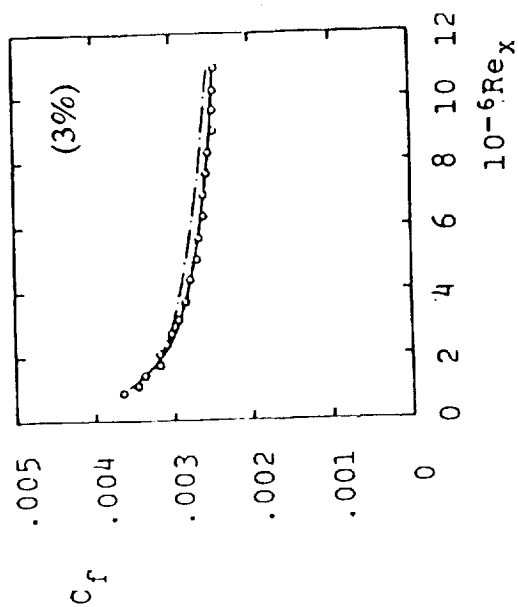
- The unmodified "cross-diffusion" term distorts predicted defect-layer structure

Model	$\tilde{\kappa}$
Wilcox	4.05
Wilcox-Rubesin	3.26
Kolmogorov	3.21
Launder	2.27
Speziale, et al	2.23

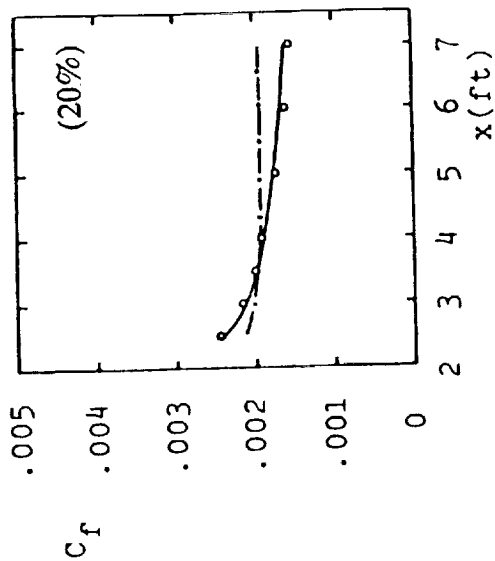
Figure 1. Comparison of computed and measured defect-layer properties: — Wilcox  $k-\omega$  model; - - - Wilcox-Rubesin  $k-\omega^2$  model; - · - Launder  $k-\epsilon$  model.



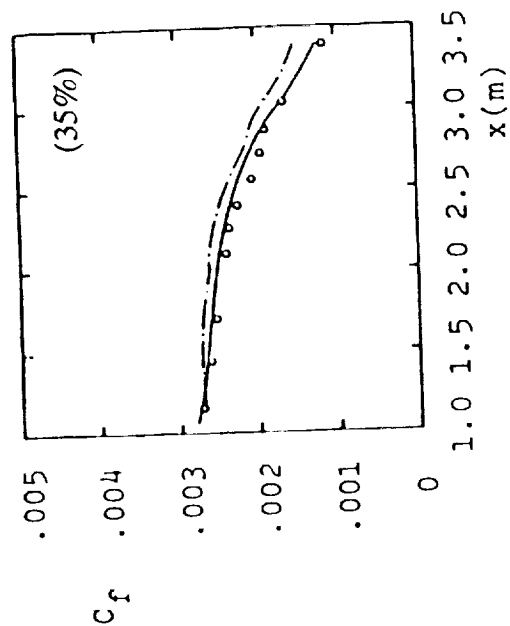
# Flat Plate



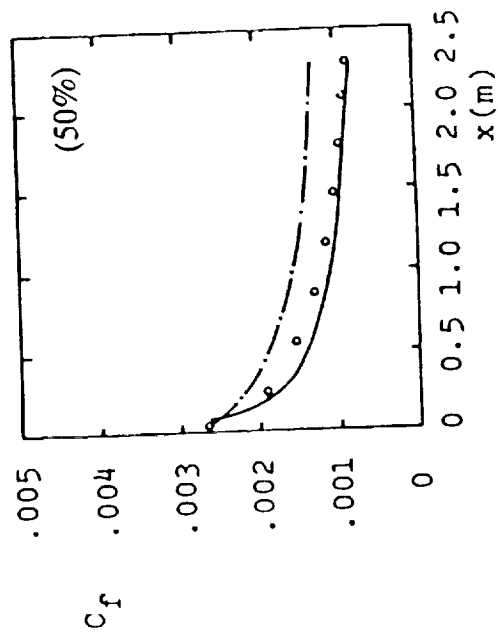
# Bradshaw Flow C



# Samuel-Joubert



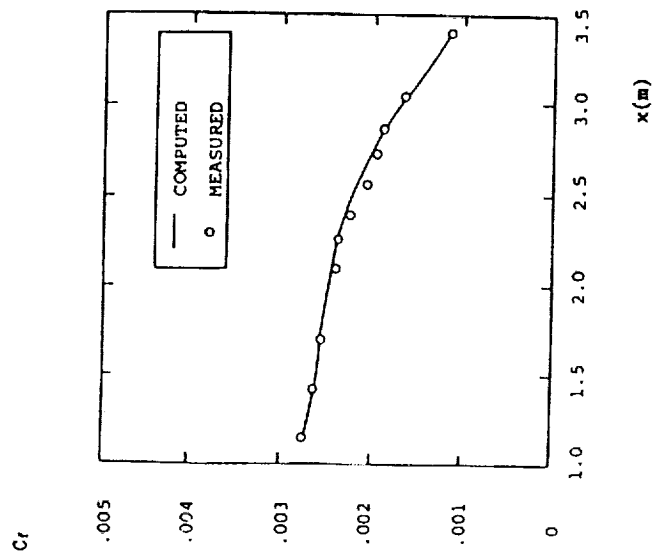
# Mass Injection



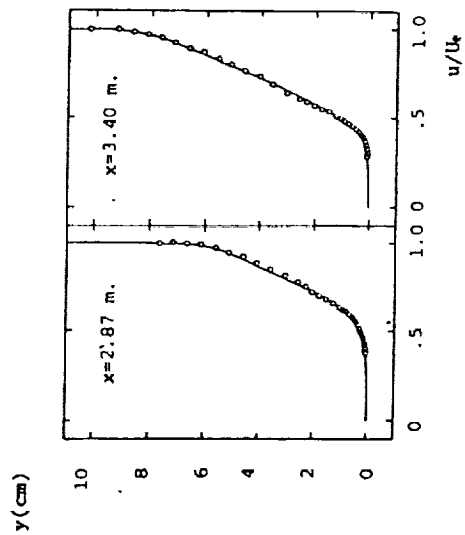
—  $k-\omega$  Model  
 - - -  $k-\epsilon$  Model

# k- $\omega$ LOVES SAMUEL-JOUBERT!

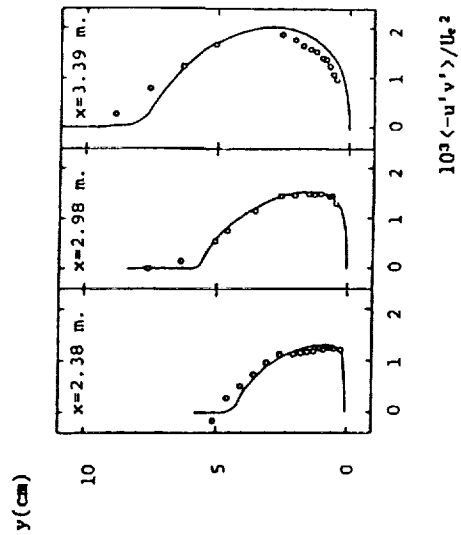
Skin Friction



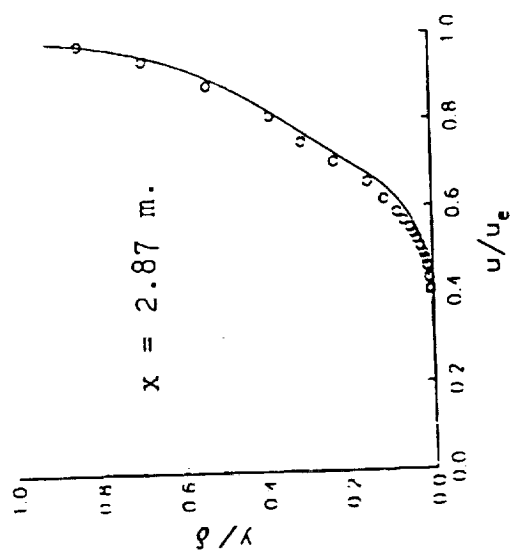
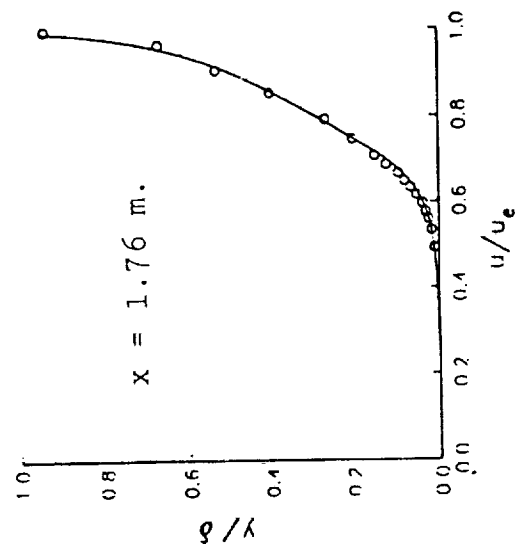
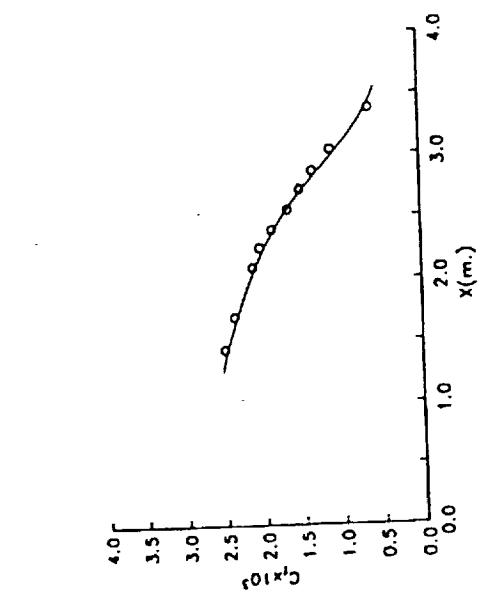
Velocity Profiles



Reynolds Stress



# AND SO DOES $k-\tau$



# Viscous Sublayer Structure

- If we simply add molecular viscosity to the  $k$ ,  $\omega$  and the mean-flow equations, we find

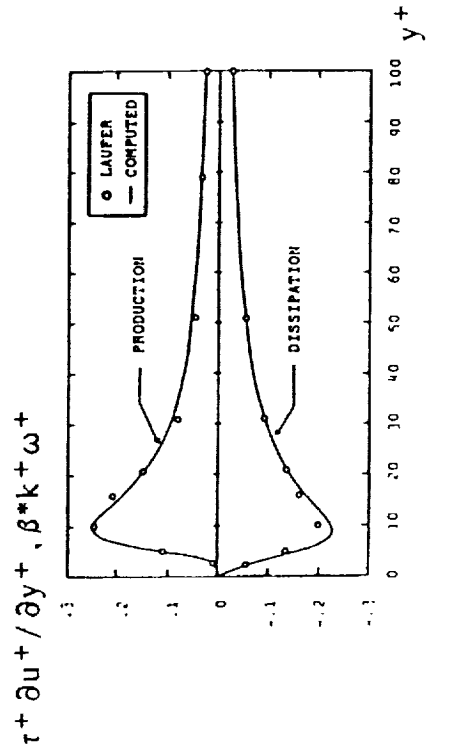
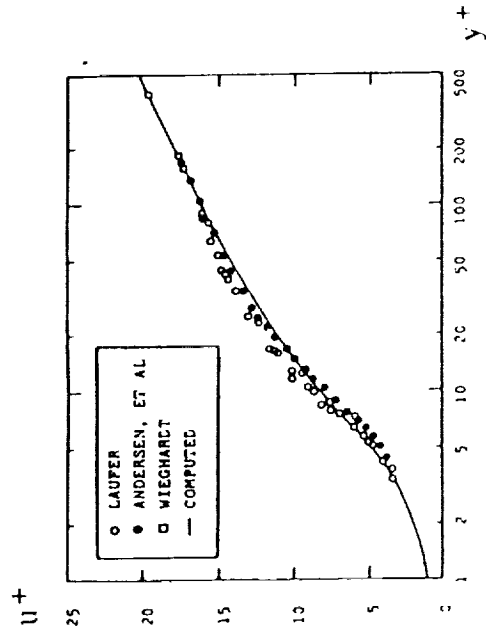
$$k \rightarrow y^n \text{ as } y \rightarrow 0 \quad \text{and} \quad u' = \kappa^{-1} \log y' \rightarrow B \text{ for } y \gg 1$$

- Most  $k$ - $\omega$  models predict  $3 < n < 4$  and  $3 < B < 7$

Kolmogorov:  $n = 3.62$  and  $B = 3.1$

Speziale and  $k$ - $\epsilon$ :  $n = 1.39$  and  $B = -2.2$

- The Wilcox model predicts  $n = 3.23$ ,  $B = 5.1$  and pretty good overall agreement with measured sublayer properties

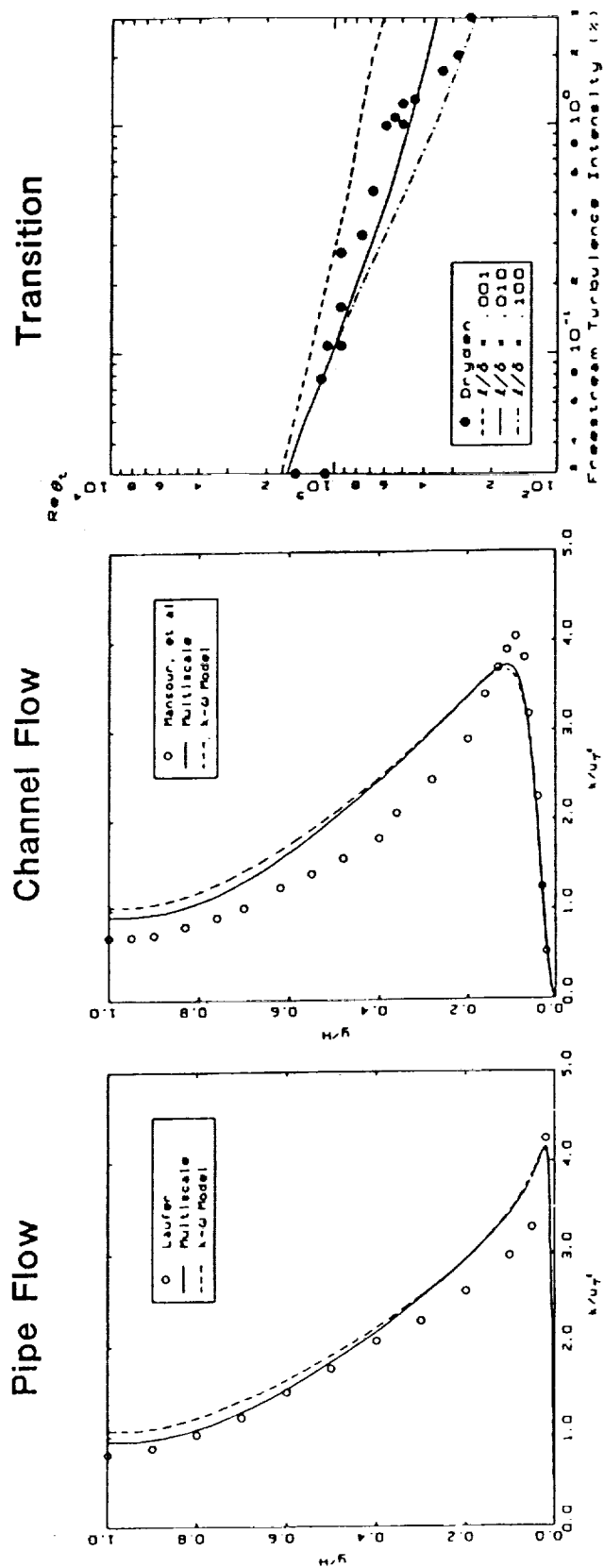


# WALL FUNCTIONS AND VISCOUS DAMPING

- Any model that fails to predict  $B \doteq 5.0$  must have either wall functions or viscous damping to achieve accurate boundary-layer results - even mixing length needs the van Driest modification
- A great deal of effort has been devoted by Viegas, Rubesin, et al toward perfecting wall functions - robust, but solutions usually are sensitive to  $y^+$
- A great deal of effort has gone into devising viscous damping functions for  $k-\epsilon$  - asymptotic consistency at the expense of stiffness
- Almost all of this effort needed because  $k-\epsilon$  says  $B = -2.2$
- Yet, there is a path of far less resistance... $k-\omega$  says  $B = 5.1$  and requires no damping!

# VISCOUS DAMPING FOR $k-\omega$

- If you want asymptotic consistency with  $k-\omega$ , using simple damping functions similar to those of Glushko...



# EFFECTS OF COMPRESSIBILITY

- Morkovin's Original Hypothesis works well for boundary layers all the way up to Mach 10 (Speziale)
- For the mixing layer, Sarkar and Zeman postulate that dissipation is the sum of solenoidal and dilatational components

$$\rho \frac{dk}{dt} = -\rho(\epsilon + \epsilon_d) + \dots$$

$$\rho \frac{d\epsilon}{dt} = -C_{\epsilon 2} \rho \epsilon^2 / k + \dots$$

$$\epsilon_d = \xi^* F(M_t) \epsilon$$

$$M_t^2 = 2k/a^2$$

*Sarkar's Model*

$$\xi^* = 1$$

$$F(M_t) = M_t^2$$

*Zeman's Model*

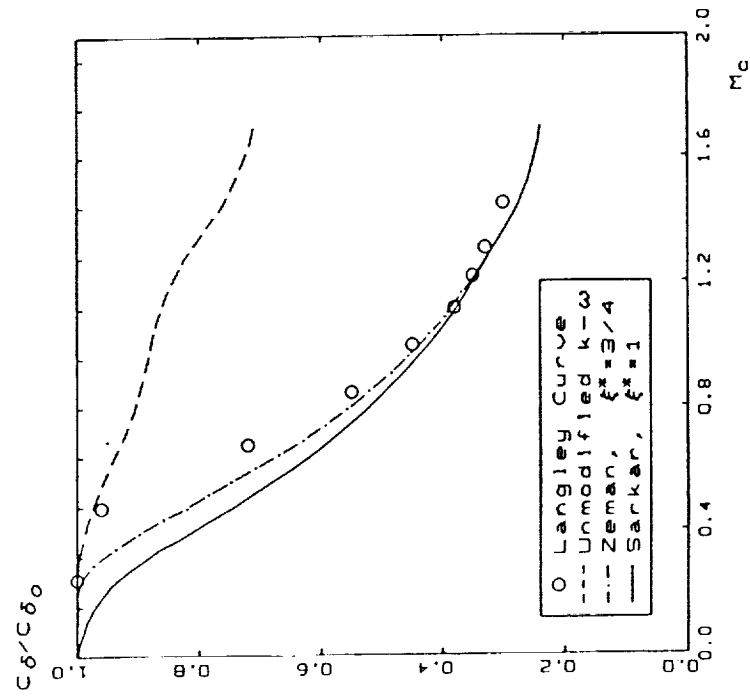
$$\xi^* = 3/4$$

$$F(M_t) = [1 - \exp\{-(M_t - M_{to})^2/\Lambda^2\}]H(M_t - M_{to})$$

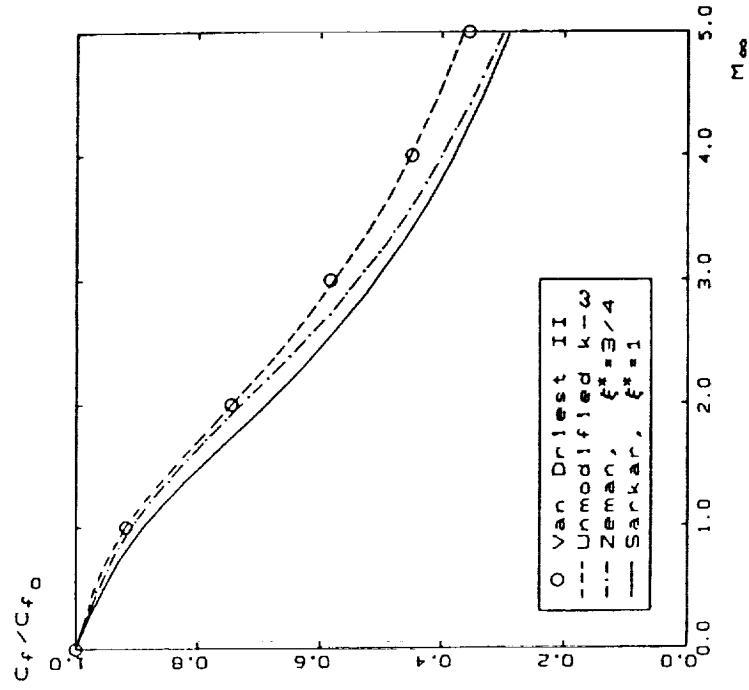
- Sarkar and Zeman modifications fix something that wasn't broken...flat-plate boundary layer
- Perturbation analysis explains why

# Applications

Compressible Mixing Layer



Adiabatic Wall Boundary Layer





# Perturbation Analysis of the Wall Layer

- For the  $k-\omega$  model...

The perturbation expansion proceeds in powers of  $M_r = u_r/a_w$ , and the velocity follows the Van Driest scaling, i.e.,

$$\frac{u^*}{U_\infty} = \frac{1}{A} \sin^{-1} \left( \frac{2A^2 v - B}{\sqrt{B^2 + 4A^2}} \right), \quad v = u/u_r$$

$$\frac{u^*}{u_r} \sim \frac{1}{K} \log(u_r y / \nu_w) + \left[ \text{constant} + \frac{1}{K} \log(\rho/\rho_w)^{1/4} \right]$$

where

$$K^2 \sim \kappa^2 [1 - (40.27\xi^* - 0.89)M_r^2 + \dots]$$

- For the  $k-\varepsilon$  model...

$$\frac{u^*}{u_r} \sim \frac{1}{K_\varepsilon} \log(u_r y / \nu_w) + \left[ \text{constant} + \frac{1}{K_\varepsilon} \log(\rho/\rho_w)^{5/4} \right]$$

where

$$K_\varepsilon^2 \sim \kappa^2 [1 - (10.23\xi^* + 2.34)M_r^2 + \dots]$$

# An Alternative Compressibility Term

- Turbulence Mach number is about twice as large in a mixing layer as it is in a boundary layer

Maximum Turbulence Mach Number,  $(M_t)_{\max}$

$M_\infty$	Boundary Layer $\xi^* = 0 \quad \xi^* = 1$		Mixing Layer $\xi^* = 0 \quad \xi^* = 1$	
0	0	0	0	0
1	.061	.061	.180	.159
2	.114	.107	.309	.227
3	.149	.135	.384	.245
4	.174	.154	.424	.254
5	.191	.171	.453	.266

- The following model for dilatational dissipation improves predictions for both the mixing layer and the boundary layer

$$\epsilon_d = \xi^* F(M_t) \epsilon$$

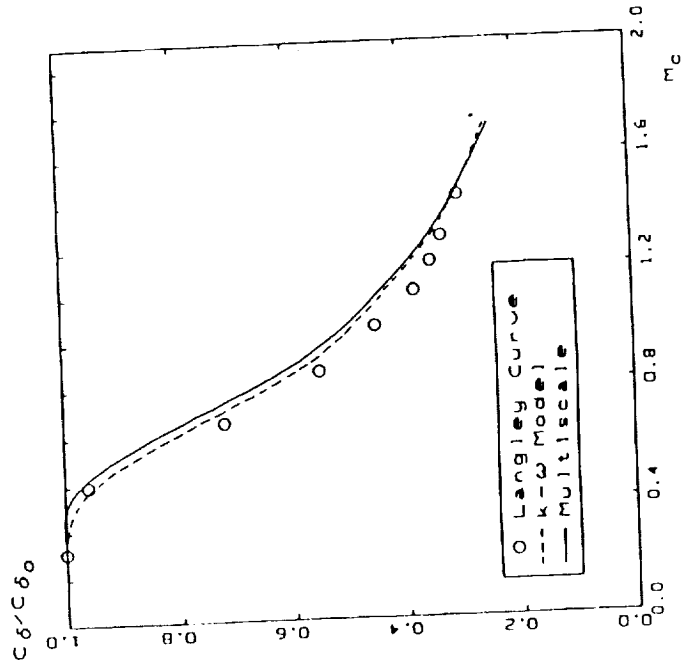
$$F(M_t) = [M_t^2 - M_{to}^2] H(M_t - M_{to})$$

$$\xi^* = 3/2, \quad M_{to} = 1/4$$

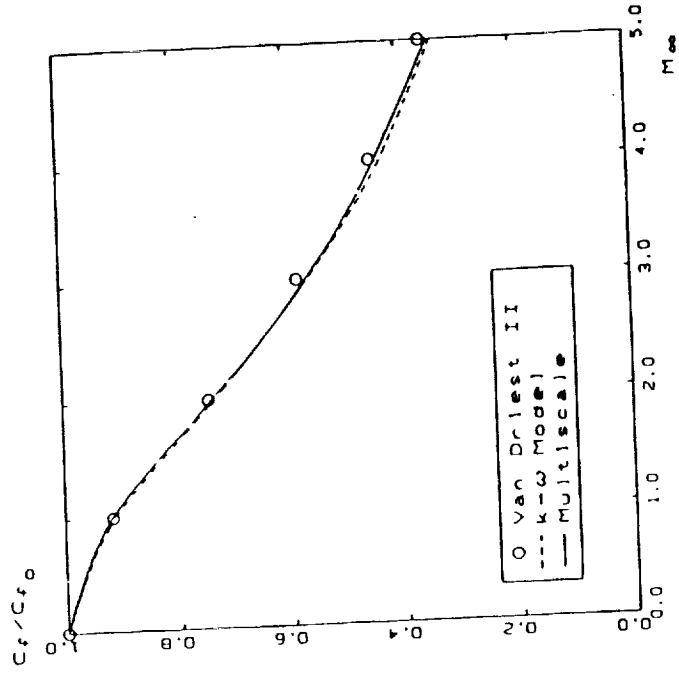
# Applications

## New Compressibility Term

Compressible Mixing Layer

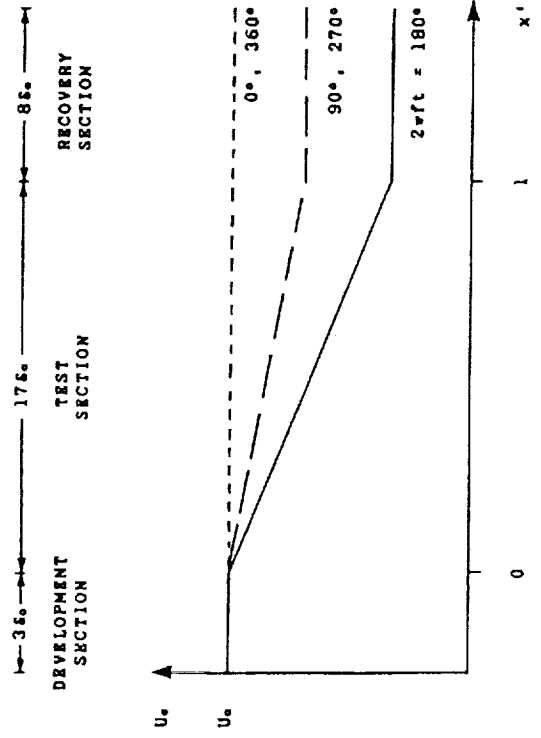


Adiabatic Wall Boundary Layer



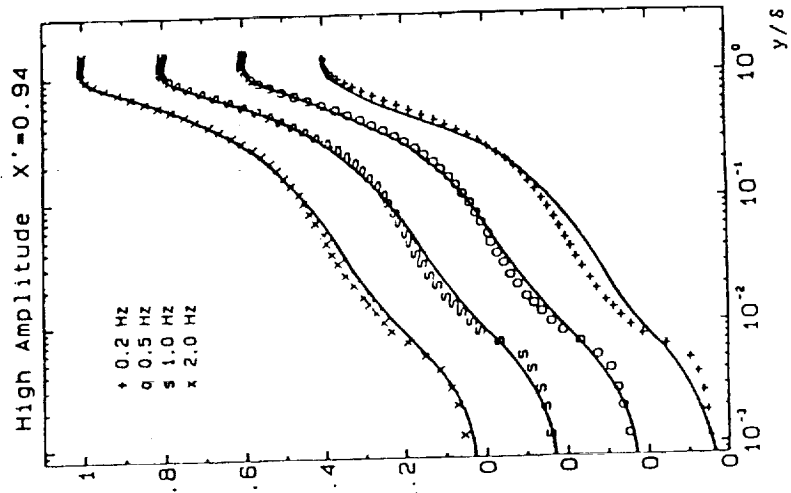
# UNSTEADY BOUNDARY LAYERS

- The  $k-\omega$  model does remarkably well for unsteady boundary layers even when periodic separation and reattachment occurs as in the experiments by Jayaraman, Parikh and Reynolds

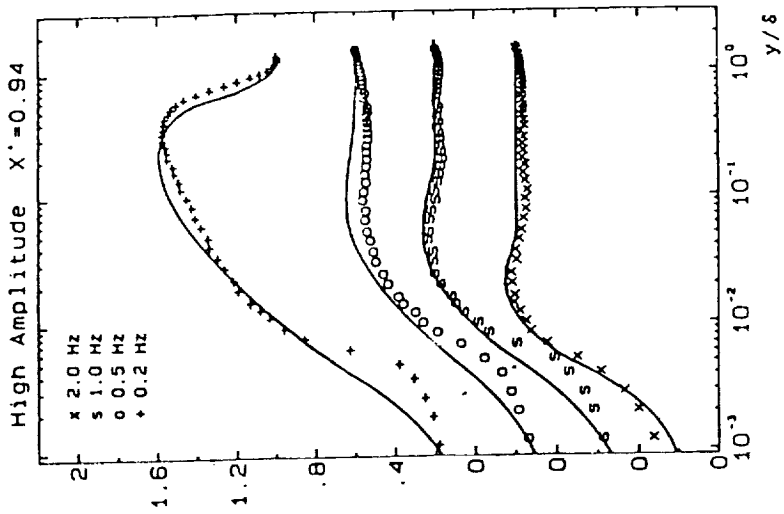


# UNSTEADY HIGH BOUNDARY LAYERS

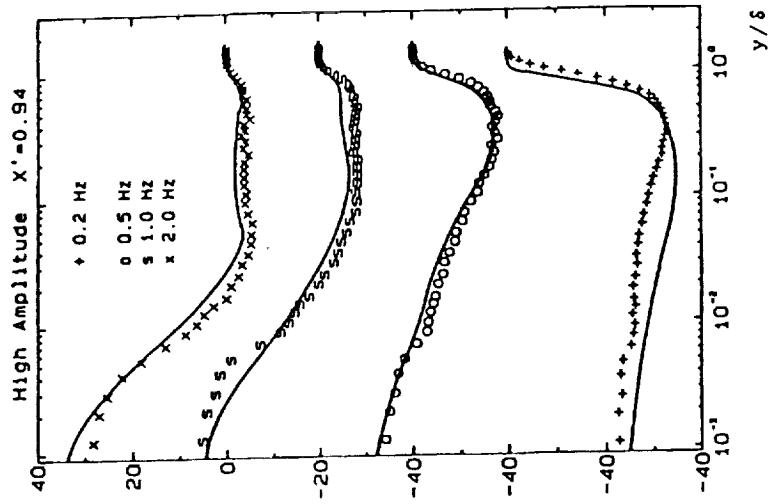
Mean Velocity



Amplitude of 1st Harmonic

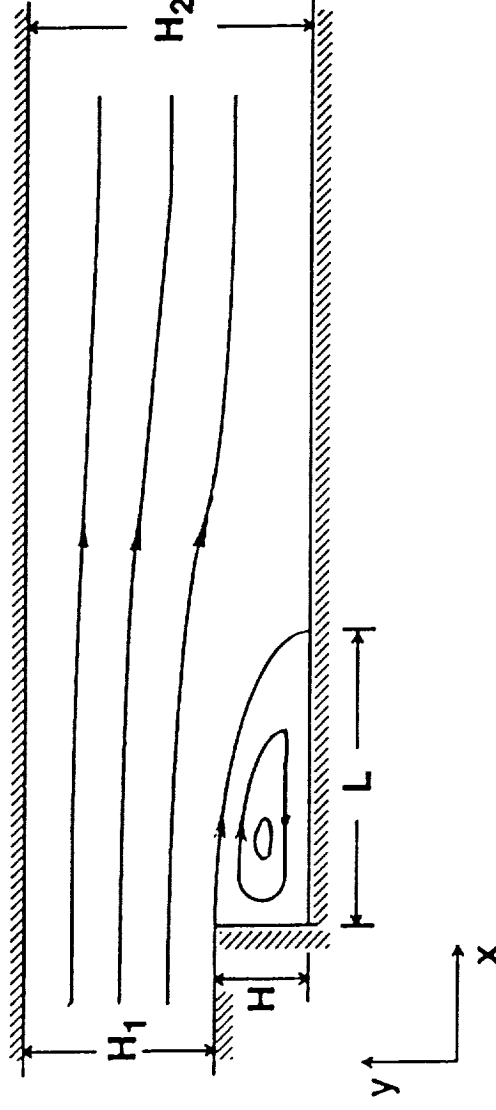


Phase Shift



# INCOMPRESSIBLE SEPARATION

- Flow past a backward-facing step has been a popular test case

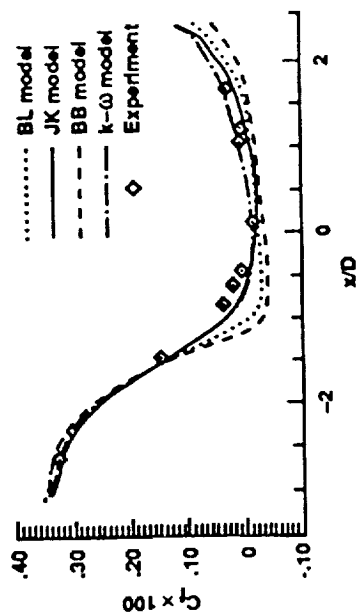
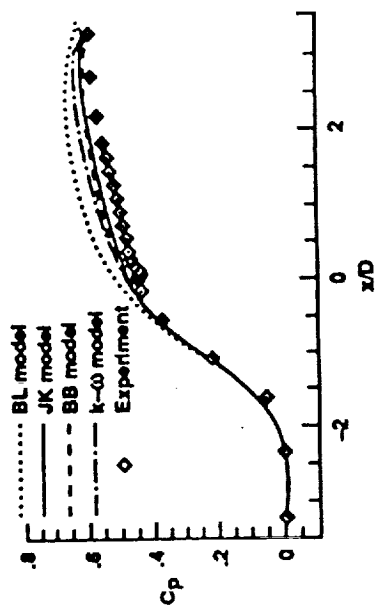
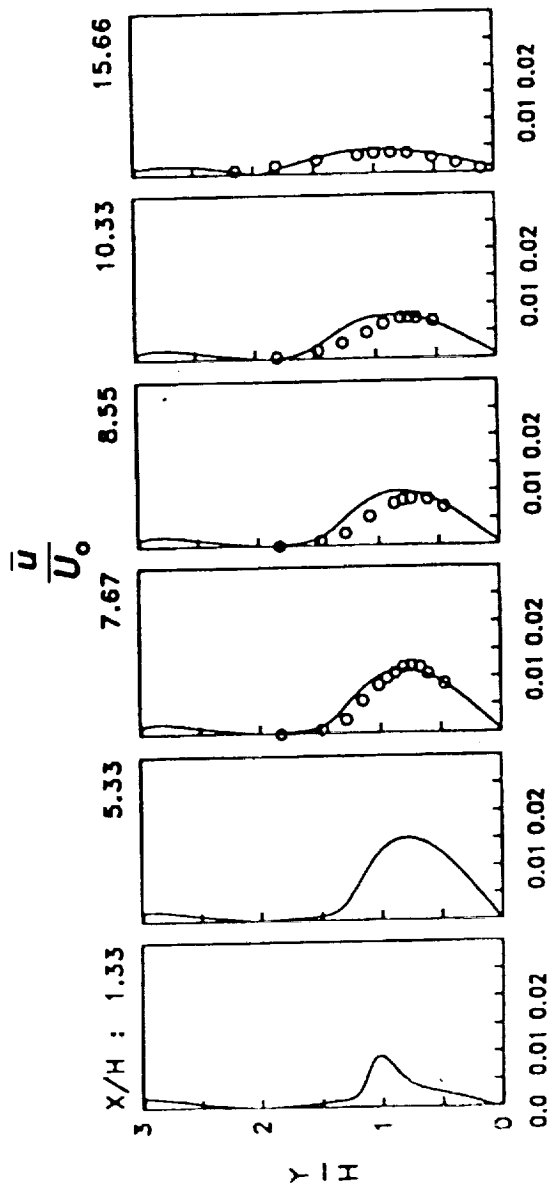
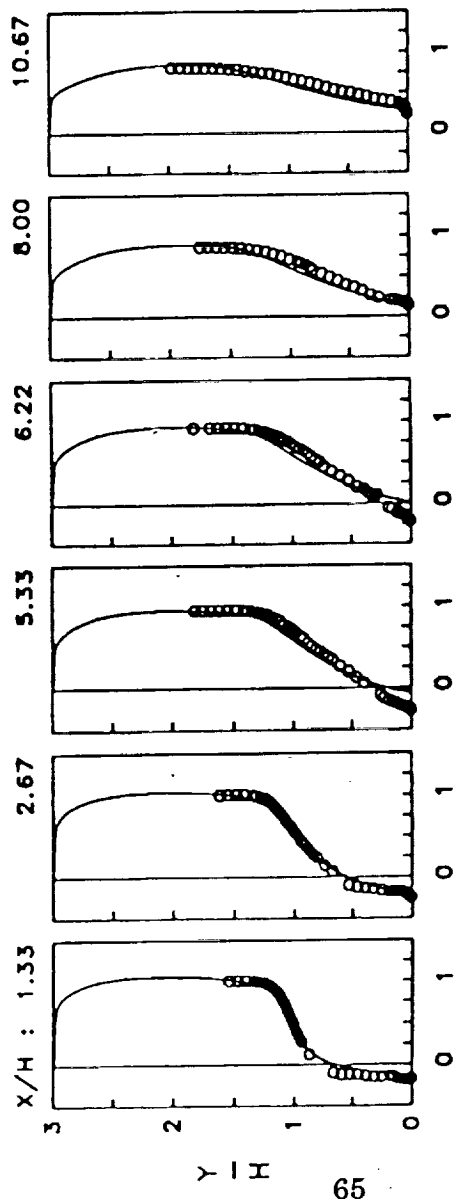


- Thangam and Speziale have shown that a  $200 \times 100$  grid is necessary for grid independence
- $k-\varepsilon$  does very well for this flow predicting reattachment at  $x/H = 6.25$  (experiments say 7.1)

# BACKSTEP RESULTS

Speziale: Standard  $k-\epsilon$   
(Eaton-Johnston Experiment)

Mentor:  $k-\omega$   
(Driver Experiment)



# COMPRESSIBLE SEPARATION

- Shock-induced boundary-layer separation is a more difficult problem
- Horstman has run many shock-separated flows with the  $k-\varepsilon$  model - separation region generally too small,  $q$  and  $c_f$  too high at reattachment - " $k-\varepsilon$  is a totally unreliable model for design purposes in 2-D and axisymmetric compressible flows with separation"
- A few cases have been done with the  $k-\omega$  model with similar results - skin friction tends to be closer to measurements
- 2-D shock-separated flows can be routinely done on a \$3,000 desktop computer - 3500 grid points  $\rightarrow$  3 to 4 hours



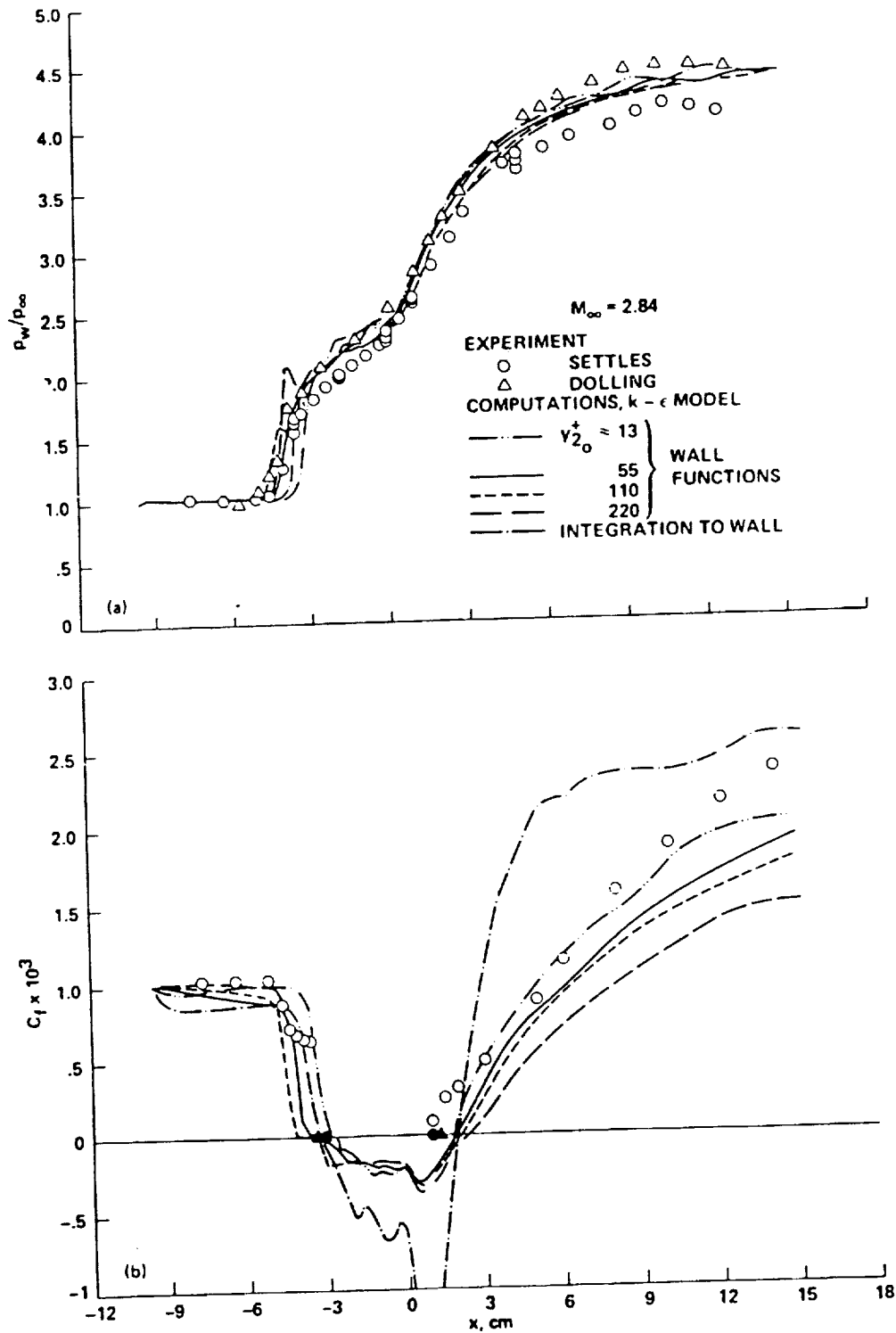
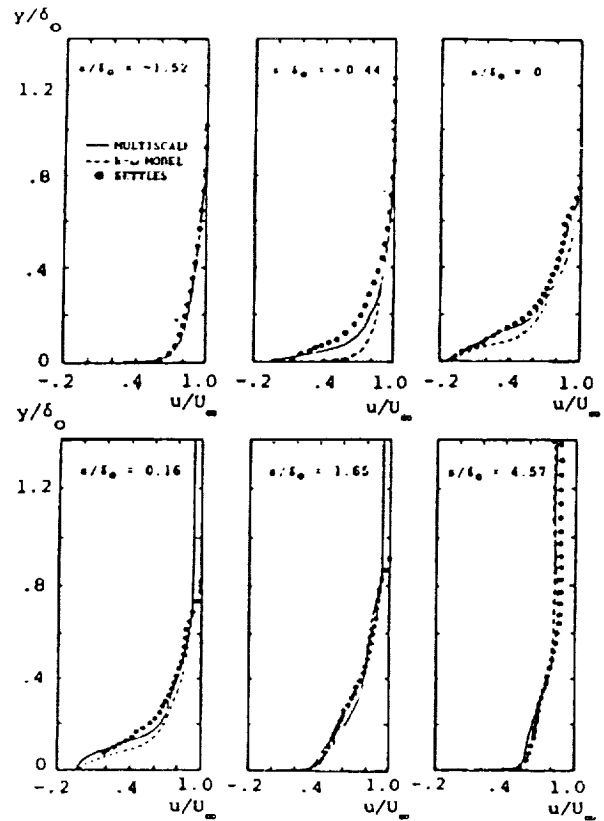
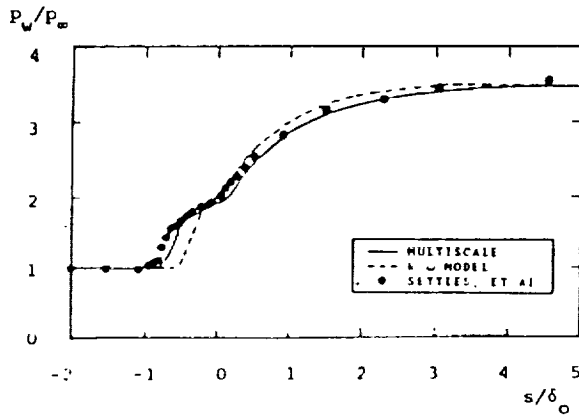
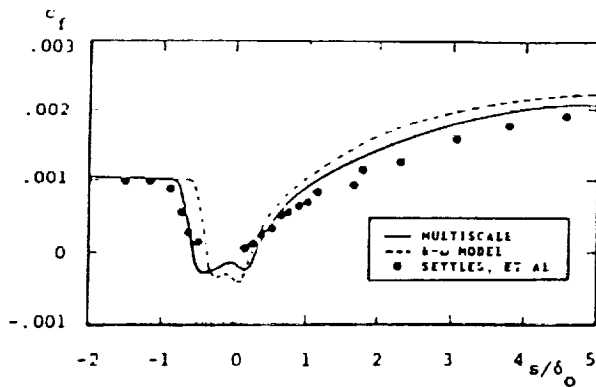
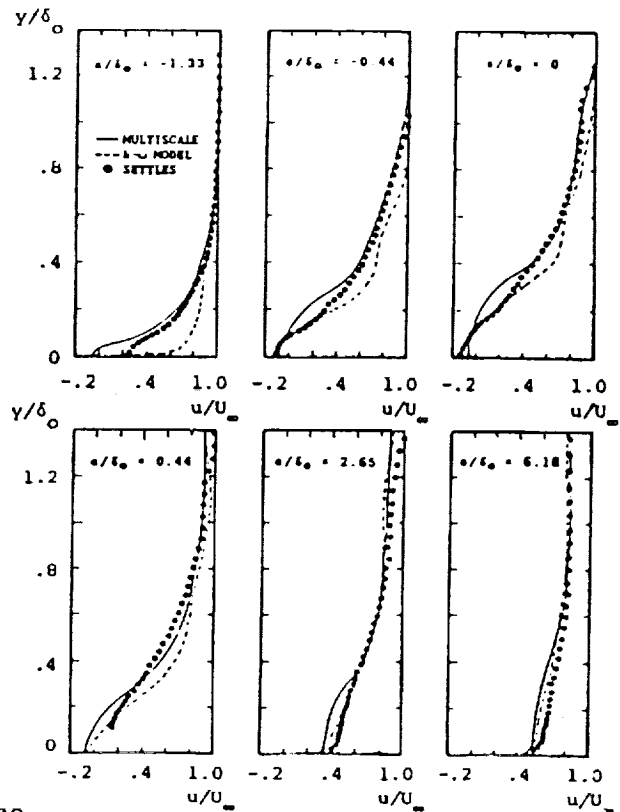
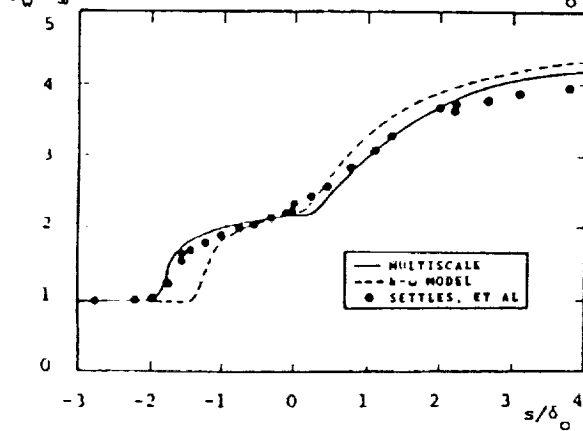
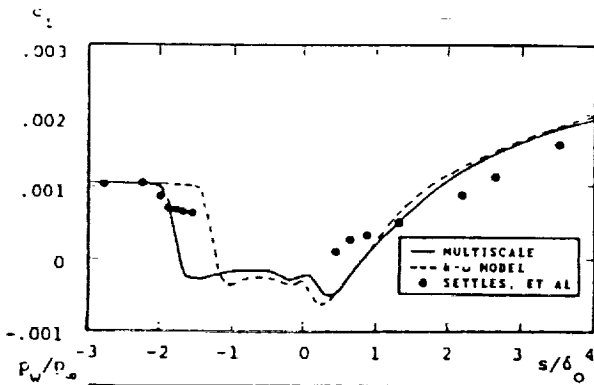


Fig. 17 Comparison of computations based on wall functions with the  $k-\epsilon$  model of turbulence, of a calculation based on the Jones-Launder near-wall terms in this  $k-\epsilon$  model, and of measurements for the two-dimensional, supersonic, 24° compression corner experiment:  $M_\infty = 2.84$ . (a) Surface-pressure distribution; (b) Skin-friction coefficient distribution.

## 20° Planar Corner



## 24° Planar Corner



# THE PRESENT STATE OF EDDY VISCOSITY MODELS

- At Stanford Olympics II,  $k-\epsilon$  and  $k-\omega^2$  did very poorly for incompressible attached boundary layers in adverse pressure gradient. Since that time:
  - $k-\epsilon$ : No universally accepted fix has evolved
  - $k=\omega$ : Changing from  $k-\omega^2$  to  $k-\omega$  fixes the problem
  - $k-\tau$ : Does well for Samuel-Joubert
- Integration to the wall continues to plague  $k-\epsilon$ 
  - $k-\epsilon$  and  $k-\tau$  say  $B = -2.2$  and must have viscous damping
  - $k-\omega$  says  $B = 5.1$  and requires no damping
- Current status of two-equation models is as follows:

Type of Flow	$k-\omega$	$k-\epsilon$	Others
Free Shear Flows	C	C	C-
Boundary Layer in $\nabla p$	A	C-	B
Low Reynolds Number Effects	A-	C+	C+
Compressibility Effects	A-	C	B
Unsteady Boundary Layers	A	I	I
Backward Facing Step	A	B+	B+
Compressible Separation	D+	D	I

# THE FUTURE DIRECTION OF EDDY VISCOSITY MODELS

- It is unlikely that a UNIVERSAL model will be of the two-equation variety - Boussinesq approximation isn't good enough.

## NEVERTHELESS...

- Development of the second equation ( $\epsilon$ ,  $\omega$ , etc.) should continue - it is part of the foundation for second-order closure models and it is easier to work with in the two-equation model context
- Perturbation methods and similarity solutions should be used more to weed out inferior formulations - the back of the envelope is still a good place to solve problems!
- Using  $k$ - $\omega$  removes the pressure gradient and integration-to-wall problems - difficulties remain for free shear flows and turbulent/nonturbulent interfaces that should be resolved
- The  $k$ - $\omega$  model warrants more attention from the turbulence community - it does many things well with the same 5 closure coefficients!

Comment paper,

Workshop on engineering turbulence modeling

NASA Lewis, August 21, 1991

P. R. Spalart,

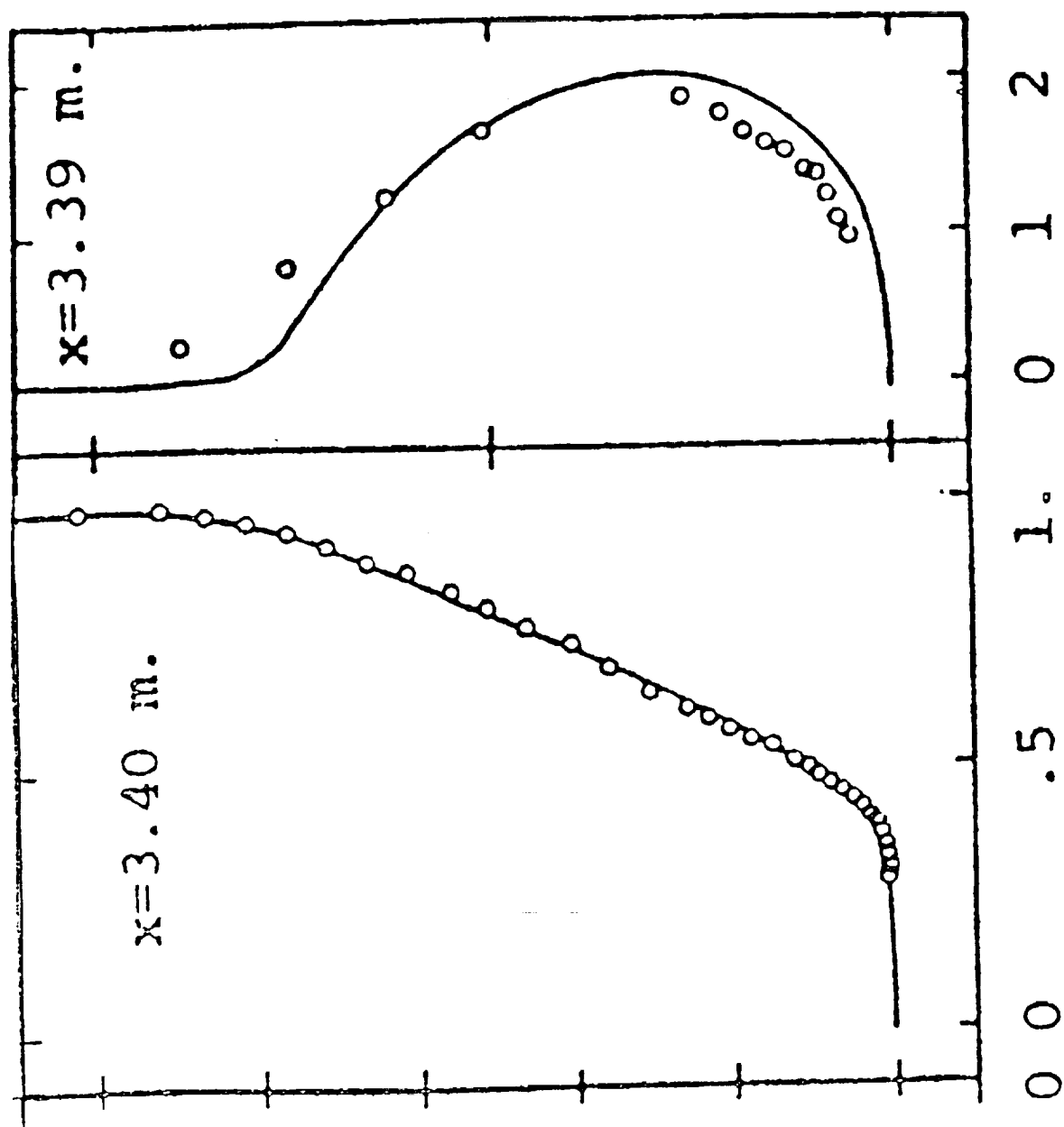
Boeing Commercial Airplane Group

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## Comments on position presentation

- What is a complete model?
- The emphasis on adverse pressure gradient is very laudable. We need to be cautious with "equilibrium boundary layers". They may be solutions of the modeled equations (which contain the same assumptions regarding the Reynolds-number dependence), and not of the full equations! Clauser's concept is not the only one around.
- This speaker is not shocked by, e.g.,  $k \propto y^{3.23}$  near the wall (as long as  $k$  is at most  $O(y^2)$ ). The near-wall behavior is *no guide* for fundamental choices such as that between  $\epsilon$  and  $\omega$ . We see nothing wrong with damping functions.
- The  $k$ - $\omega$  results in the Samuel-Joubert flow are inconsistent ( $\delta$  at different heights in  $U$  and in  $\overline{u'v'}$ ).

Samuel-Joubert flow, computed with  $k-\omega$



## Boeing (commercial) position

- The push is towards complex geometries, unstructured or at least very distorted and multi-block grids, separation, and we must have high Reynolds number.
- We have experience with Mellor-Herring and derivatives, Baldwin-Lomax, Johnson-King, RNG, Baldwin-Barth, and one shot at  $k-\epsilon$ . Johnson-King has the best press in Navier-Stokes codes.
- Viscous-inviscid coupling and algebraic models are endangered (at least on the research side), but we are very unlikely to give up eddy viscosity for years.
- We find that a correct first  $y^+$  (say  $\approx 5$ ) does **not** ensure grid convergence at Reynolds numbers of interest, because of artificial dissipation.
- The range of flow types is not as wide as in mechanical engineering ("industrial flows"). For instance we have not jets (except when on the ground!) and certainly no homogeneous turbulence. Boundary layers under numerous influences (shock, sweep, curvature) and approaching separation are everything.



## Speaker's position

- The true  $k$  and  $\epsilon$  not very reliable:  $f(y^+, R_\theta, PG)$ . There is no proof that the asymptote as  $R_\theta \rightarrow \infty$  is a finite function of  $y^+$ .
- $k-\epsilon$  requires  $k^+ = C_\mu^{-1/2}$  in the log layer, which is not confirmed by **ANY** experiment nor DNS.
- The  $k$  problem can be hidden by viscous damping functions ( $f(y^+)$ ) and DNS Reynolds numbers are too low to clearly expose it.
- In some models  $k$  is “not really”  $k$ , but then why does it get the whole benefit of the production term? (which is the only exact one!)
- The problem with using  $y^+$  is not  $y$ . It's  $u_\tau$ . See new model.
- Because of the Reynolds number we use DNS results for guidance, but not for calibration.
- Five DNS's are cooking (slowly): 2D APG (Watmuff experiment); 3D APG (Sendstad); 2D bubble (Coleman); 2D with suction (Mariani); 2D with heat transfer (Bell). The Reynolds numbers are no higher but the physics are more interesting.

## New one-equation model

• Accepted for Reno '92. Preprints available. Closer to Nee-Kovasznay and Secundov than to Baldwin-Barth.

• A one-equation model, transporting  $\tilde{\nu}$  ( $\approx \nu_t$ ), has been developed from scratch.

$$\frac{D\tilde{\nu}}{Dt} = c_{b1}[1 - f_{t2}] \tilde{S} \tilde{\nu} + \frac{1}{\sigma} \left[ \nabla \cdot [(\nu + \tilde{\nu}) \nabla \tilde{\nu}] + c_{b2}(\nabla \tilde{\nu})^2 \right] - \left[ c_{w1} f_w - \frac{c_{b1}}{\kappa^2} f_{t2} \right] \left[ \frac{\tilde{\nu}}{d} \right]^2 + \Delta U^2 f_{t1}$$

• "Rate of change = production + diffusion - destruction + trip".

• It is "semi-local": it uses the distance from the wall,  $d$ , but not  $u_\tau$ ,  $\delta^*$ , nor  $F_{max}$ . It is compatible with unstructured grids. The  $d$ -dependent term is a destruction of  $\nu_t$ . It involves a finely-tuned function  $f_w$ .

• The goal is not a universal model, but one that is useful in aerodynamics.

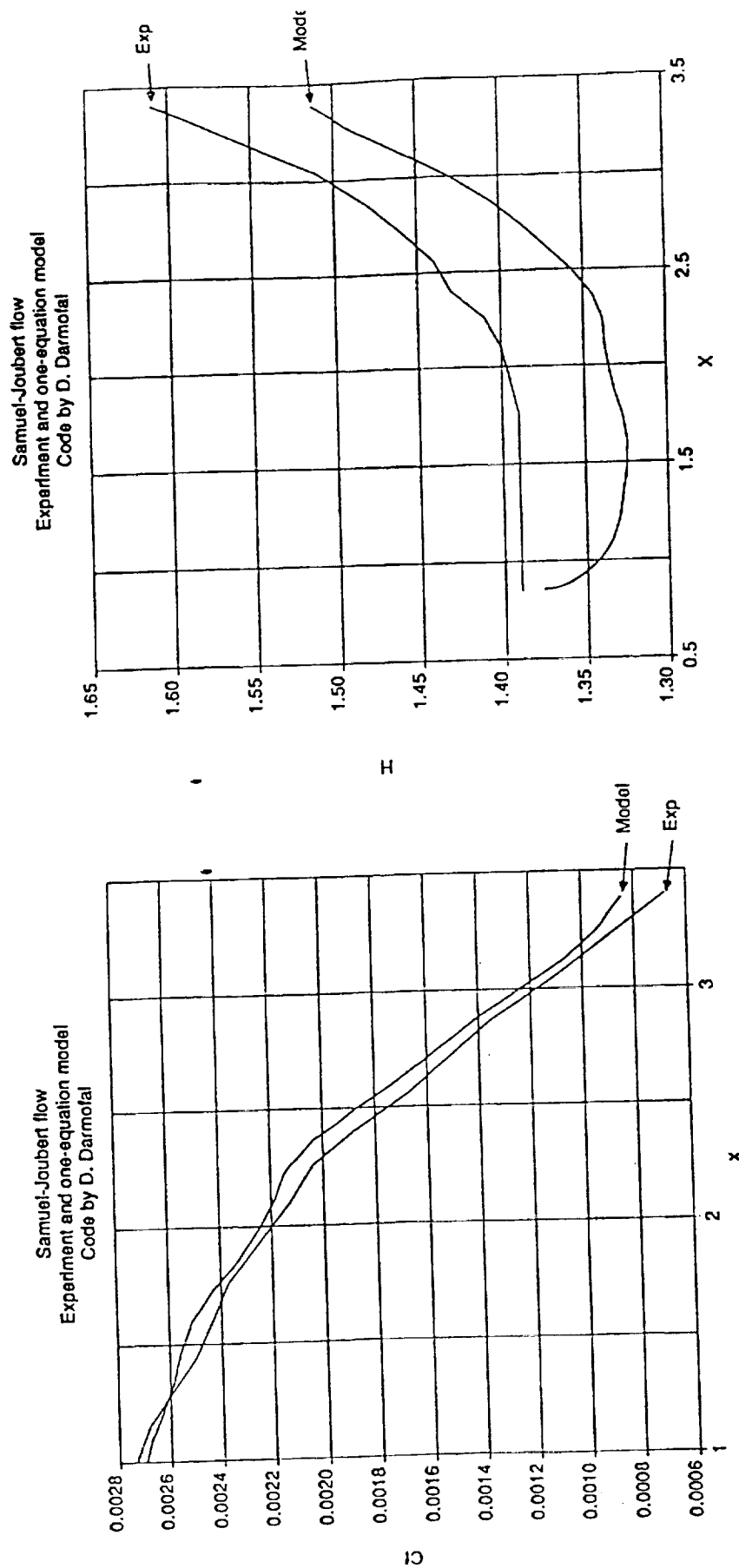
• The model was calibrated to give the right stress in the 2D mixing layer, the 2D wake, and the flat-plate boundary layer. It matches the classical log layer.

• The transported quantity is linear ( $\kappa u_\tau y$ ) from the log layer all the way to the wall, making it easier to resolve than  $U$ .

• The results are insensitive to the freestream value of  $\tilde{\nu}$  (0 is best).

• The model does better than CS, BL and  $k-\epsilon$  in adverse pressure gradient (lower  $C_f$ , higher  $H$ ). On Samuel-Joubert the  $C_f$  is fine, but  $H$  is low (how did they get  $H = 1.39$  with  $Re_\theta \approx 5000$  and almost no PG?). In transonic flow the model seems to place the shock slightly *in front* of the experimental shock.

# Samuel-Joubert flow, computed with the new model





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**Comment on : The Present State and Future Direction of  
Eddy Viscosity Models**

**T. J. Coakley  
NASA-Ames Research Center  
Moffett Field, CA 94035**

**Workshop on Engineering Turbulence Modeling  
August 21-22, 1991**

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## Summary and Comments

*Wall bounded flows:* The  $k - \omega$  model is probably best

Adverse pressure gradient flows

Compressible flat plate flows, esp. with heat transfer

Roughness - blowing - transition

Simplicity - no damping functions

Numerical stability - eg leading edge start

But: Separation and reattachment still are problems  
(all models)

*Free Shear Flows:* The  $k - \epsilon$  is probably best

$k - \omega$  solutions depend on free stream  $\omega$

$k - \epsilon$  more corrections and improvements available

This presentation: Compressible-hypersonic flows (NASP)

General discussion of  $k - \omega$  model with corrections

Comparison of model prediction for flat plate flows  
( $k - \epsilon$ ,  $k - \omega$ ,  $q - \omega$ )

Comparison of model predictions for a separated ramp  
flow

## Generalizations of the $k - \omega$ Model

$$\rho \frac{dk}{dt} = \mu_i S - \frac{2}{3} \rho k D - \beta^* \rho \omega k + [(\mu + \sigma_k \mu_T) k_{,j}]_{,j}$$

$$\rho \frac{d\omega}{dt} = \alpha \frac{\omega}{k} (\mu_i S - C_1 \rho k D) - \beta \rho \omega^2 + [(\sigma_L \mu + \sigma_\omega \mu_T) \omega_{,j}]_{,j}$$

$$\mu_T = \alpha^* \rho k / \omega, \quad S = (u_{i,j} + u_{j,i} - \frac{2}{3} \delta_{ij} u_{k,k}) u_{i,j}, \quad D = u_{k,k}$$

### Baseline Model

$$\beta^* = 9/100, \quad \beta = 3/40, \quad \alpha = 5/9, \quad \alpha^* = 1,$$

$$\sigma_L = 1, \quad \sigma_k = \sigma_\omega = \frac{1}{2}, \quad C_1 = \frac{2}{3}$$

### Low Re Model (Transition)

$$\alpha, \alpha^* = fns(\frac{k}{\nu \omega}), \quad \sigma_L = 5/18$$

### Compressible Dissip. (Compressible free shear layers)

$$\beta, \beta^* = fns(\sqrt{k}/c)$$

### Compression Mod. (Compressible separation)

$$C_1 = 2.4, \quad (d\rho L/dt = 0, \quad L = \sqrt{k}/\omega)$$

### Algebraic length scale (Reattachment heat transfer)

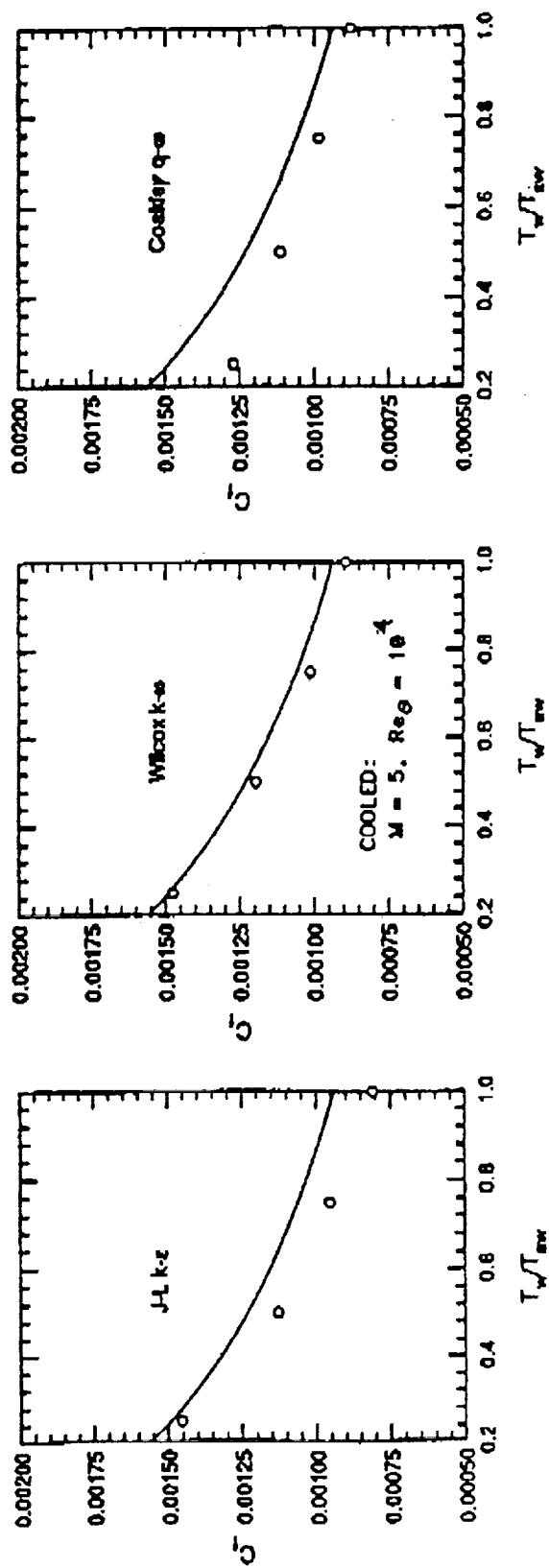
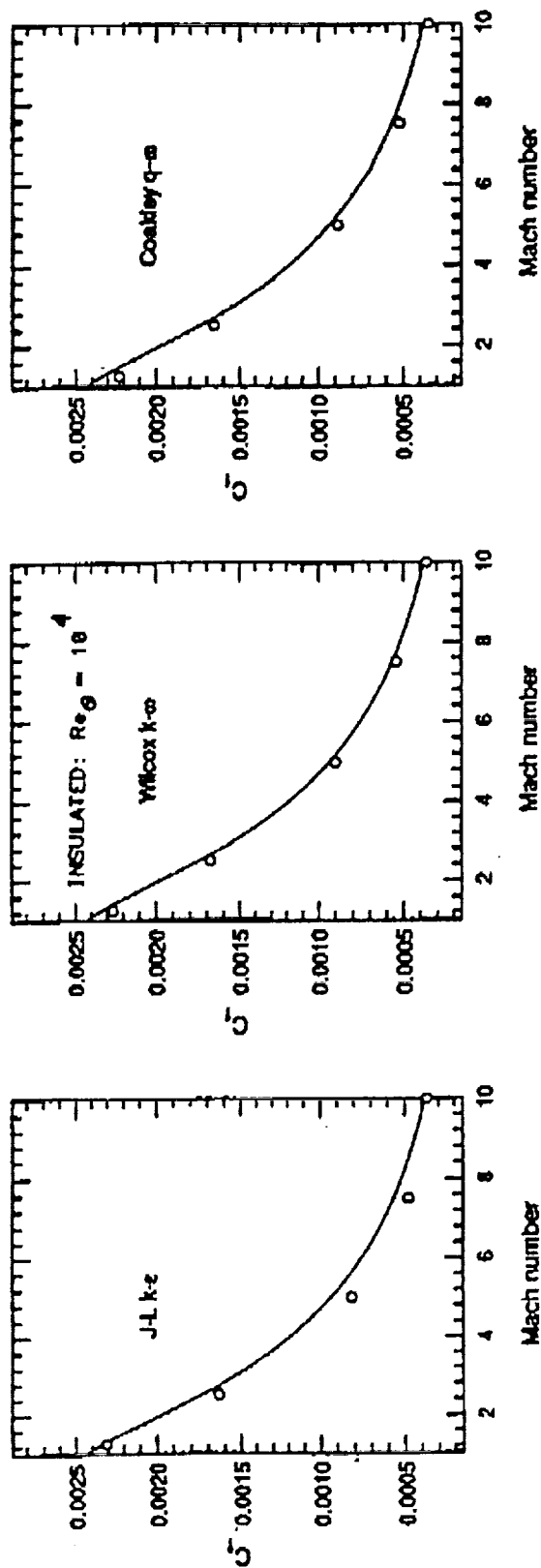
$$\mu_T = \alpha^* \rho \sqrt{k} L, \quad L = \min(\sqrt{k}/\omega, .225y)$$

### Vorticity length scale (Incompressible separation)

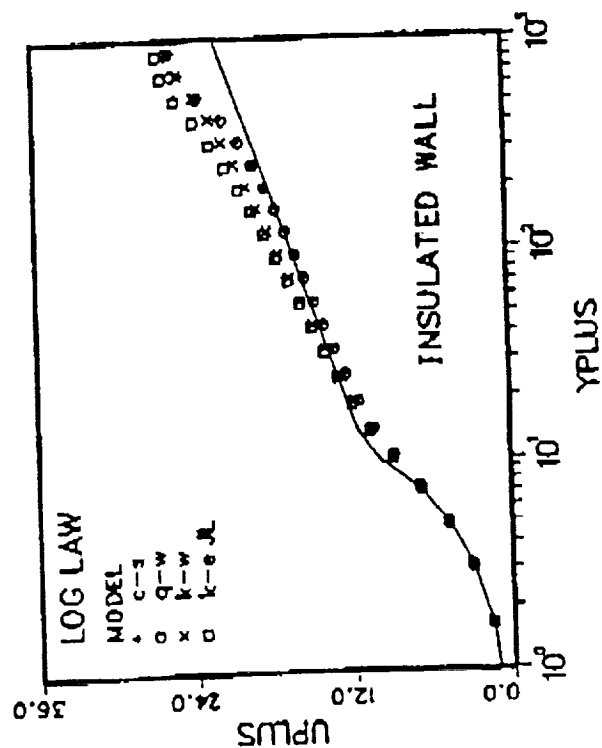
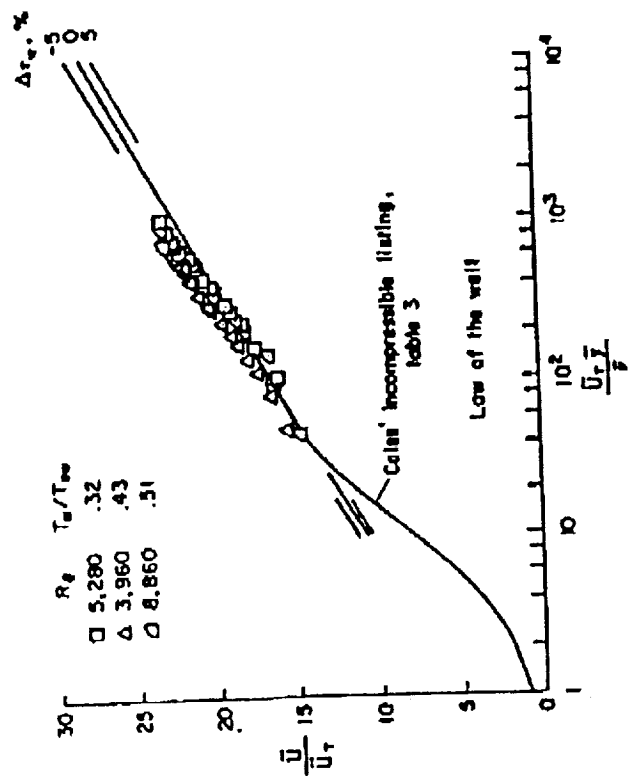
$$\mu_T = \alpha^* \rho \sqrt{k} L, \quad L = \min(\sqrt{k}/\omega, .3\sqrt{k}/|\vec{\omega}|)$$

# SKIN FRICTION ON INSULATED AND COOLED PLATES

## COMPARED WITH THE VAN DRIEST CORRELATION

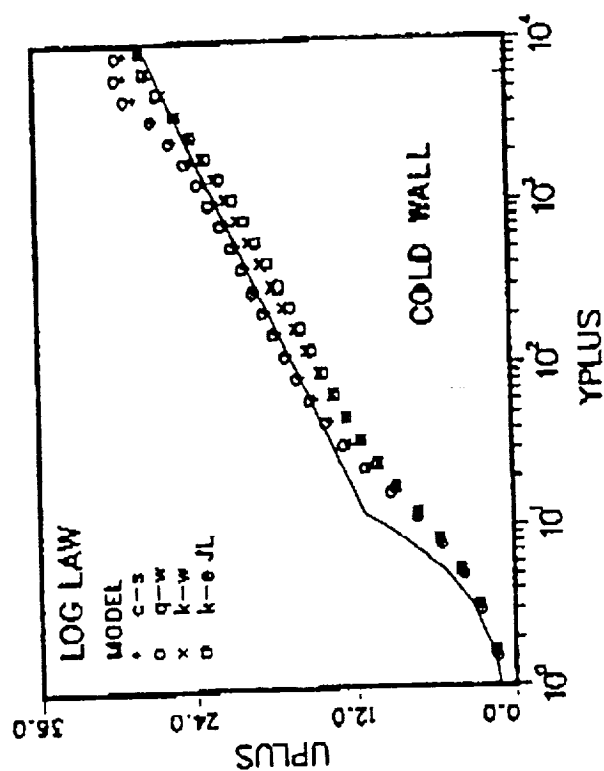






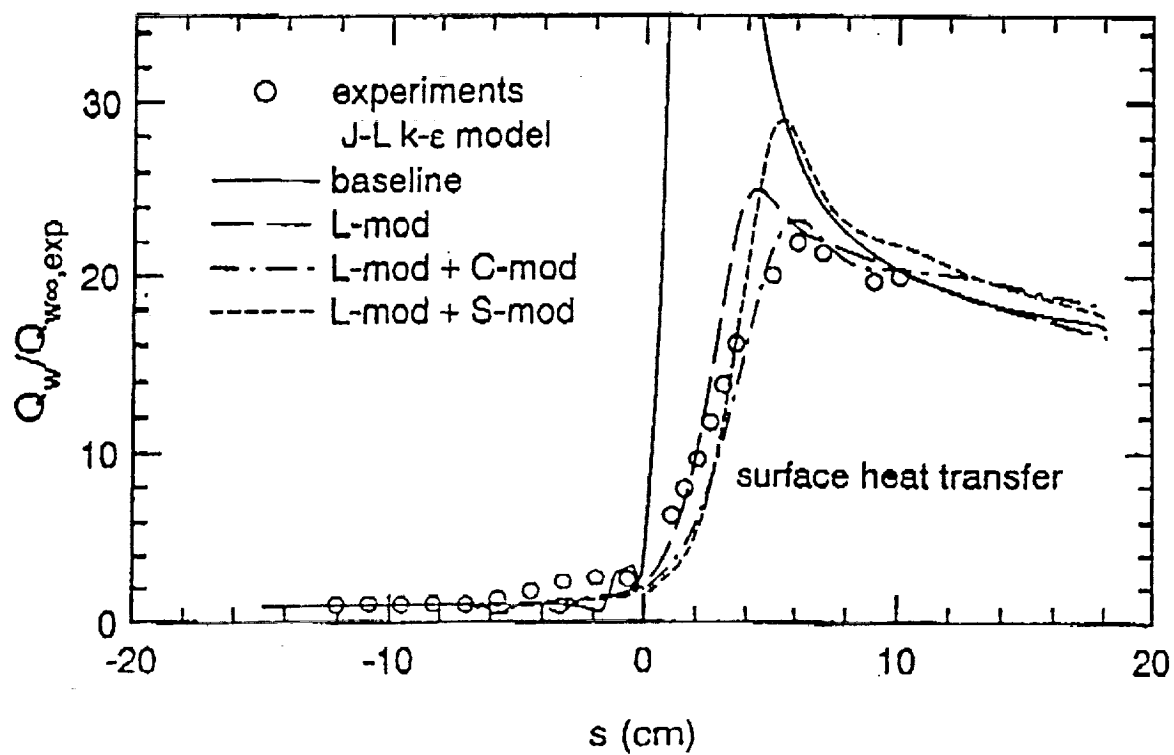
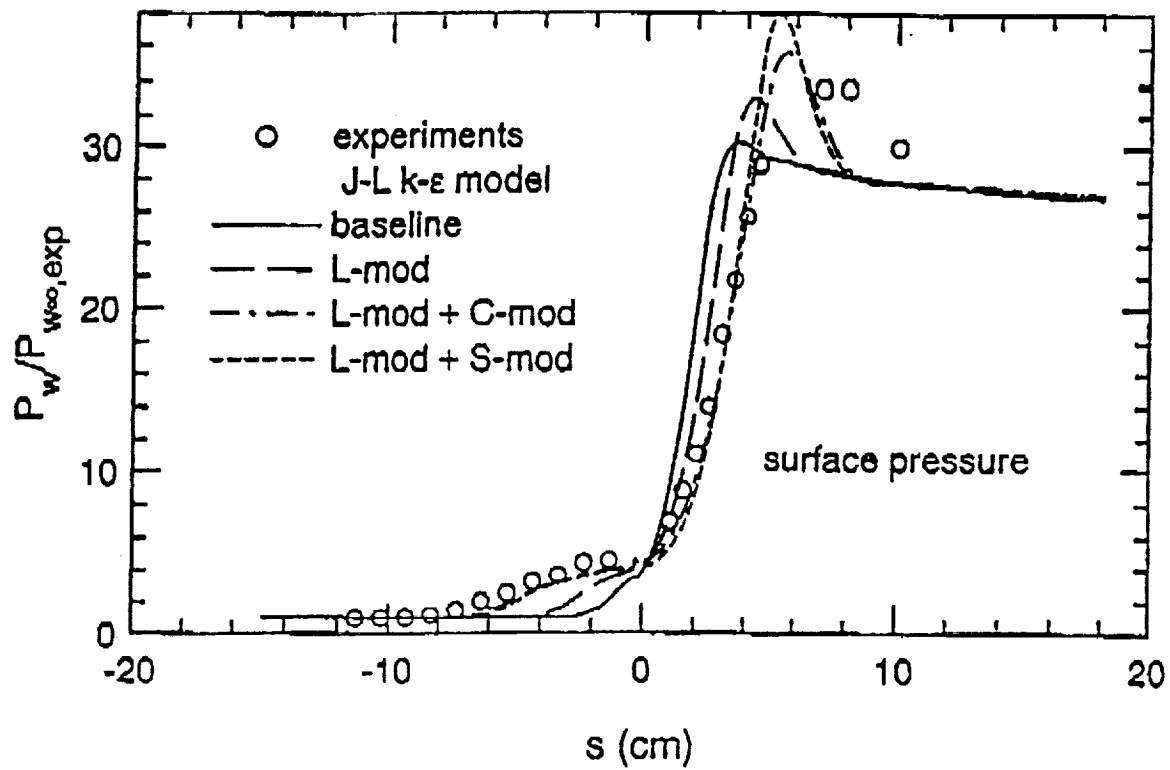
# COMPARISON OF VELOCITY PROFILES WITH THE COMPRESSIBLE LAW OF THE WALL

EXPERIMENT: KEENER AND HOPKINS,  $M = 6.5$   
 COMPUTATION:  $M = 5$ ,  $TW/TAW = 0.2$ ,  $1.0$



# Hypersonic Cylinder-Flare

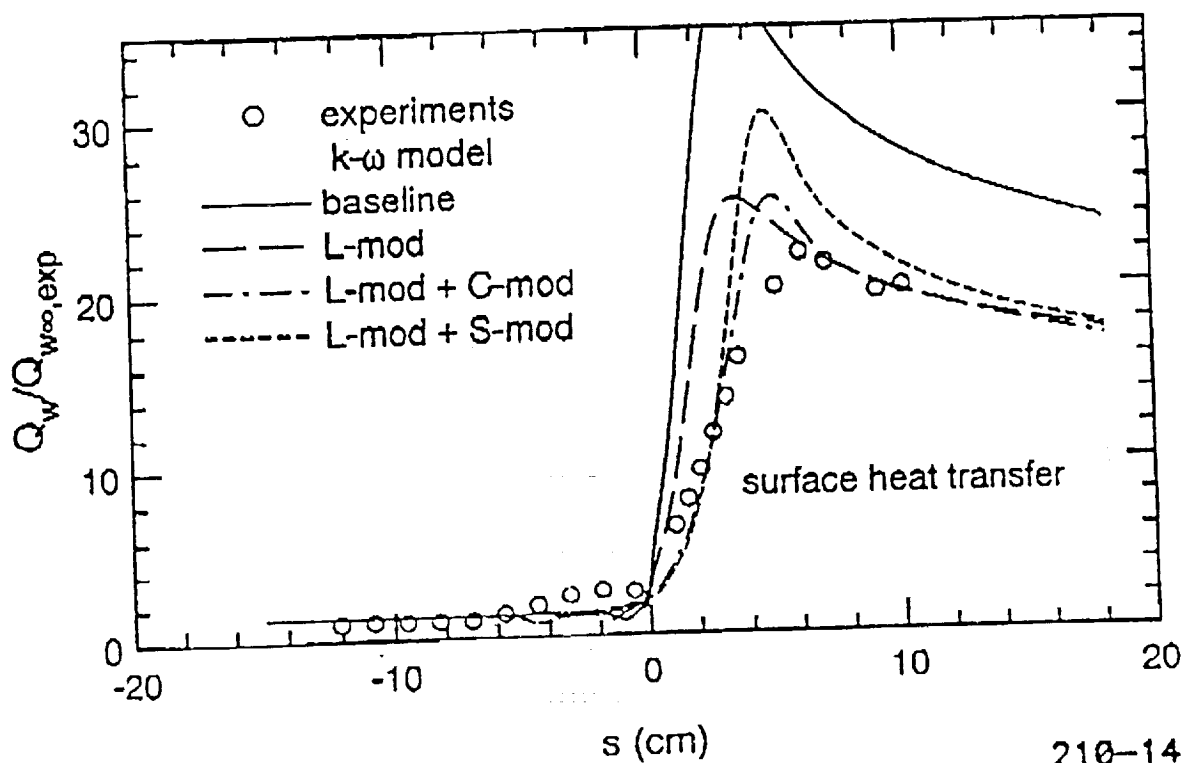
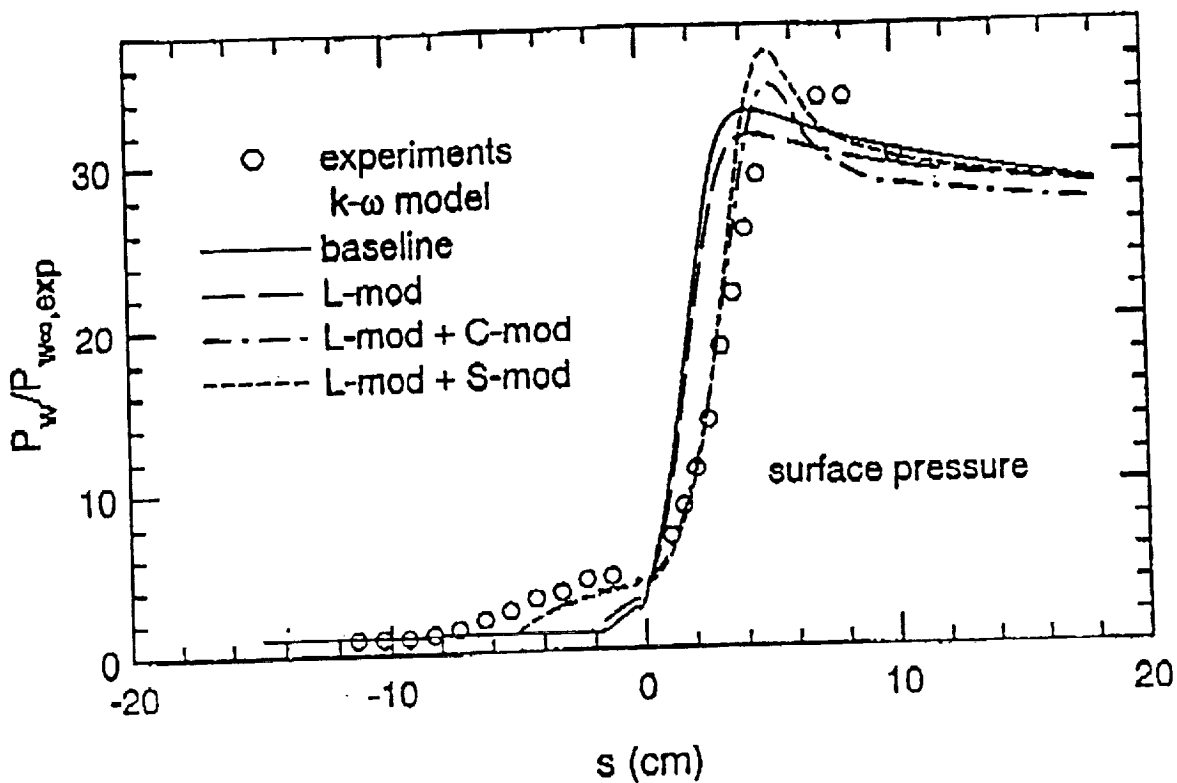
35° flare angle  $M = 7.05$



210-13

# Hypersonic Cylinder-Flare

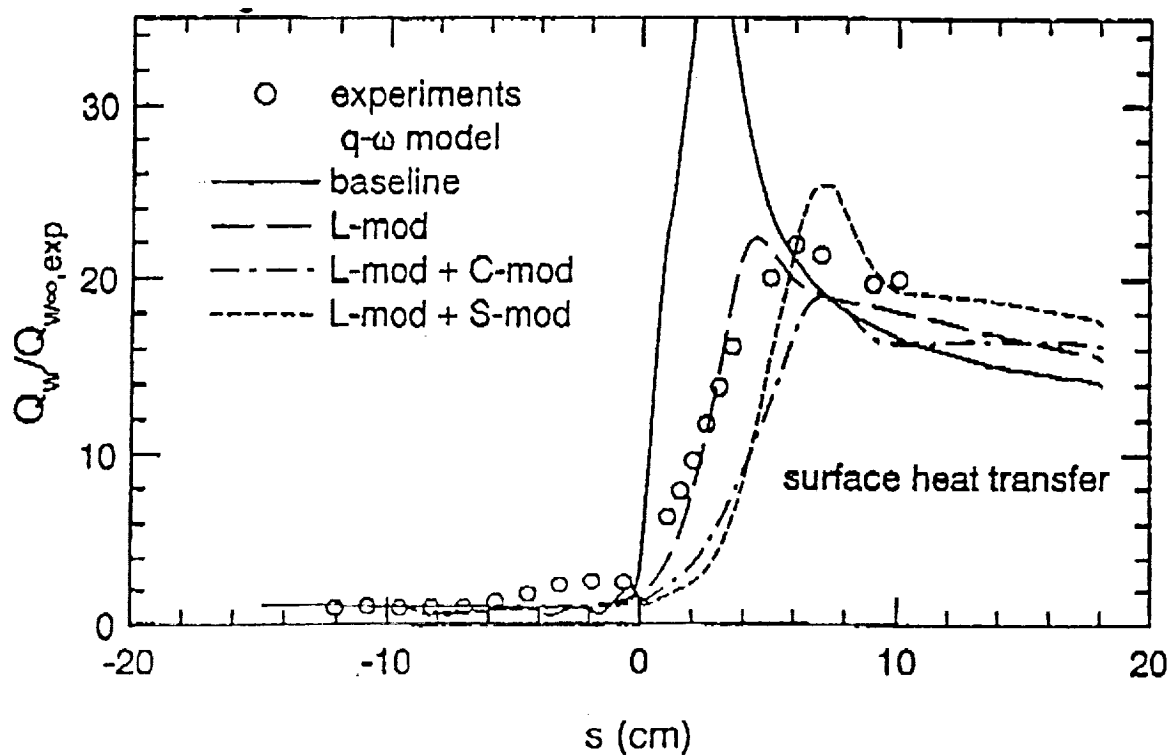
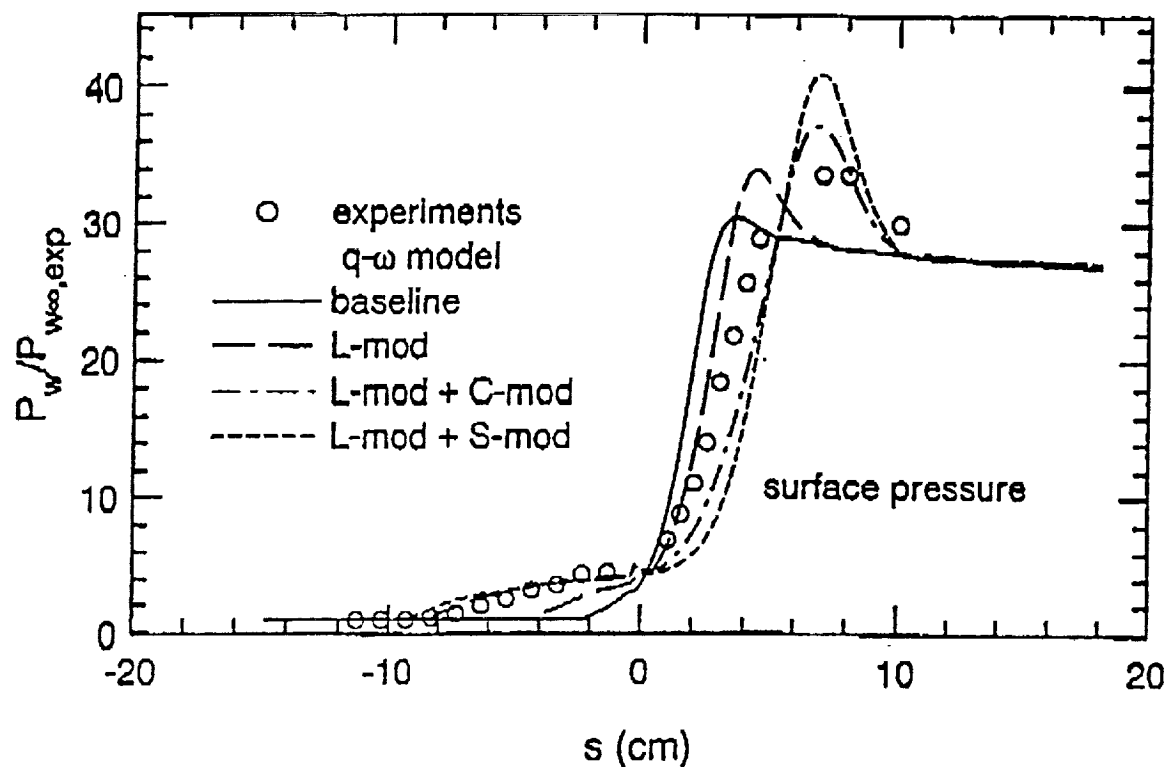
35° flare angle  $M = 7.05$



210-14

# Hypersonic Cylinder-Flare

35° flare angle  $M = 7.05$



210-15

# COMMENT ON TWO-EQUATION MODELS

N. Lang and T. Chitsomboon

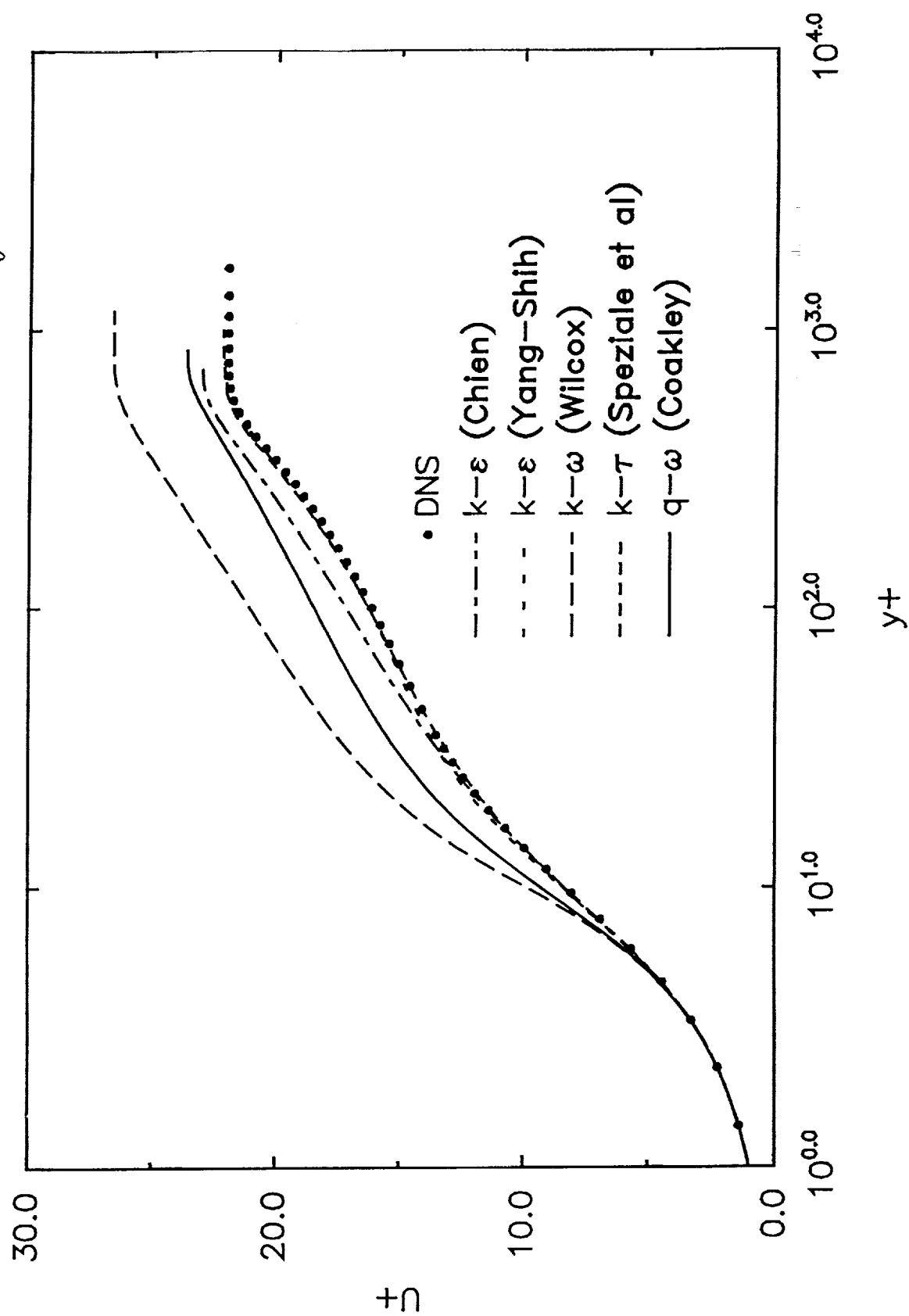
Center for Modeling of Turbulence and Transition  
ICOMP/NASA Lewis Research Center

## • SEVERAL NEAR-WALL TWO-EQUATION MODELS WERE STUDIED:

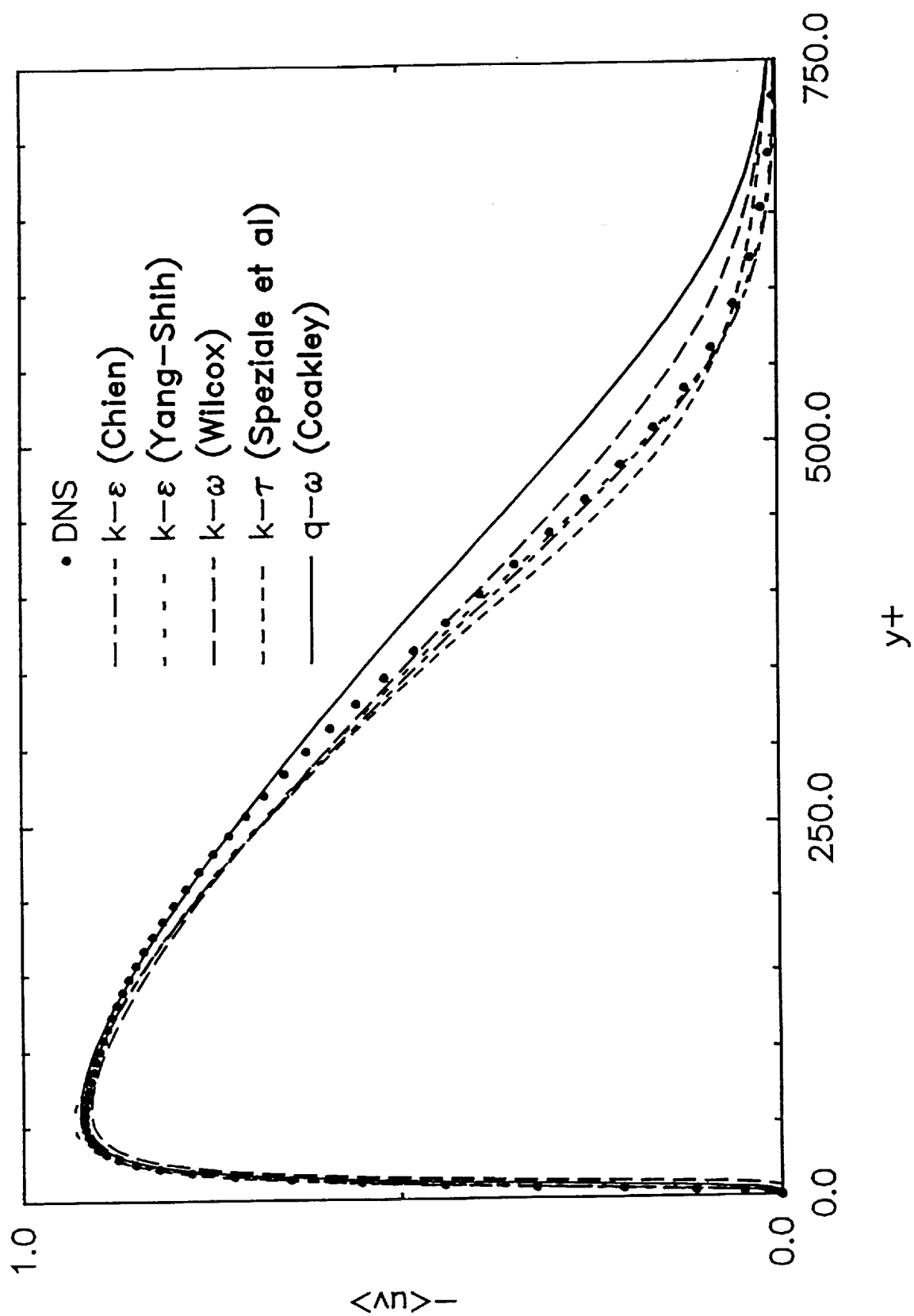
- $k-\epsilon$  (9 models)
- $k-\omega$
- $q-\omega$
- $k-\tau$

36-34  
36-347  
P-13  
N92-24520

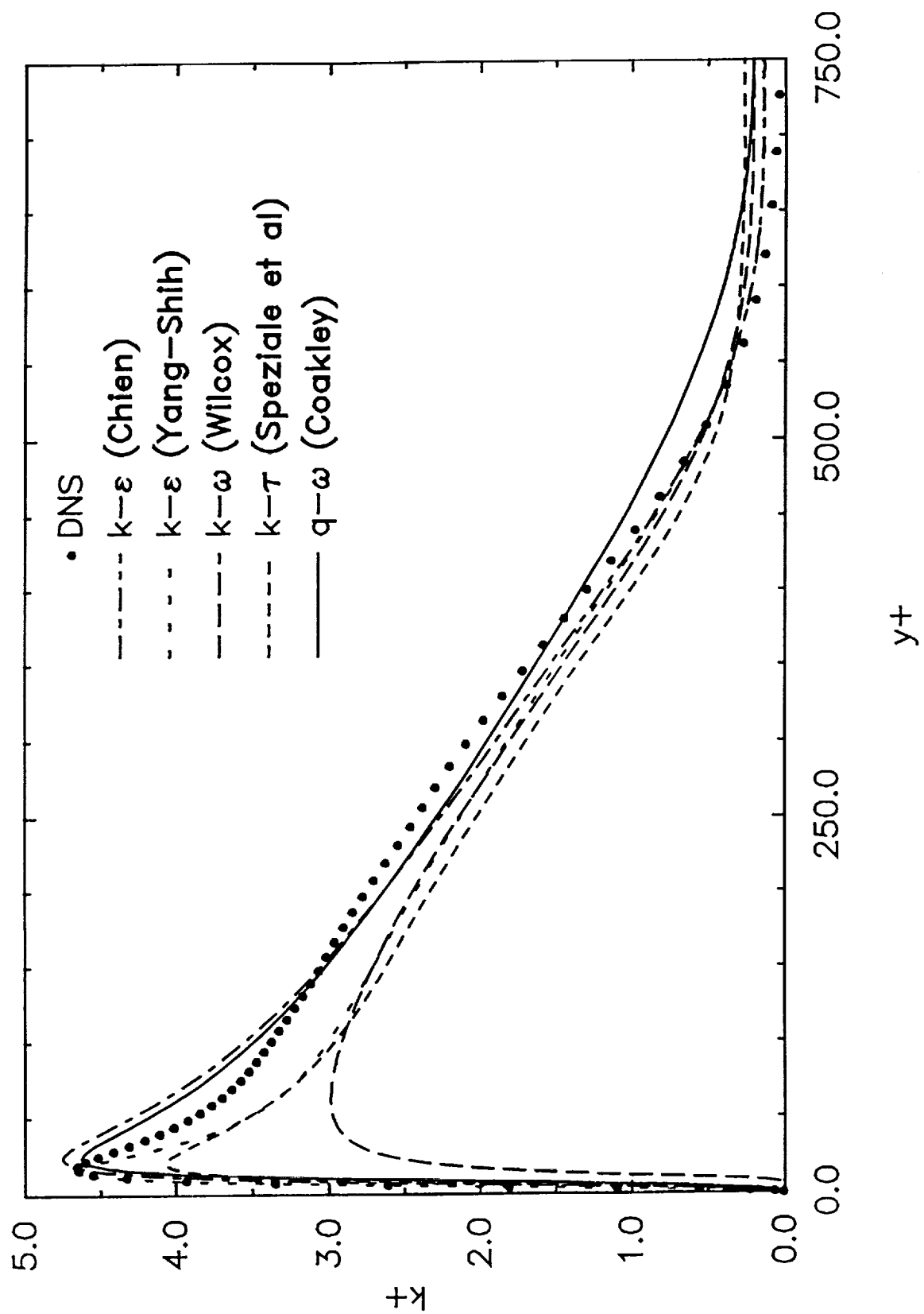
# FLAT PLATE BOUNDARY LAYER: $Re_\theta = 1410$



# FLAT PLATE BOUNDARY LAYER: $Re_\theta = 1410$

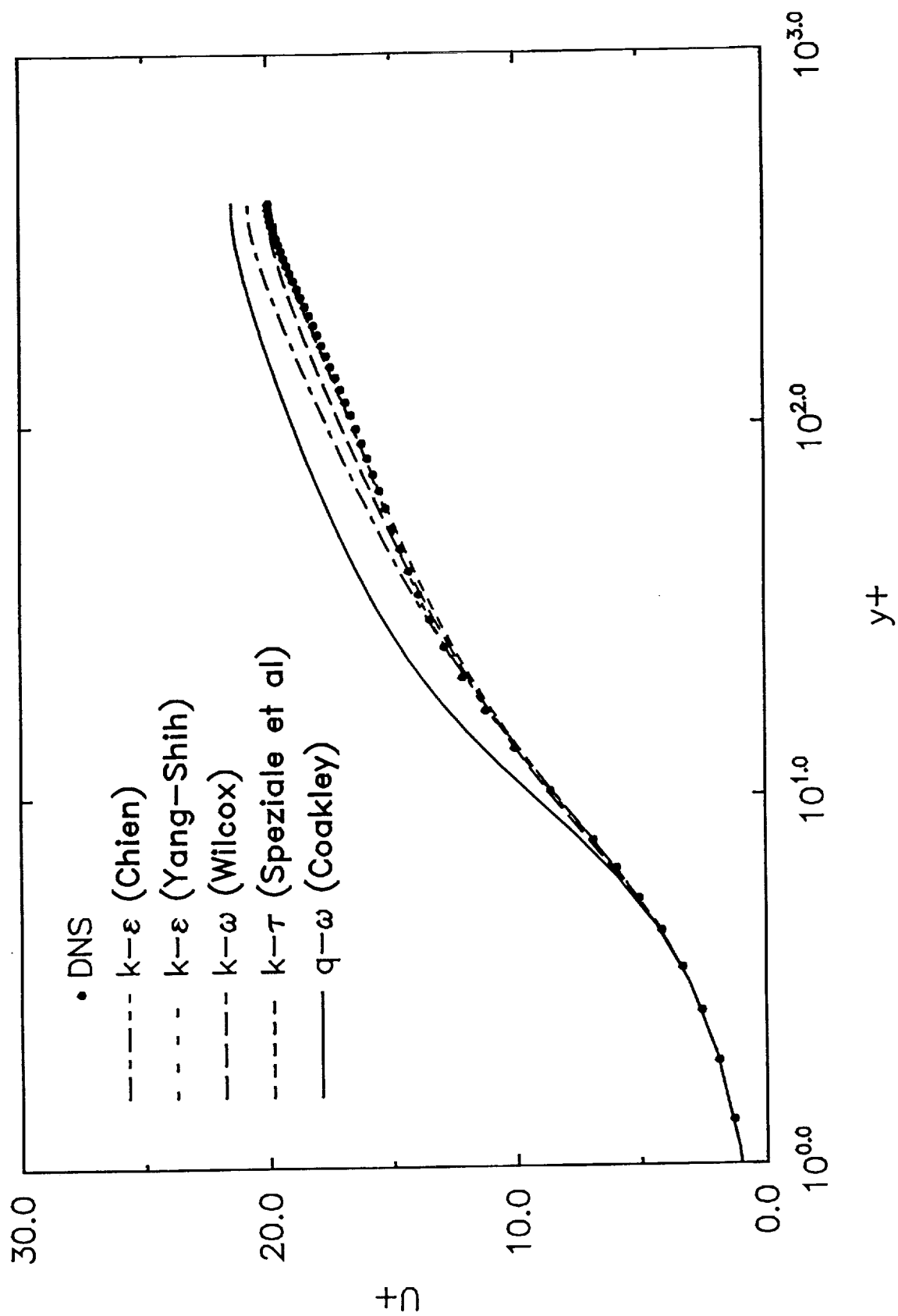


# FLAT PLATE BOUNDARY LAYER: $Re_\theta = 1410$

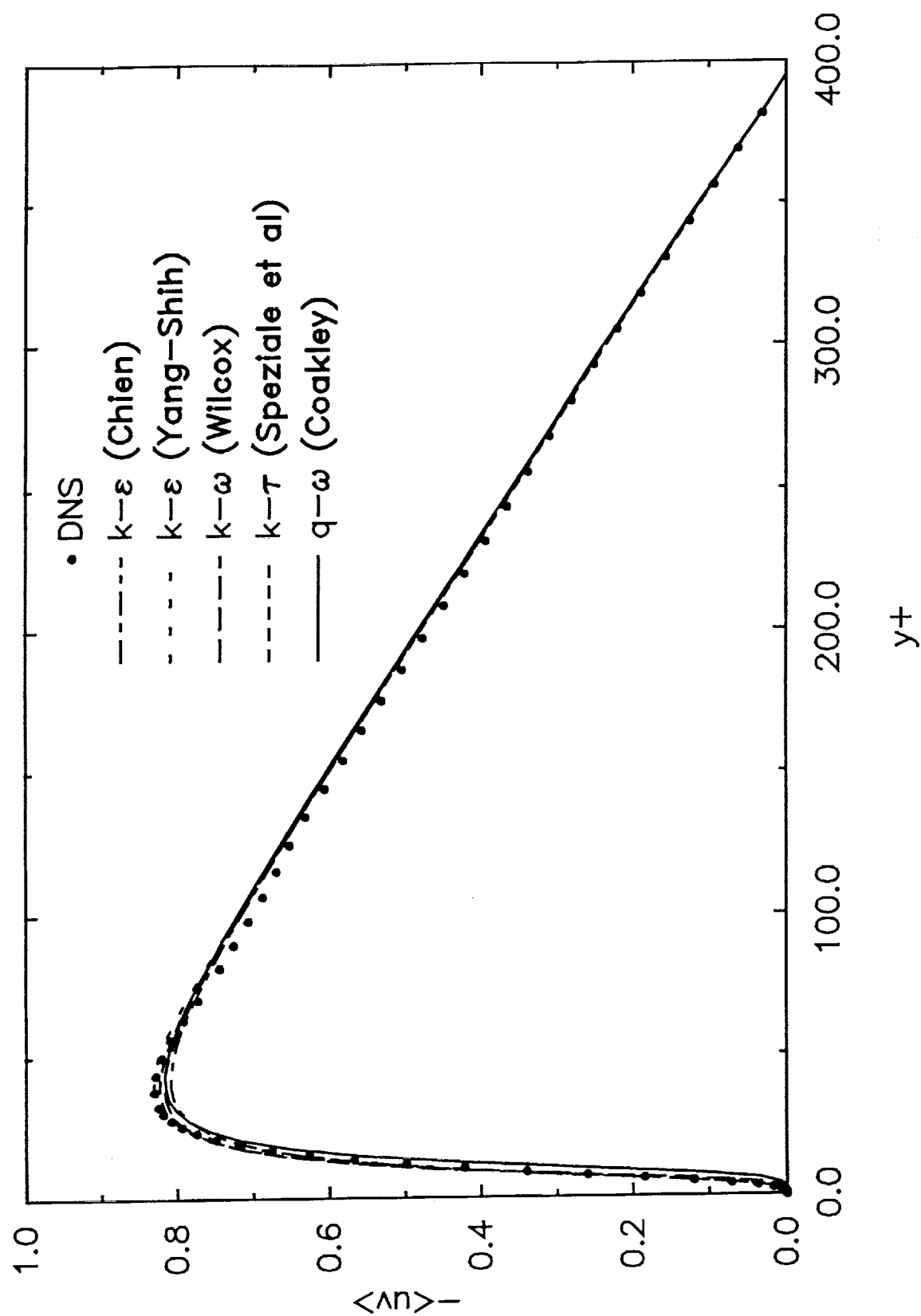




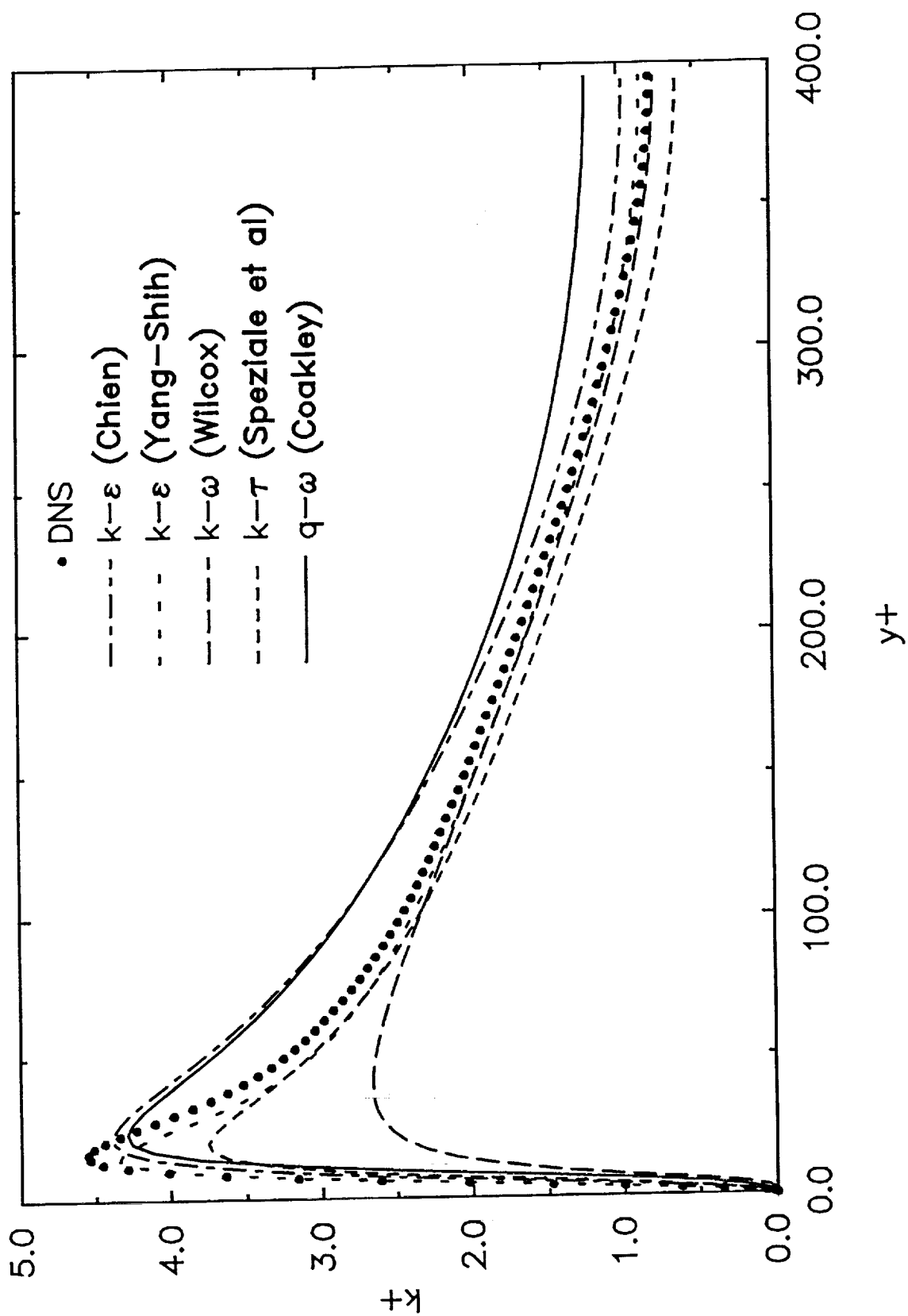
# CHANNEL FLOW: $Re_\tau = 395$



# CHANNEL FLOW: $Re_\tau = 395$



# CHANNEL FLOW: $Re_\tau = 395$



## SUMMARY

- STUDIED SEVERAL MODELS WITH SIMPLE FLOWS.
- COMPARED RESULTS WITH DNS.
- ALL MODELS PREDICTED GOOD TURBULENT SHEAR STRESSES.
- THEY DIFFERED IN COMPARISONS OF 'k' AND 'u'.
- A GOOD  $k-\epsilon$  MODEL IS AS GOOD AS ANY MODEL.
- A MODEL SHOULD BE ABLE TO HANDLE A SIMPLE FLOW IF IT WOULD HAVE A CHANCE FOR A MORE COMPLEX FLOW

## DISCUSSION

D.M. Bushnell

I would like to make a comment pertaining to the problem of numerical resolution and numerical fidelity. People are showing all kinds of results with various models without keeping track of how well they are doing numerically. There should be, at some stage, in turbulence modeling community some agreement of some ? calculation with some standard code and then stick with that kind of quality and fidelity the whole way through. Without this I am not sure what I am looking at quite honestly.

R. Mankabadi (to D. Wilcox)

I noticed Wilcox in his talk gave  $k - \epsilon$  an incomplete grade for the case of unsteady boundary layers. You may like to know that Howell (1980) and Ramaprian (1983) used  $k - \epsilon$  model to calculate unsteady boundary layer and found that  $k - \epsilon$  model could not predict this if the amplitude of oscillation or frequency is high.

D. Wilcox (to T.Chitsomboom)

I have a comment about the T. Chitsomboom's slide we just saw for  $Re = 1410$ . I have grid independent solutions for that so I'll be very suspect of these results.

T. Chitsomboom (reply)

I didn't mention, and you didn't either, that  $k - \omega$  model is quite sensitive to  $\omega$  boundary condition. Difficulty we encounter is how to specify  $\omega$  at the wall. Solutions we have shown are about in the middle range; we can get some better results than this and also some worse.

D. Wilcox (to T. Chitsomboom)

You have to be careful with the  $\omega$  boundary condition because if this is messed up then the boundary layer is all whacked up and I am sure that that is the case in this computation.

M.S. Anand (to D. Wilcox)

I like to preface my question by mentioning that we do pdf methods and for this purpose we need the time scale information. For this purpose we solve  $\omega$  equation (either a mean  $\omega$  equation or a stochastic  $\omega$  equation). In the limited calculations I have done, I have not noticed the sensitivity to free stream  $\omega$  you talked about. I have done calculations of single axial jets with or without co-flow; non-turbulent and very low turbulence co-flow. If there is sensitivity could you clarify what the sensitivity is due to and shouldn't there be sensitivity to  $k - \epsilon$  models too ?

D. Wilcox (reply)

No sensitivity in  $k - \epsilon$  because it's just not there. You can vary freestream dissipation all over the place and get the same answer. In  $k - \omega$  model I am cheating a little in one regard. I am always doing the similarity solutions and not marching. There are only two values of  $\omega$  which satisfy this. I vary these a little bit in calculations. Neither of these values gives good spreading rate but these more or less bracket it. You are not seeing the sensitivity because you are calculating the free stream  $\omega$ . If you change the initial value of  $\omega$  to start your calculation you'll see it.

M.S. Anand (to D. Wilcox)

That's what I am doing.

D. Wilcox (reply)

I'll look into numerics. I believe my similarity solutions are really good.

W.K. George

I would like to focus our attention away from the nitty gritty of solutions and focus on when should we expect eddy viscosity models to work. Tennekes and Lumely remind you that it was included in their book to show why eddy viscosity has problems. The fact that you have a local model, it can not handle separation or flow pass separation. Although Reynolds stress models are a straightforward increase in complexity but it's a quantum leap in adding physics.

B.E. Launder (to D. Wilcox and T. Gatski)

I would like to clarify a thing on Dave's and Tom's talk. Last time I saw the  $k - \omega$  model it seemed to me that the second equation had in it a supplementary source term. I see Dave nodding and Tom saying no.

D. Wilcox (to B.E. Launder)

There ain't none.

B.E. Launder (to D. Wilcox)

So it's cleaned up in the current marketed version. It is very interesting step. As I indicated in my talk if for separated flows one wants accurate results we need to use an extra source term in dissipation equation which will remove difficulty with adverse pressure gradients. It's not a problem of high Mach number. It is intrinsically a problem of separated flow; we get too big of a length scale near the wall and too high of heat transfer coefficient. And

if separation is provoked by a shock wave, the separated flow could be a zero Mach number or a seven Mach number. Survey shows when one shifted attention from adverse to favorable pressure gradients  $k - \omega$  did better in the adverse pressure gradient, and  $k - \epsilon$  predicted much better transition and re-laminarization. Tom Coakley said if you want to predict transition using  $k - \omega^2$  model you have to put  $Re$  effects which are absent.

D. Wilcox (to Spalart)

Several points were made that I should answer to. First, a complete model refers to terminology used by ?? a few years ago. It simply means you can use it with out knowing anything whatsoever about the flow, like an appropriate mixing length. Thus a two equation model is about as simple of a model as you can get.

P. Spalart

Why then isn't a one equation model complete?

D. Wilcox (to Spalart)

Because you still need to specify a length scale.

You also seemed alarmed that I was using Clauser's data to tune the  $k - \omega$  model. The perturbation method solution yields a similarity solution that demands that  $\beta_T$  be constant. You must compare with data where  $\beta_T$  is constant. Whether or not this is a limited data set, it is the only way that is formally consistent with the perturbation solution. When I go to a non-similar solution, the perturbation solution results are certainly bourn out. That is why I use Clauser's data. It's dictated by a mathematical necessity.



P. Spalart

How many of the moments of the turbulence satisfy this right kind of scaling?  
Scaling which is  $U_\tau$  equals a constant?

D. Wilcox

Well, I don't know but this shouldn't invalidate the analysis.

You were worried about whether the velocity profiles and the shear stress data came from the same calculation-absolutely! You're only talking about one data point out near the edge.

C. Speziale (to Spalart)

Are you saying that there is no destruction term if there's no wall? Essentially the destruction term disappears?

P. Spalart (reply)

Yes.

C. Speziale (to Spalart)

How then would you do in a non-equilibrium shear flow? According to this, the only thing that the eddy viscosity term can do is grow. I was thinking specifically of a situation where diffusion effects are small.

P. Spalart (reply)

It wouldn't work there.



57-34  
210888  
p. 44

N92-24521

The Present Status  
and Future Direction of  
Algebraic Reynolds Stress Models

Workshop on Engineering Turbulence Modeling  
ICOMP, NASA Lewis Research Center  
August 21-22, 1991

Dale B. Taulbee  
University at Buffalo

## OUTLINE

- The need for algebraic stress models  
– deficiencies of the linear gradient model.
- Classical algebraic Reynolds stress models.
- Nonlinear stress-strain relations.
- Critique of the models and some new developments.

## LINEAR GRADIENT MODEL

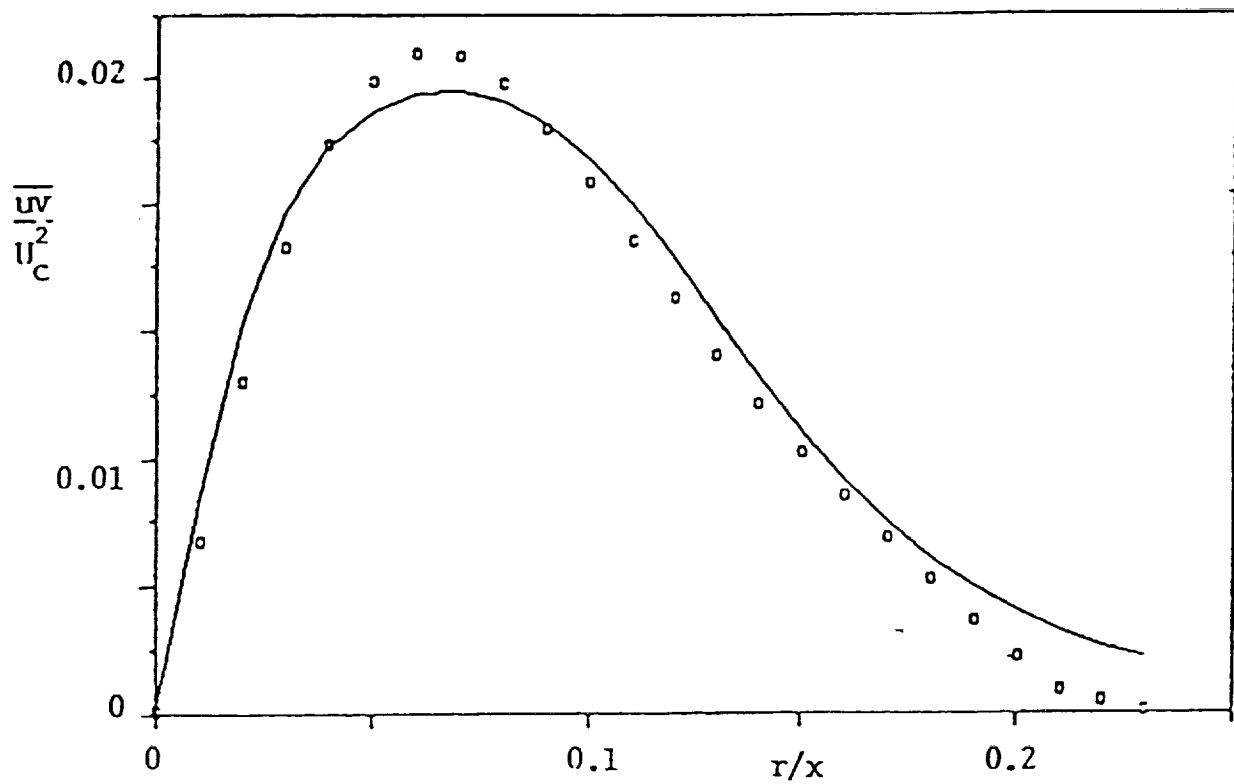
$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} - 2\nu_t S_{ij}$$

$$S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$

$$\nu_t = C_\nu k l = C_\mu k^2 / \epsilon$$

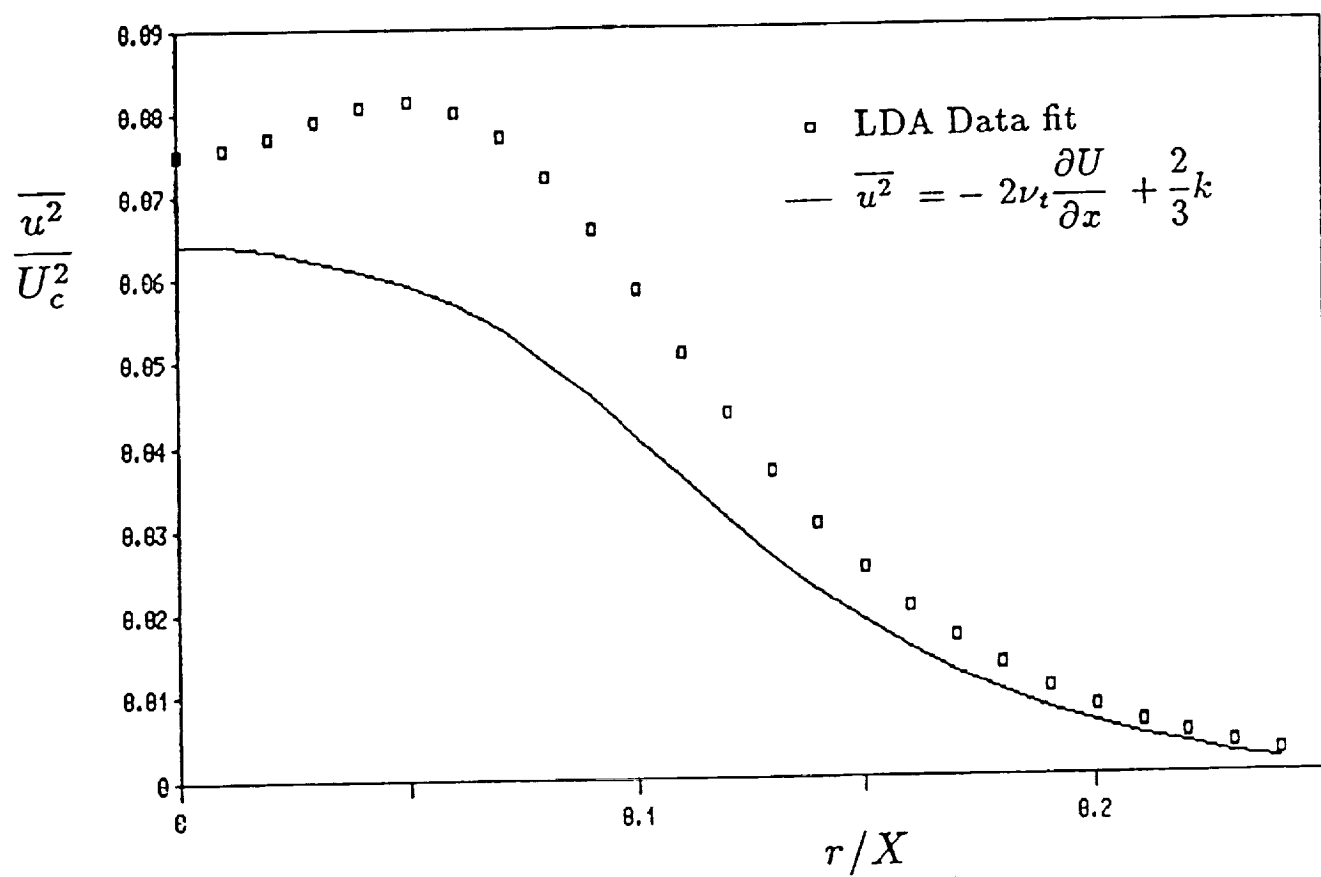
$$C_\mu = 0.09$$

- Works reasonably well for near-parallel shear flows, however,  $C_\mu$  is not a constant.
- Poor representation of the normal stresses and, hence, does not work well for many multidimensional flows.



Shear Stress in the Axisymmetric Jet  
 □ LDA Data Fit, — Gradient Hypothesis

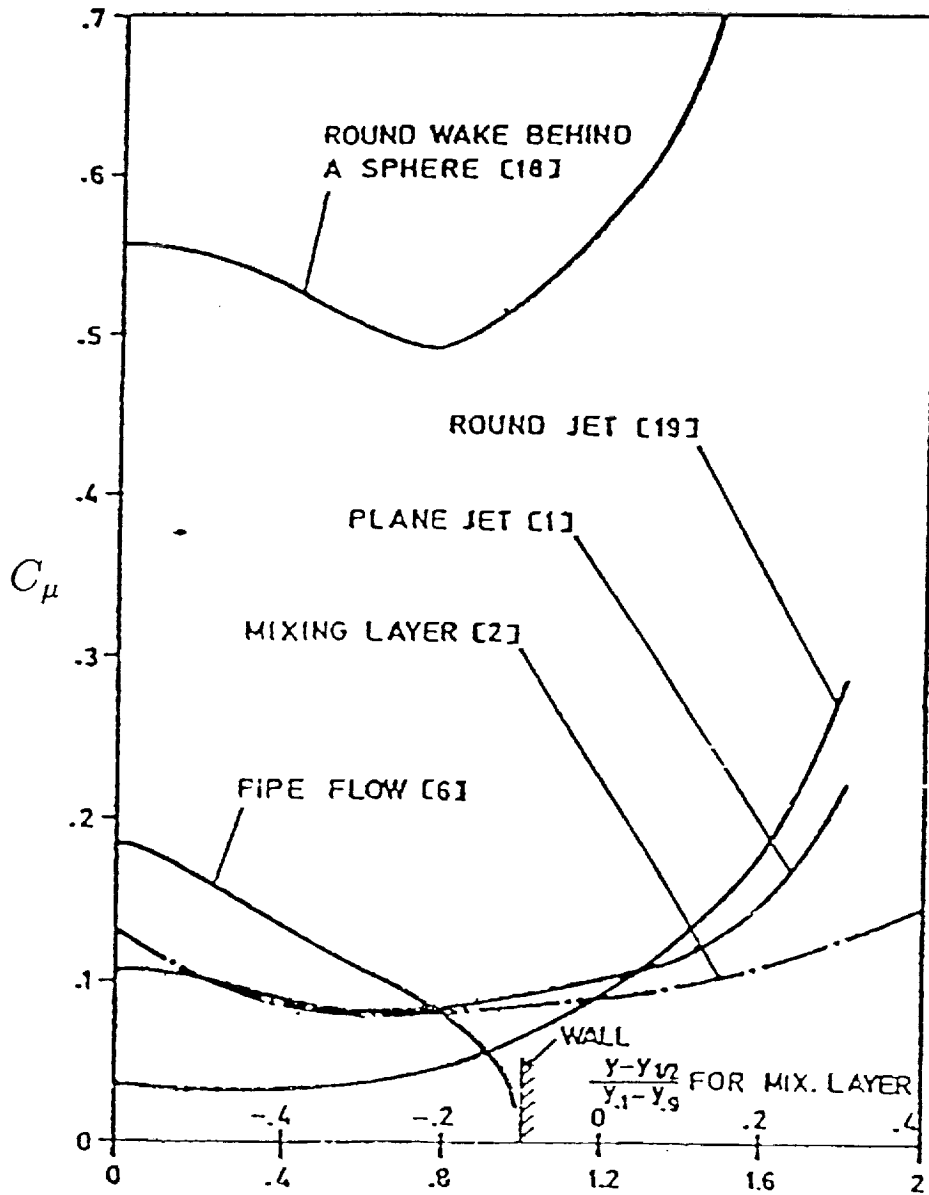
# Normal Reynolds Stress - Round Jet



# VARIATION OF $C_\mu$

MODEL PARAMETERS SHOULD NOT BE TAKEN AS UNIVERSAL CONSTANTS

$$\nu_t = C_\mu \frac{k^2}{\epsilon} = -\frac{\overline{uv}}{\partial U / \partial y}$$



$y/a$  FOR PIPE .  $y/y_{1/2}$  FOR JETS AND WAKES

RODI (1975)



# DEFICIENCIES OF LINEAR GRADIENT MODEL FOR $\overline{u_i u_j}$

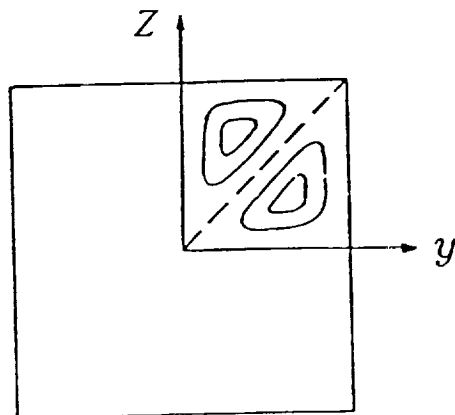
- Poor Representation of Normal Stresses

- Homogeneous Shear Flow,  $U = U(y)$

$$\overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{2}{3}k$$

$$\overline{uv} = -\nu_t \frac{dU}{dy}$$

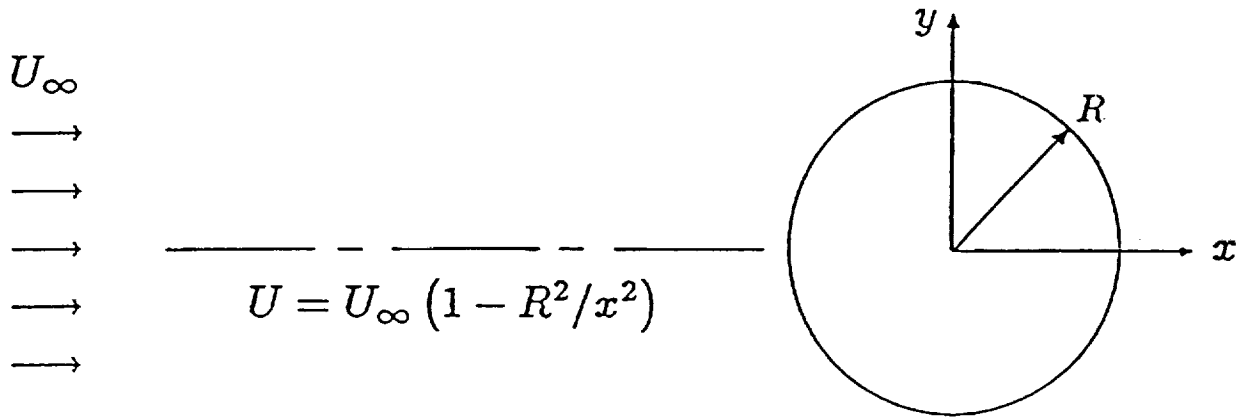
- Secondary Flow in a Non-Circular Duct



Driven by  $(\overline{w^2} - \overline{v^2})$

- Linear Gradient Model :  $\overline{v^2} = \overline{w^2} = 2k/3$

# STAGNATION STREAMLINE TURBULENCE



$$U \frac{dk}{dx} = P - \epsilon$$

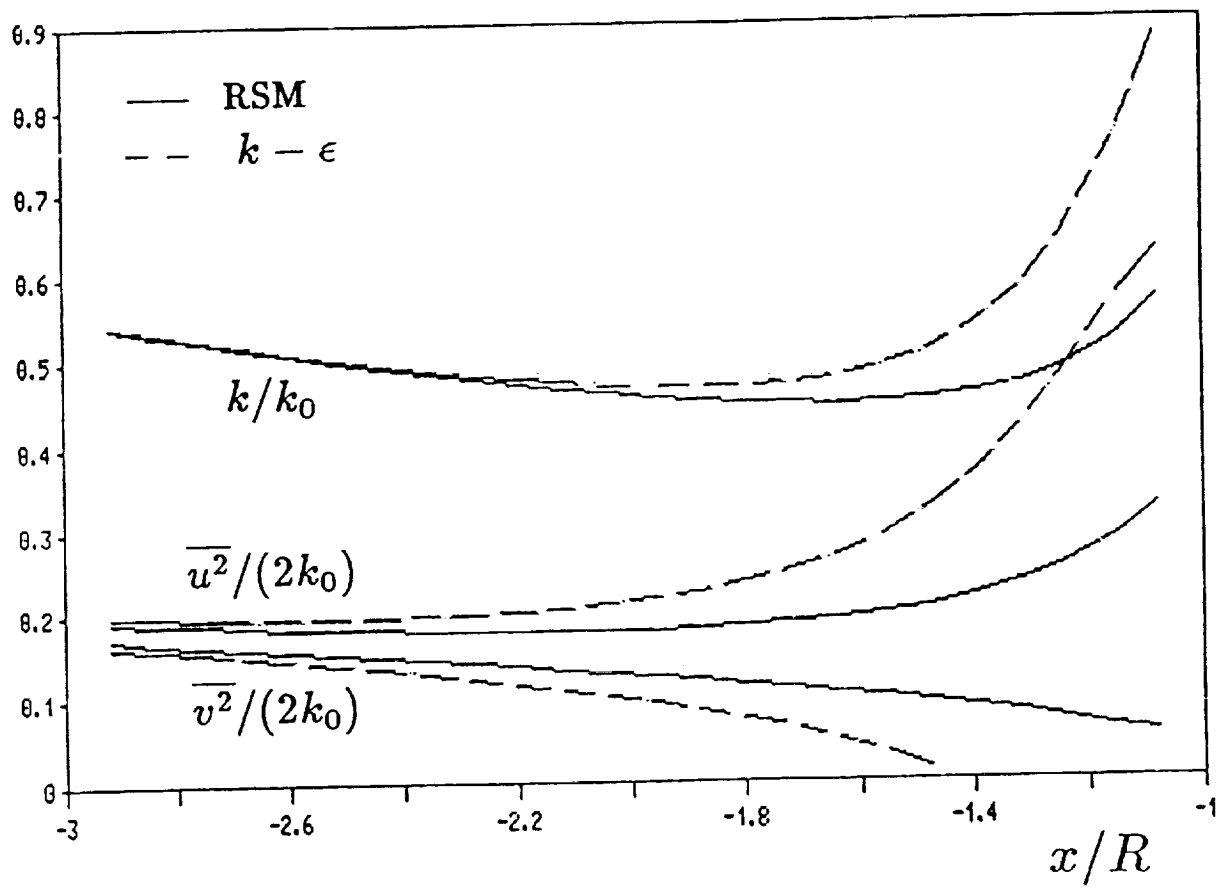
$$U \frac{d\epsilon}{dx} = \frac{\epsilon}{k} (C_{\epsilon_1} P - C_{\epsilon_2} \epsilon)$$

$$\text{where : } P = -\overline{u^2} \frac{\partial U}{\partial x} - \overline{v^2} \frac{\partial V}{\partial y} = -(\overline{u^2} - \overline{v^2}) \frac{\partial U}{\partial x}$$

$$k - \epsilon \text{ model : } \overline{u^2} = 2k/3 - \nu_t \frac{\partial u}{\partial x}$$

$$\overline{v^2} = 2k/3 - \nu_t \frac{\partial v}{\partial y}$$

$$P = \nu_t \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) \frac{\partial u}{\partial x} = 2\nu_t \left( \frac{\partial u}{\partial x} \right)^2$$



Stagnation Streamline Flow

## ALGEBRAIC REYNOLDS STRESS MODELS

$$f(\overline{u_i u_j}, k, \epsilon, S_{ij}) = 0$$

More accurately describes the anisotropy of the turbulence.

Account for the effects of :

Longitudinal surface curvature

Corner Geometry

Swirl

Buoyancy

Rotation

Maintains the simplicity of a two-equation ( $k - \epsilon$ ) model calculation

Basic Assumption = Local Formulation :

Reynolds stress depends only on local conditions – turbulence is anisotropic only if maintained by velocity gradient.

$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} \quad \text{if } S_{ij} = 0$$

# DERIVATION OF ALGEBRAIC STRESS MODELS

## Classical ASM

- Derived from modeled Reynolds stress equation

## Nonlinear stress-strain models

- Derived from :
  - Two-point closure theories
  - Continuum mechanics
  - Expansion of classical ASM formulation

## CLASSICAL ASM

Algebraic Stress Model formulated from Modeled Reynolds Stress Equation (Rodi, 1972)

$$\frac{D\overline{u_i u_j}}{Dt} = -\frac{\partial T_{ijk}}{\partial x_l} + P_{ij} + \Phi_{ij} + \epsilon_{ij}$$

- 1) Convection and Diffusion Neglected
- 2) Convection – Diffusion proportioned to that of the kinetic energy equation

$$\begin{aligned}\frac{D\overline{u_i u_j}}{Dt} - \frac{\partial T_{ijl}}{\partial x_l} &= \frac{\overline{u_i u_j}}{k} \left( \frac{Dk}{Dt} - \frac{\partial T_l}{\partial x_l} \right) \\ &= \frac{\overline{u_i u_j}}{k} (P - \epsilon)\end{aligned}$$

$$\frac{\overline{u_i u_j}}{k} (P - \epsilon) = P_{ij} + \Phi_{ij} + \epsilon_{ij}$$

## Algebraic Stress Model Continued

$$\frac{D\overline{u_i u_j}}{Dt} = \frac{\partial T_{ijl}}{\partial x_l} + P_{ij} + \Phi_{ij} + \epsilon_{ij}$$

$$\frac{\overline{u_i u_j}}{k} (P - \epsilon) = P_{ij} + \Phi_{ij} + \epsilon_{ij}$$

Rodi (1976)

$$\Phi_{ij} = -C_1 \frac{\epsilon}{k} \left( \overline{u_i u_j} - \frac{2}{3} k \delta_{ij} \right) - C_2 \left( P_{ij} - \frac{2}{3} P \delta_{ij} \right)$$

$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} + \frac{1 - C_2}{C_1} \frac{P_{ij}/\epsilon - \frac{2}{3} P/\epsilon \delta_{ij}}{1 + \frac{1}{C_1} (P/\epsilon - 1)} k$$

Shear Flow

$$\overline{u^2} = \frac{2}{3} k + \frac{4}{3} (1 - C_2) \frac{C_\mu}{P/\epsilon + C_1/2 - 1} \frac{k^3}{\epsilon^2} \left( \frac{\partial U}{\partial y} \right)^2$$

$$\overline{v^2} = \frac{2}{3} k - \frac{2}{3} (1 - C_2) \frac{C_\mu}{P/\epsilon + C_1/2 - 1} \frac{k^3}{\epsilon^2} \left( \frac{\partial U}{\partial y} \right)^2$$

$$\overline{w^2} = \frac{2}{3} k - \frac{2}{3} (1 - C_2) \frac{C_\mu}{P/\epsilon + C_1/2 - 1} \frac{k^3}{\epsilon^2} \left( \frac{\partial U}{\partial y} \right)^2$$

$$-\overline{uv} = C_\mu \frac{k^2}{\epsilon} \frac{\partial U}{\partial y}$$

$$C_\mu = \frac{2}{3} (1 - C_2) \frac{C_1/2 - 1 + C_2 P/\epsilon}{(P/\epsilon + C_1/2 - 1)^2}$$

## ALGEBRAIC STRESS MODEL CONTINUED

$$\begin{aligned}\Phi_{ij} = & -C_1 \frac{\epsilon}{k} (\overline{u_i u_j} - \frac{2}{3} k \delta_{ij}) - C_2 (P_{ij} - \frac{2}{3} P \delta_{ij}) \\ & - C_3 (D_{ij} - \frac{2}{3} P \delta_{ij}) - C_4 k \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)\end{aligned}$$

$$C_2 = \frac{C'_2 + 8}{11} \quad , \quad C_3 = \frac{8C'_2 - 2}{11} \quad , \quad C_4 = \frac{30C'_2 - 2}{55}$$

Shear Flow

Pope (1975)

$$\overline{u^2} = \frac{2}{3} k - \frac{2}{3} \frac{4C}{P/\epsilon + C_1/2 - 1} C_\mu \frac{k^3}{\epsilon^2} \left( \frac{\partial U}{\partial y} \right)^2$$

$$\overline{v^2} = \frac{2}{3} k + \frac{2}{3} \frac{10C + 1}{P/\epsilon + C_1/2 - 1} C_\mu \frac{k^3}{\epsilon^2} \left( \frac{\partial U}{\partial y} \right)^2$$

$$\overline{w^2} = \frac{2}{3} k - \frac{2}{3} \frac{6C + 1}{P/\epsilon + C_1/2 - 1} C_\mu \frac{k^3}{\epsilon^2} \left( \frac{\partial U}{\partial y} \right)^2$$

$$\overline{uv} = -C_\mu \frac{k^2}{\epsilon} \frac{\partial U}{\partial y}$$

$$C_\mu = \frac{C_1/2 - 1 - \frac{5}{6} (11C^2 - 4C - 1) P/\epsilon}{(P/\epsilon + C_1/2 - 1)^2}$$

$$C = -\frac{1}{11} \left( 1 + \frac{3}{2} C'_2 \right)$$



# ALGEBRAIC STRESS MODEL

Comparison with data for Homogeneous Shear Flow

Data from Harris, Graham, and Corrsin (1977)

Neglect Diffusion :

$$\frac{D \overline{u_i u_j}}{Dt} = \frac{\overline{u_i u_j}}{k} \frac{Dk}{Dt}$$

	$U \frac{d\overline{u_i u_j}}{dx}$	$\frac{\overline{u_i u_j}}{k} U \frac{dk}{dx}$
$\overline{u^2}$	1.96	2.09
$\overline{v^2}$	0.88	0.83
$\overline{w^2}$	1.32	1.24
$\overline{uv}$	0.62	0.62

Stress Values :

	$C_1 = 1.8$	$C_1 = 1.5$	$C_1 = 1.8$	
	$C_2 = 0.6$	$C'_2 = 0.4$	$C'_2 = 0.54$	Exp.
$\overline{u^2}/k$	1.195	1.050	0.965	1.004
$\overline{v^2}/k$	0.402	0.366	0.372	0.398
$\overline{w^2}/k$	0.402	0.580	0.662	0.598
$-\overline{uv}/k$	0.356	0.393	0.305	0.298
$C_\mu$	0.068	0.075	0.058	0.057

## CONVECTION-DIFFUSION ASSUMPTION

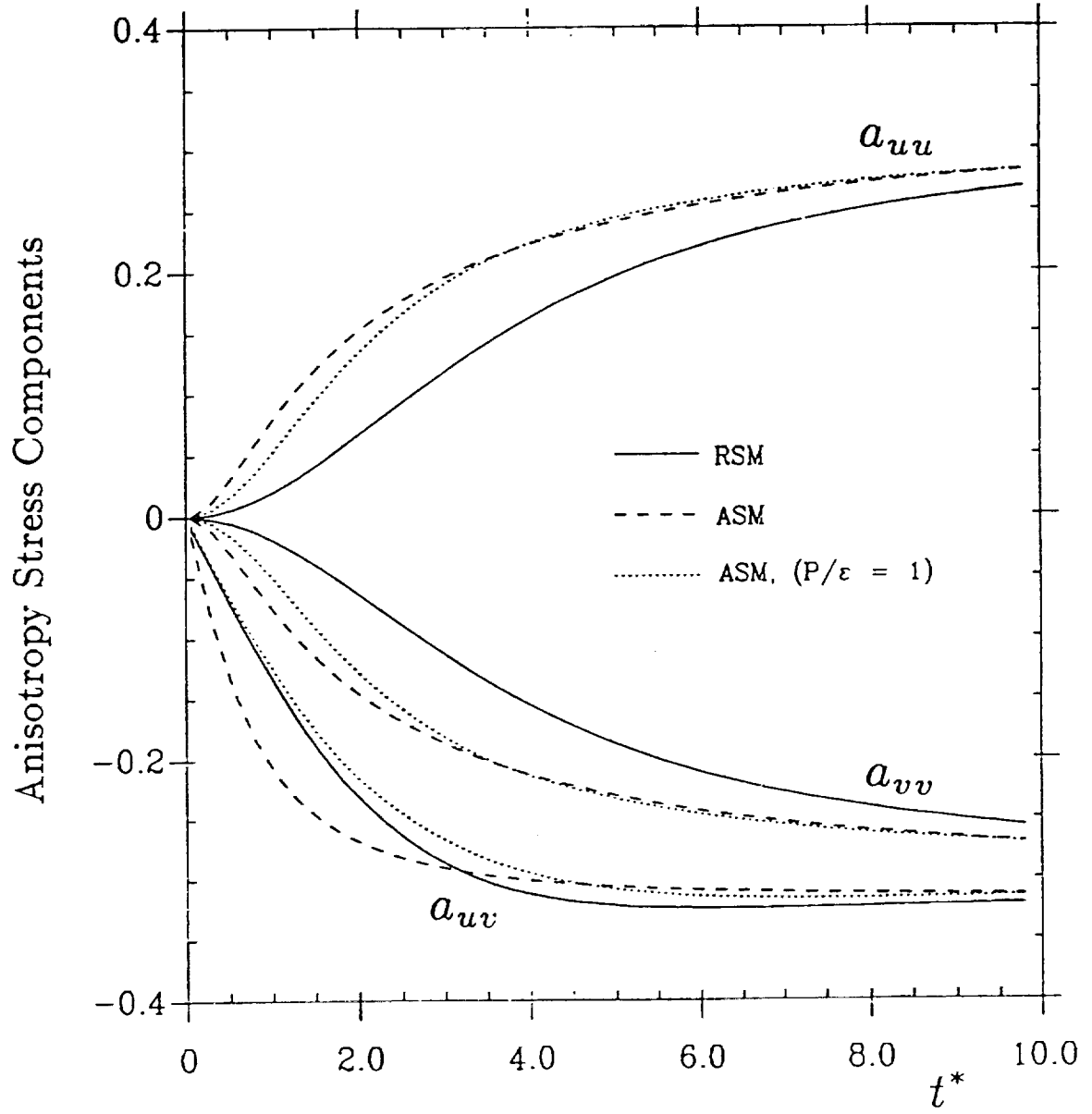
$$\begin{aligned} \frac{D\overline{u_i u_j}}{Dt} + \frac{\partial T_{ijk}}{\partial x_l} = & k \frac{D a_{ij}}{Dt} + \frac{\partial T_{ijl}}{\partial x_l} - \frac{\overline{u_i u_j}}{k} \frac{\partial T_l}{\partial x_l} \\ & + \frac{\overline{u_i u_j}}{k} (p - \epsilon) \end{aligned}$$

where :  $a_{ij} = \frac{\overline{u_i u_j}}{k} - \frac{2}{3} \delta_{ij}$

$$1) \quad \frac{\partial T_{ijk}}{\partial x_l} - \frac{\overline{u_i u_j}}{k} \frac{\partial T_l}{\partial x_l} \approx 0$$

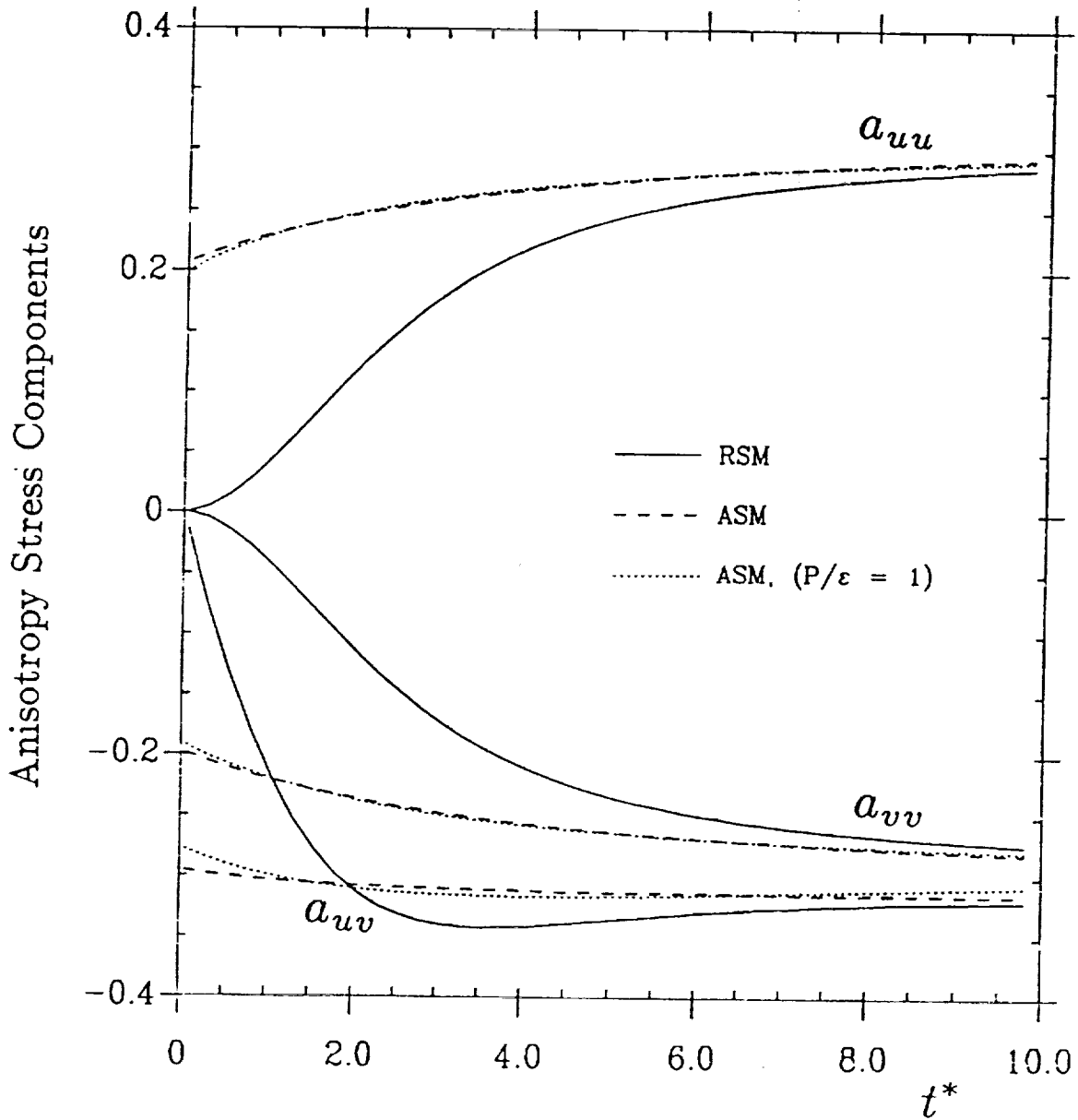
$$2) \quad \frac{D a_{ij}}{Dt} = 0$$

Then the ASM represents the asymptotic solution of the modeled Reynolds stress equation. However,  $\tau = k/\epsilon$  is the local flow value as determined from the  $k$  and  $\epsilon$  transport equation.



ASM Prediction for Homogeneous Shear Flow :

$$a_{ij0} = 0 \quad ; \quad \left( \frac{k}{\epsilon} \frac{dU}{dy} \right)_0 = 0$$



ASM Prediction for Homogeneous Shear Flow :

$$a_{ij0} = 0 \quad , \quad \left( \frac{k}{\epsilon} \frac{dU}{dy} \right)_0 = 2.5$$

## SOLUTION WITH ASM FOR MULTIDIMENSIONAL PROBLEMS

- Solve set of nonlinear algebraic equations as part of the solution – causes numerical problems.
- Simplify the ASM formulation
  - $P/\epsilon = 1$
  - Neglect certain terms
  - Neglect certain gradient terms for specific problems
- Expand into an explicit nonlinear stress-strain form (NLM)

$$\frac{\overline{u_i u_j}}{k} = f ( \tau S_{ij} , \tau^2 S_{ij}^2 , \dots )$$

# EXPLICIT STRESS-STRAIN RELATION FROM ALGEBRAIC STRESS MODEL

$$\frac{\overline{u_i u_j}}{k} (P - \epsilon) = P_{ij} + \Phi_{ij} - \frac{2}{3} \delta_{ij}$$

$$\Phi_{ij} = -C_1 \frac{\epsilon}{k} (\overline{u_i u_j} - \frac{2}{3} k \delta_{ij}) - C_2 (P_{ij} - \frac{2}{3} P \delta_{ij}) - \dots$$

$$\frac{P}{\epsilon} = -\frac{\overline{u_i u_j}}{\epsilon} \frac{\partial U_i}{\partial x_j} = -\frac{\overline{u_i u_j}}{k} \frac{k}{\epsilon} \frac{\partial U_i}{\partial x_j}$$

$$\text{Expansion : } \frac{\overline{u_i u_j}}{k} = \frac{2}{3} \delta_{ij} + a_{ij}^{(1)} \tau + a_{ij}^{(2)} \tau^2 + \dots$$

Ahmadi (1988)

Rubinstein & Barten (1990)

Taulbee (1989)

Horiuti (1990)

$$\frac{\overline{u_i u_j}}{k} = \frac{2}{3} \delta_{ij} - 2C_\mu \frac{k}{\epsilon} S_{ij}$$

$$+ \alpha \frac{k^2}{\epsilon^2} \left[ S_{ij} \frac{\partial U_j}{\partial x_l} + S_{jl} \frac{\partial U_i}{\partial x_l} - \frac{2}{3} S_{lm} \frac{\partial U_l}{\partial x_m} \delta_{ij} \right]$$

$$- \beta \frac{k^2}{\epsilon^2} \left[ S_{il} \frac{\partial U_l}{\partial x_j} + S_{jl} \frac{\partial U_l}{\partial x_i} - \frac{2}{3} S_{lm} \frac{\partial U_l}{\partial x_m} \delta_{ij} \right]$$

$\alpha$  and  $\beta$  are directly determined from the constants of the RSM. However, if only terms through second order are retained, the expanded formulation does not accurately represent the original ASM.

## NONLINEAR STRESS-STRESS MODELS

General form (Speziale, 1991)

$$\begin{aligned} a_{ij} = & -2 C_{\mu} \tau S_{ij} \\ & -4 \alpha_1 \tau^2 (S_{il} S_{lj} - \frac{1}{3} S^2 \delta_{ij}) \\ & -4 \alpha_2 \tau^2 (S_{il} \Omega_{lj} + S_{jl} \Omega_{li}) \\ & -4 \alpha_3 \tau^2 (\Omega_{il} \Omega_{lj} - \frac{1}{3} \Omega^2 \delta_{ij}) \\ & -2 \alpha_4 \tau^2 D S_{ij} / Dt \end{aligned}$$

$$\tau = \text{time scale } (\tau = k/\epsilon)$$

Lumley (1969)

Saffman (1974)

Yoshizawa (1984) DIA two-point closure theory

Speziale (1987) continuum mechanics

Ahmadi (1988) expansion of ASM

Rubinstein & Barten (1990) RNG theory

## NLM COEFFICIENT VALUES

	$C_\mu$	$\alpha_1$	$\alpha_2$	$\alpha_3$
Nisiziwa/Yoshizawa (Channel Flow)	0.090	0.0709	0.0159	0.0960
Speziale (Channel Flow)	0.090	-0.0138	0.0138	0
Rubinstein & Barten (RNG Theory)	0.0845	-0.0570	0.0120	-0.047
Rubinstein & Barten (ASM Expansion)	0.090	-0.0523	0.0198	0
Comparison with RSM	0.090	-0.0015	0.012	0

Homogeneous shear flow :

$$C_\mu = -a_{12}/(\tau \frac{dU}{dy})$$

$$\alpha_1 - \alpha_3 = -\frac{3}{2}(a_{11} + a_{22})/(\tau \frac{dU}{dy})^2$$

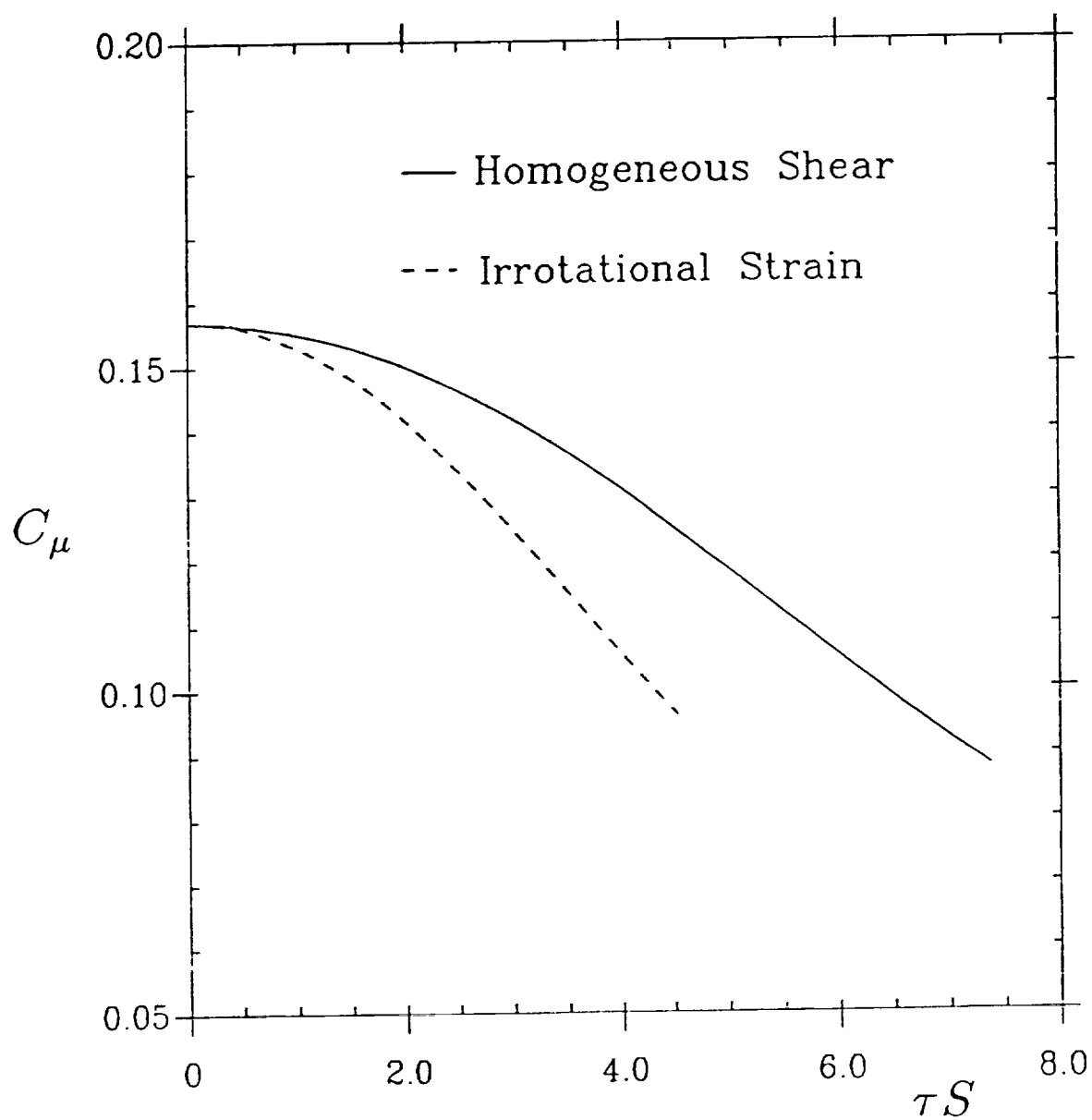
$$\alpha_2 = -\frac{1}{4}(a_{11} - a_{22})/(\tau \frac{dU}{dy})^2$$

Irrotational strain flow :

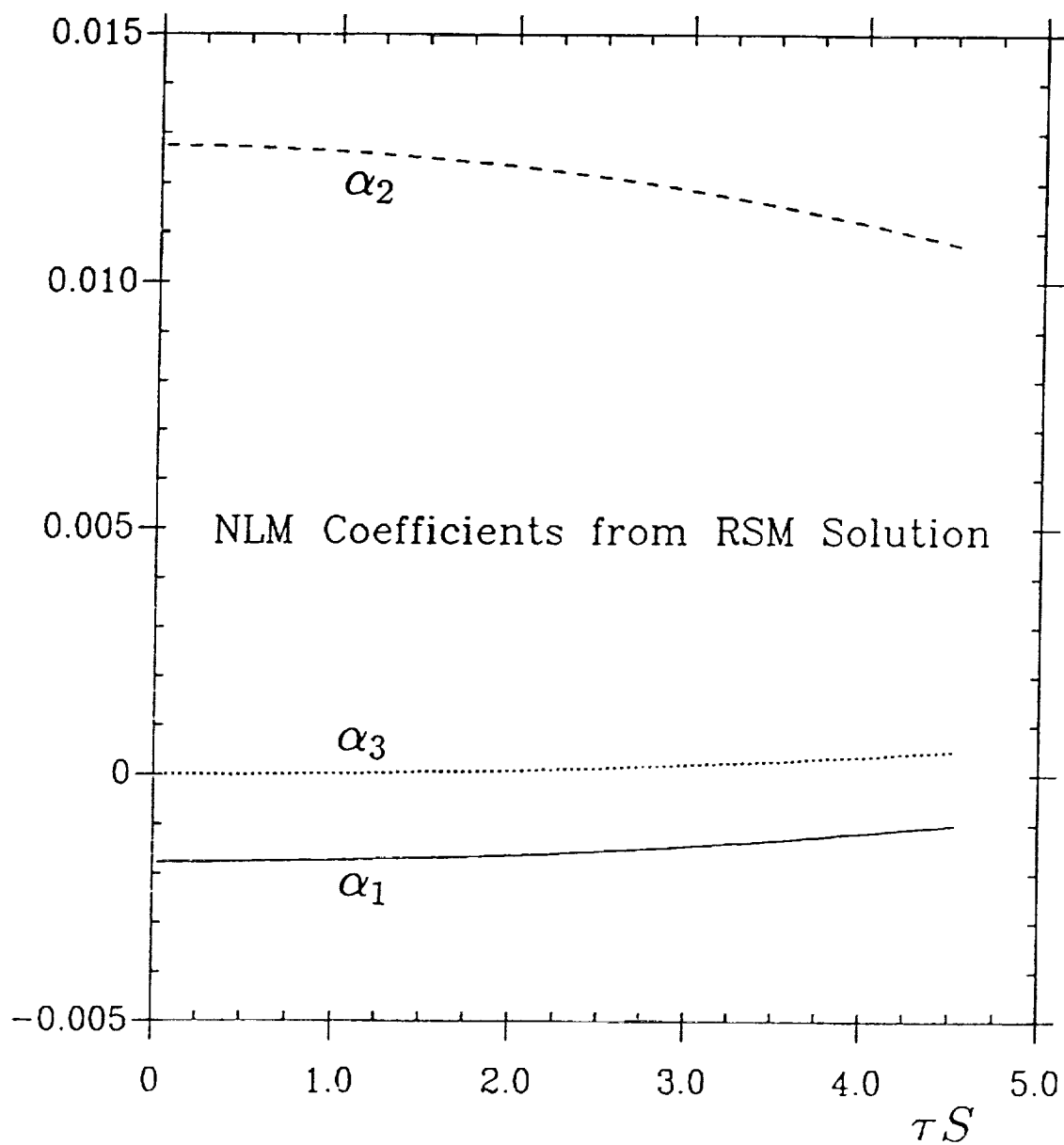
$$C_\mu = \frac{1}{4}(a_{22} - a_{11})/(\tau \frac{dU}{dx})$$

$$\alpha_1 = -\frac{3}{8}(a_{11} + a_{22})/(\tau \frac{dU}{dx})^2$$

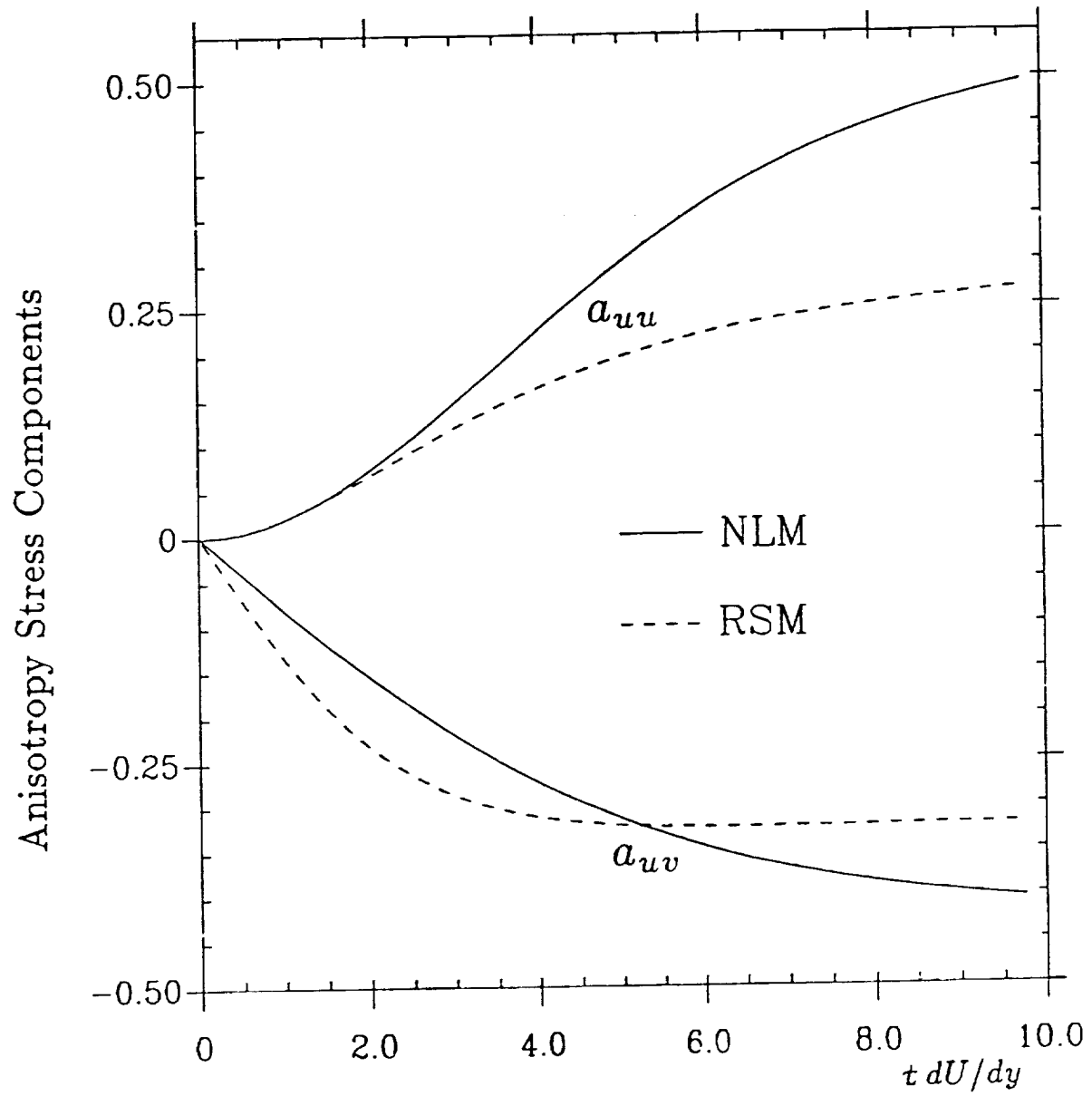




Stress-Strain Model Constants from RSM Solution

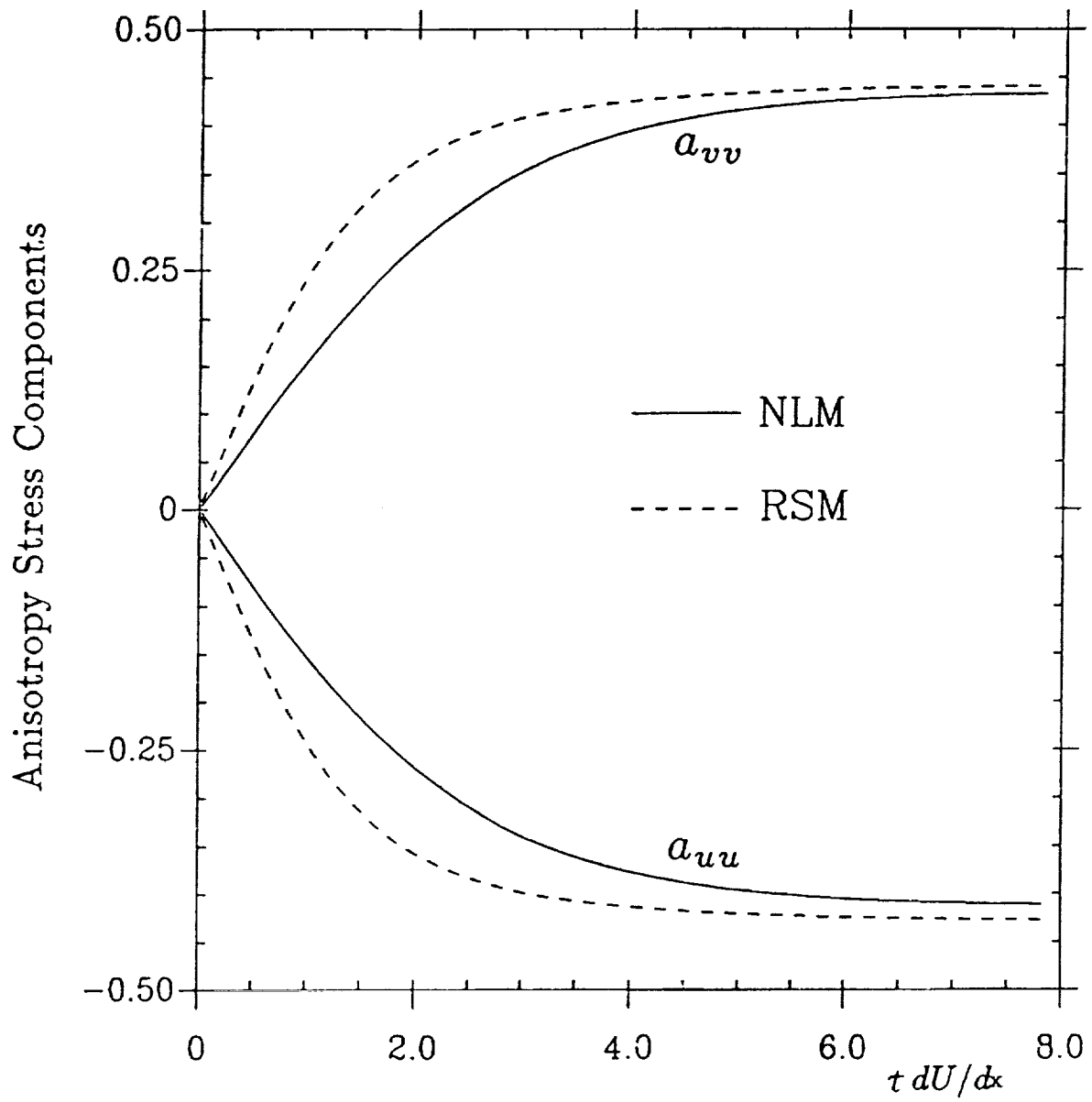


Stress-Strain Model Constants from RSM Solution



Nonlinear Stress-Strain Model Solution for  
Homogeneous Shear Flow :

$$C_\mu = 0.09, \quad \alpha_1 = -0.0015, \quad \alpha_2 = 0.012, \quad \alpha_3 = 0$$



Nonlinear Stress-Strain Model Solution for  
Irrotational Strain Flow :

$$C_\mu = 0.09, \quad \alpha_1 = -0.0015, \quad \alpha_2 = 0.012, \quad \alpha_3 = 0$$

# STRESS-STRAIN RELATION FROM EXPANSION OF REYNOLDS STRESS MODEL

Reynolds Stress Equation :

$$\frac{D\overline{u_i u_j}}{Dt} = -\frac{\partial T_{ijk}}{\partial x_l} + P_{ij} + \Phi_{ij} - \frac{2}{3}\epsilon \delta_{ij}$$

$$\begin{aligned} \Phi_{ij} = & -C_1 \frac{\epsilon}{k} \left( \overline{u_i u_j} - \frac{2}{3}k \delta_{ij} \right) - \alpha \left( P_{ij} - \frac{2}{3}P \delta_{ij} \right) \\ & - \beta \left( P_{ij} - \frac{2}{3}P \delta_{ij} \right) - 2\gamma k S_{ij} \end{aligned}$$

$$\alpha = \frac{C_2 + 8}{11} \quad , \quad \beta = \frac{8C_2 - 2}{11} \quad , \quad \gamma = \frac{30C_2 - 2}{55}$$

Dissipation equation :

$$\frac{D\epsilon}{Dt} = -\frac{\partial \overline{u_l \epsilon'}}{\partial x_l} + C_{\epsilon_1} \frac{k}{\epsilon} P - C_{\epsilon_2} \frac{k^2}{\epsilon}$$

## EXPANSION Continued

$$a_{ij} = \frac{\overline{u_i u_j}}{k} - \frac{2}{3} \delta_{ij} \quad \text{and} \quad \tau = \frac{k}{\epsilon}$$

$$\begin{aligned} \dot{a}_{ij} = & -\frac{1}{k} \left[ \frac{\partial T_{ijl}}{\partial x_l} - \left( a_{ij} + \frac{2}{3} \delta_{ij} \right) \frac{\partial T_l}{\partial x_l} \right] \\ & - \frac{P}{\epsilon} \frac{a_{ij}}{\tau} - (C_1 - 1) \frac{a_{ij}}{\tau} - \frac{8}{15} S_{ij} \\ & - (1 - \alpha - \beta) (a_{il} S_{lj} + a_{jl} S_{li} - \frac{2}{3} a_{kl} S_{lk} \delta_{ij}) \\ & - (1 - \alpha + \beta) (a_{il} \Omega_{lj} + a_{jl} \Omega_{li}) \end{aligned}$$

$$\dot{\tau} = -\tau \left[ \frac{1}{k} \frac{\partial \overline{u_l k'}}{\partial x_l} - \frac{1}{\epsilon} \frac{\partial \overline{u_l \epsilon'}}{\partial x_l} \right] + (C_{\epsilon_2} - 1) - (C_{\epsilon_1} - 1) \frac{P}{\epsilon}$$

Expansion :  $a_{ij} = a_{ij}^{(1)} \tau + a_{ij}^{(2)} \tau^2 + \dots$

Requires  $\tau (S_{kl} S_{lk})^{1/2}$  be small

Transport terms lead to small second order contributions and are neglected.

## EXPANSION Continued

$$\dot{a}_{ij} = a_{ij}^{(1)} \dot{\tau} + \dot{a}_{ij}^{(1)} \tau + 2a_{ij}^{(2)} \tau \dot{\tau} + \dots$$

$$\text{where } \dot{\tau} = (C_{\epsilon_2} - 1) - (C_{\epsilon_1} - 1) \frac{P}{\epsilon}$$

$$\begin{aligned} a_{ij} = & -2C_{\mu} \tau S_{ij} - 4\alpha_1 \tau^2 (S_{ik} S_{kj} - \frac{1}{3} S^2 \delta_{ij}) \\ & - 4\alpha_2 \tau^2 (S_{ik} \Omega_{kj} + S_{jk} \Omega_{ki}) - 2\alpha_4 \tau^2 \dot{S}_{ij} \end{aligned}$$

$$C_{\mu} = \frac{4/15}{C_1 + C_{\epsilon_2} - 2 + (2 - C_{\epsilon_1})P/\epsilon}$$

$$\alpha_1 = - \frac{(1 - \alpha - \beta) C_{\mu}}{C_1 + 2C_{\epsilon_3} - 3 + (3 - 2C_{\epsilon_1})P/\epsilon}$$

$$\alpha_2 = \frac{(1 - \alpha + \beta) C_{\mu}}{2(C_1 + 2C_{\epsilon_3} - 3 + (3 - 2C_{\epsilon_1})P/\epsilon)}$$

$$\alpha_4 = - \frac{C_{\mu}}{2(C_1 + 2C_{\epsilon_3} - 3 + (3 - 2C_{\epsilon_1})P/\epsilon)}$$

## EXPANSION Continued

- Same general form as nonlinear stress-strain relation.
- Model coefficients are determined from Reynolds-stress model parameters.
- There is no  $(\Omega_{il}\Omega_{lj} - \frac{1}{3}\Omega^2\delta_{ij})$  term. Possibly the model for the rapid part of the pressure strain term in the RSM is incomplete. Stress-strain models derived from DIA or RNG theory contain this term.
- Series is valid for small  $\tau S$ . Solution with this model, even with more terms, does not give good results near the asymptotic state in homogeneous shear or irrotational strain flows.



SOLUTION VALID FOR  
SMALL AND LARGE  $kS/\epsilon$

$$\begin{aligned}\tau \dot{\tau} \frac{D}{D\tau} \left( \frac{a_{ij}}{\tau} \right) = & - (2 - C_{\epsilon_1}) \frac{P}{\epsilon} \frac{a_{ij}}{\tau} - (C_1 + C_{\epsilon_1} - 2) \frac{a_{ij}}{\tau} - \frac{8}{15} S_{ij} \\ & - (1 - \alpha - \beta)(a_{il} S_{lj} + a_{jl} S_{li} - \frac{2}{3} a_{kl} S_{lk} \delta_{ij}) \\ & - (1 - \alpha + \beta)(a_{il} \Omega_{lj} + a_{jl} \Omega_{li})\end{aligned}$$

For  $\frac{D}{D\tau} \left( \frac{a_{ij}}{\tau} \right) = 0$  :

- Correct form for small  $\tau S$

$$\frac{a_{ij}}{\tau} = a_{ij}^{(1)} + \dots$$

- Right side closely represents the asymptotic solution to the Reynolds stress equation,  $Da_{ij}/Dt = 0$

Linear algebraic equations ( $P/\epsilon$  retained implicitly) solved by the method given by Pope (1975).

# ALGEBRAIC STRESS MODEL

**IMPROVED:**

$$\begin{aligned} \left[ C_1 + C_{\epsilon_1} - 2 + (2 - C_{\epsilon_1}) \frac{P}{\epsilon} \right] a_{ij} &= -\frac{8}{15} \tau S_{ij} \\ &- (1 - \alpha - \beta) \left( a_{il} S_{lj} + a_{jl} S_{li} - \frac{2}{3} a_{kl} S_{lk} \delta_{ij} \right) \tau \\ &+ (1 - \alpha + \beta) (a_{il} \Omega_{lj} + a_{jl} \Omega_{li}) \tau \end{aligned}$$

**STANDARD:**

$$\begin{aligned} \left[ C_1 - 1 + \frac{P}{\epsilon} \right] a_{ij} &= -\frac{8}{15} \tau S_{ij} \\ &- (1 - \alpha - \beta) \left( a_{il} S_{lj} + a_{jl} S_{li} - \frac{2}{3} a_{kl} S_{lk} \delta_{ij} \right) \tau \\ &+ (1 - \alpha + \beta) (a_{il} \Omega_{lj} + a_{jl} \Omega_{li}) \tau \end{aligned}$$

# NEW EXPLICIT ALGEBRAIC STRESS MODEL

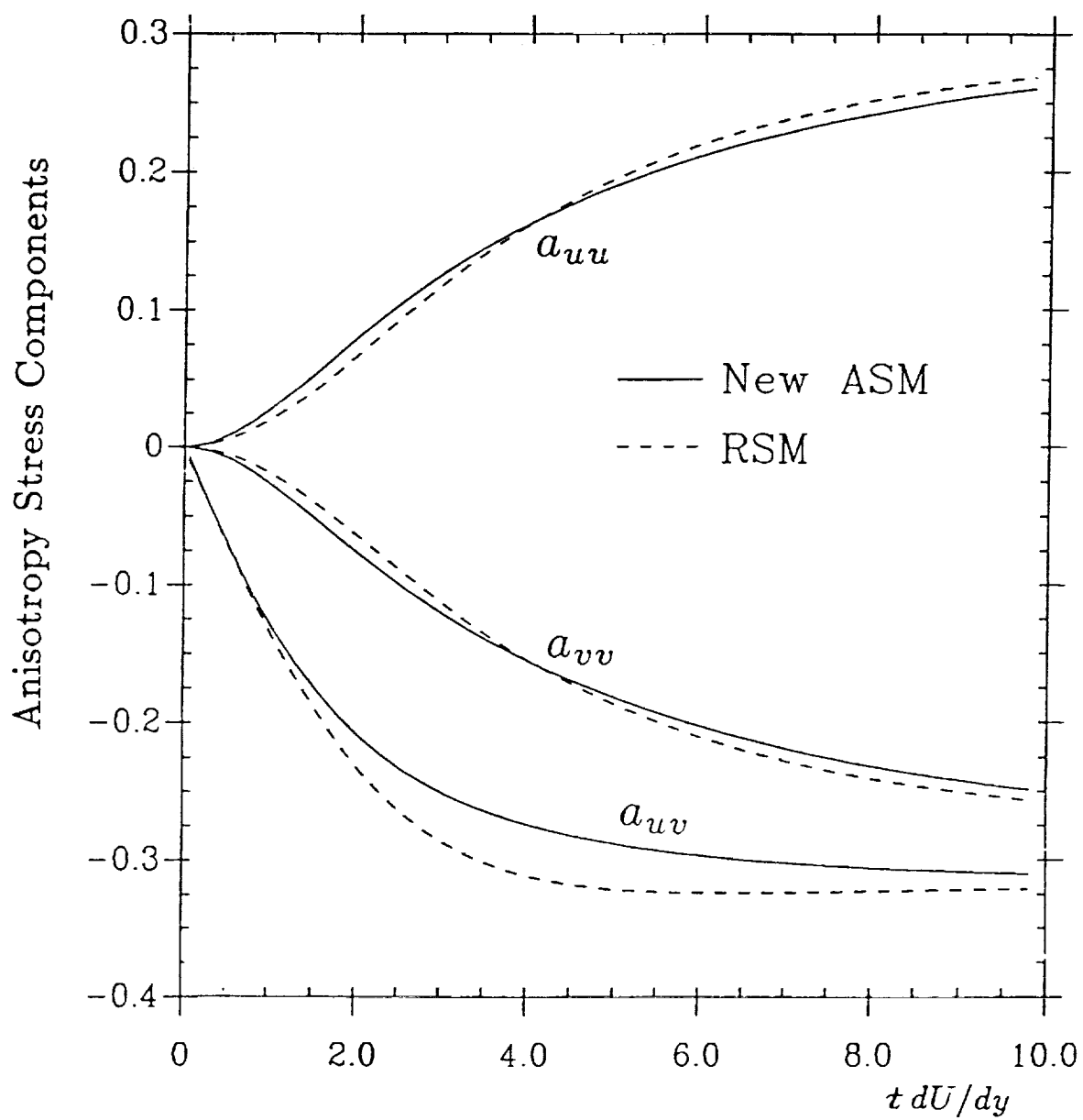
## For Two Dimensions

$$\begin{aligned}
 a_{ij} = & -2C_\mu \tau S_{ij} \\
 & -2C_\mu(1 - \alpha - \beta) g \tau^2 S^2 \left( \frac{2}{3} \delta_{ij}^{(3)} - \delta_{ij}^{(2)} \right) \\
 & -2C_\mu(1 - \alpha + \beta) g \tau^2 (S_{il} \Omega_{lj} + S_{jl} \Omega_{li})
 \end{aligned}$$

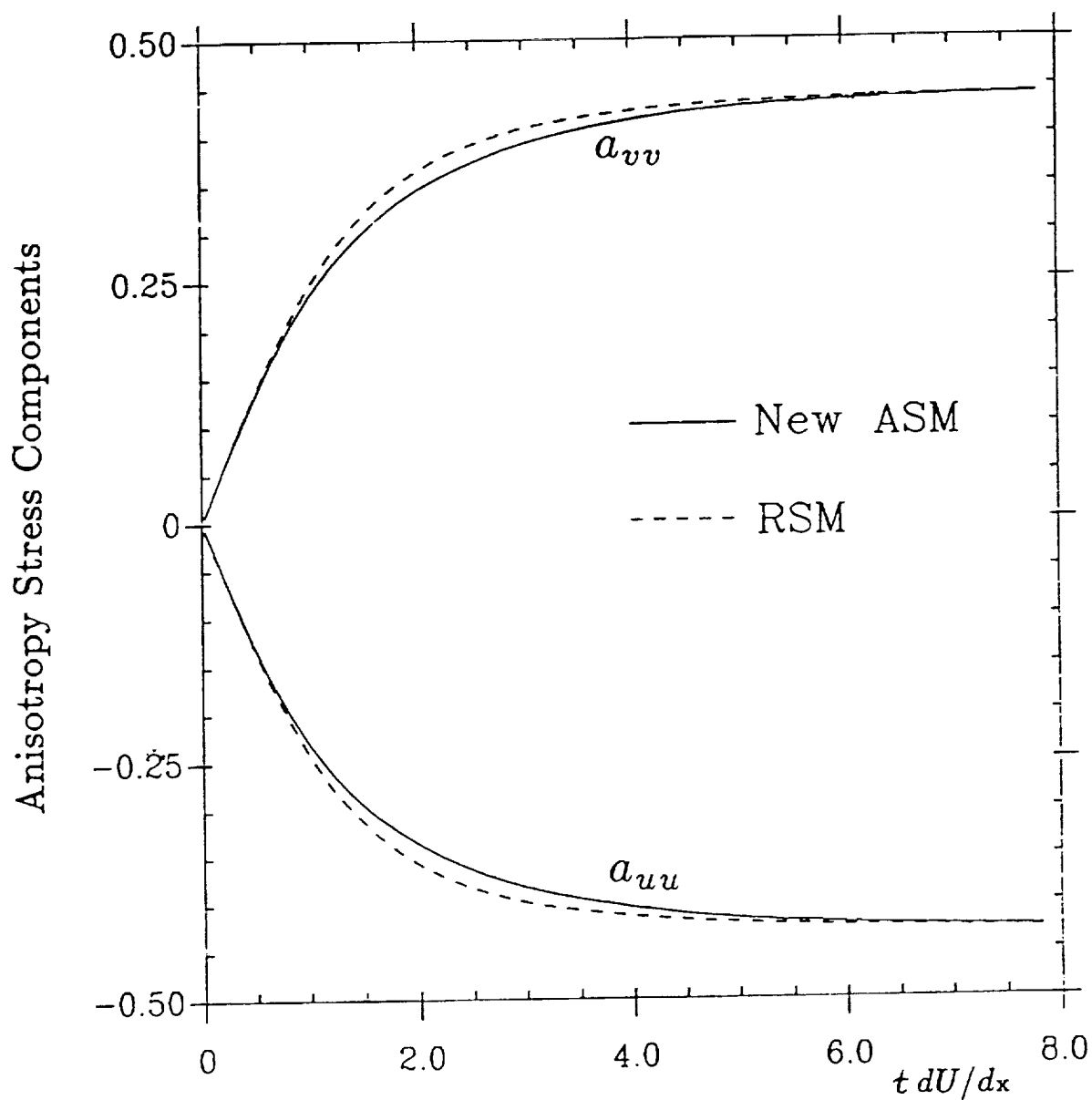
$$C_\mu = \frac{4g/15}{1 - \frac{2}{3}(1 - \alpha - \beta)^2 g^2 S^2 - 2(1 - \alpha + \beta)^2 g^2 \Omega^2}$$

$$g = \frac{1}{C_1 + C_{\epsilon_2} - 2 + (2 - C_{\epsilon_1})P/\epsilon}$$

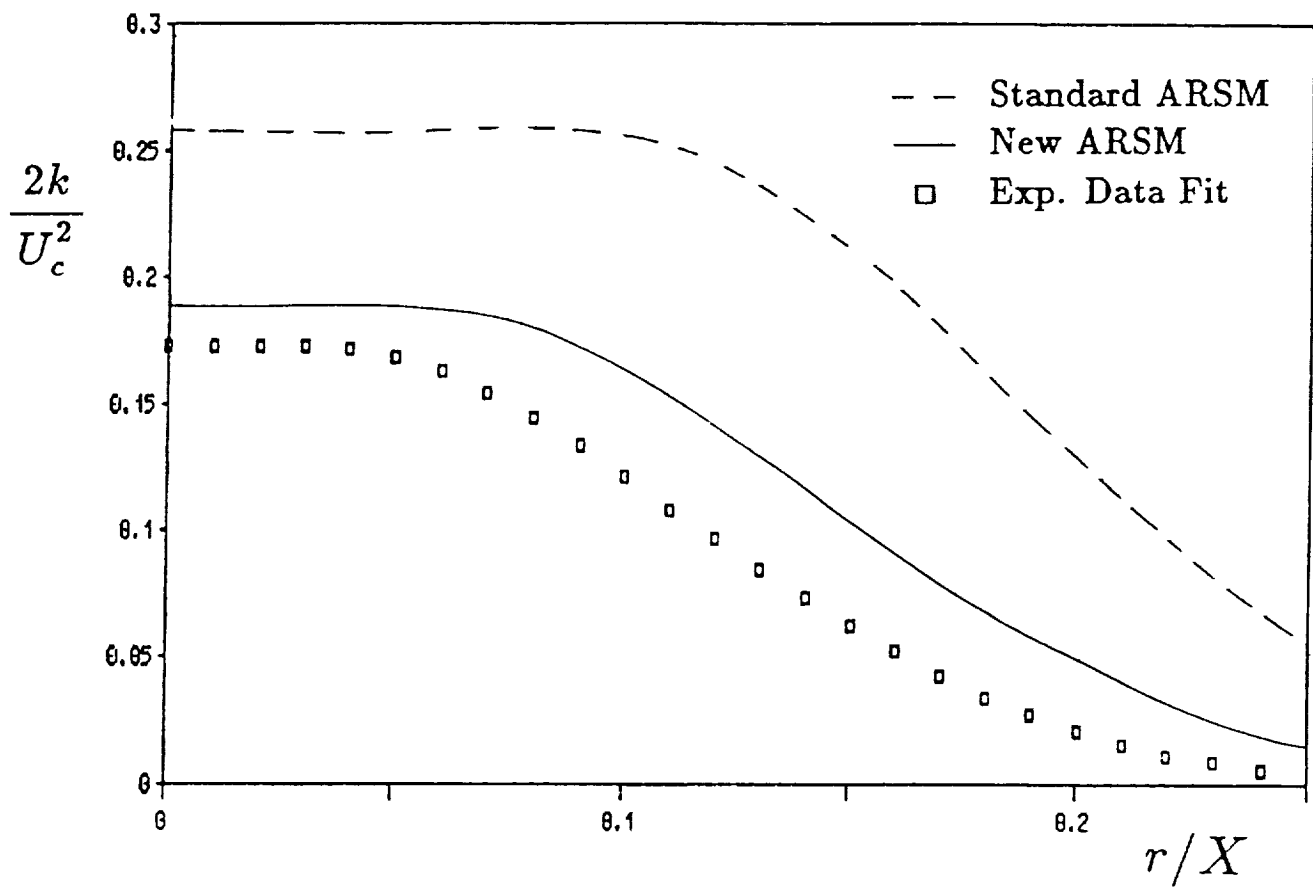
Model coefficients are determined in terms of the Reynolds-stress Model parameters.



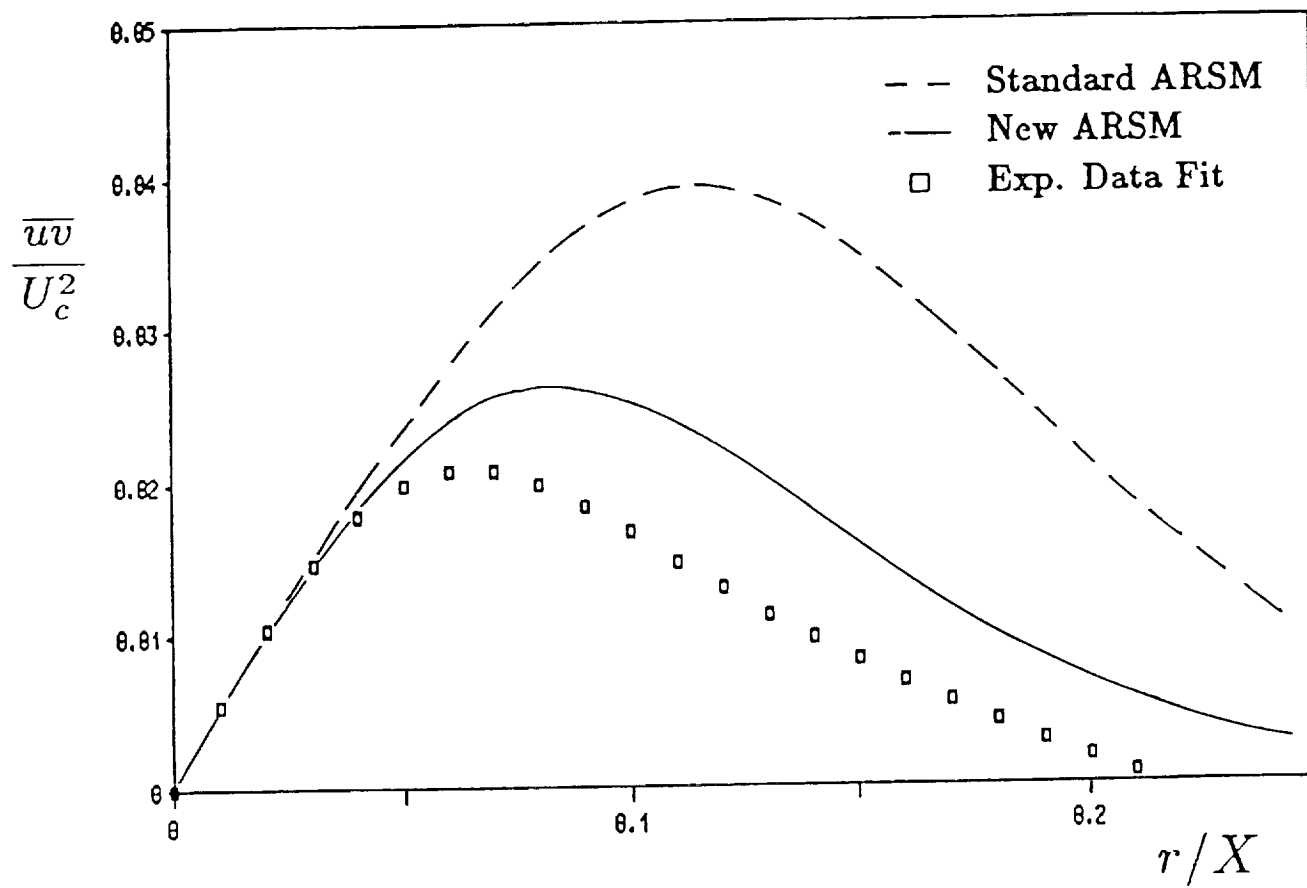
New Algebraic Stress Model Solution for  
Homogeneous Shear Flow



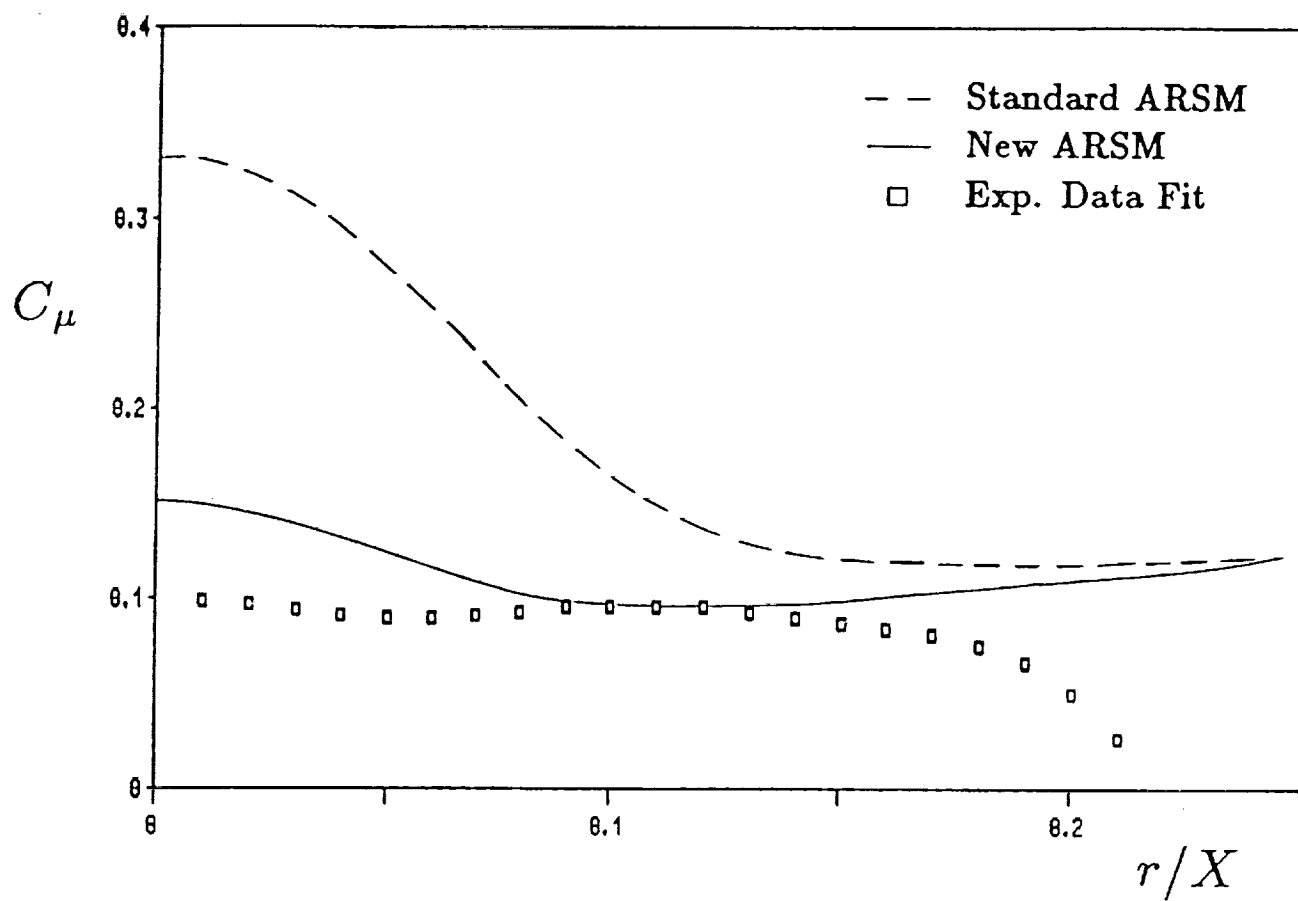
New Algebraic Stress Model Solution for  
Irrotational Strain Flow



Axisymmetric Jet – Kinetic Energy

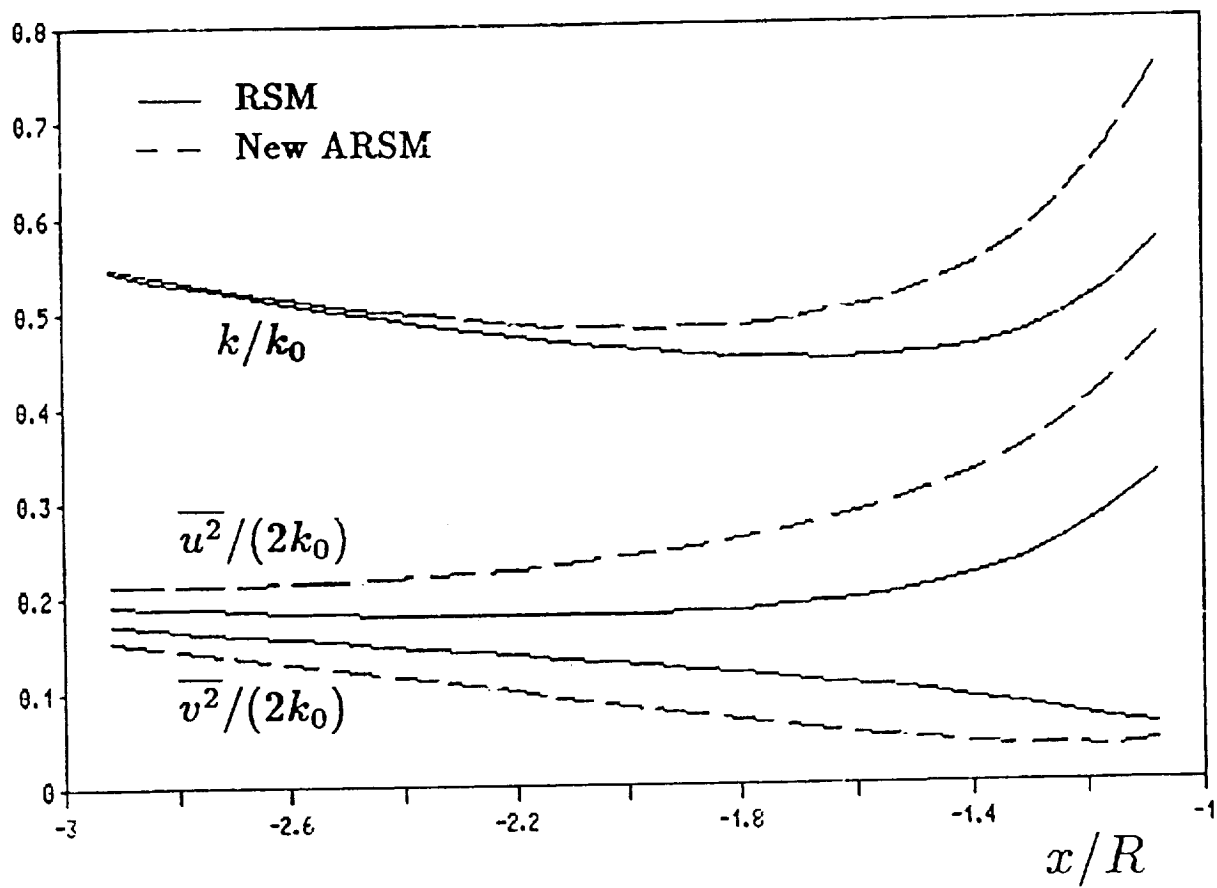


Axisymmetric Jet – Shear Stress



Axisymmetric Jet – Eddy Viscosity Coefficient





Stagnation Streamline Flow

## CONVECTIVE EFFECTS

WRITE EQUATIONS IN TERMS OF  $\tau S$

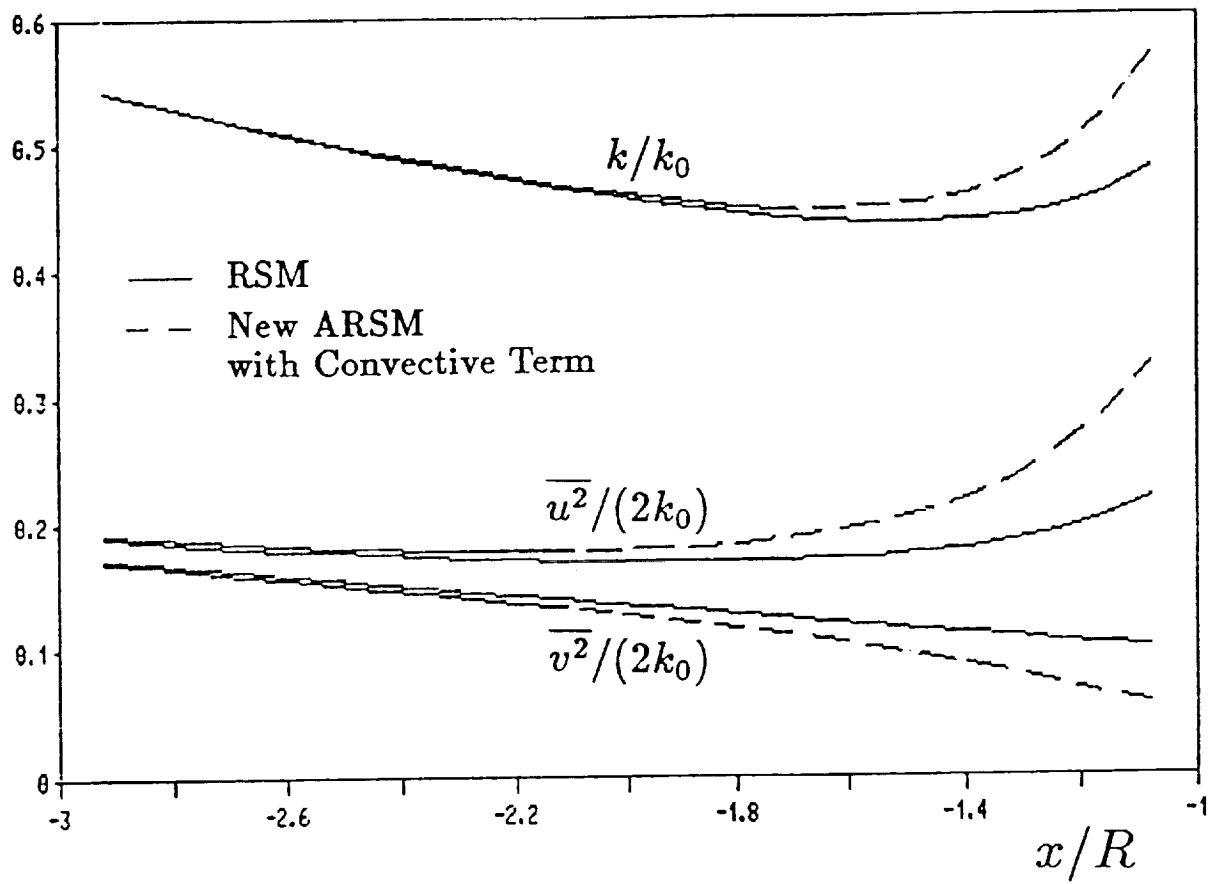
where,  $\tau = k/\epsilon$  and  $S = (S_{kl}S_{lk})^{\frac{1}{2}}$

$$\tau \frac{Da_{ij}}{Dt} = \tau^2 S \frac{D}{Dt} \left( \frac{a_{ij}}{\tau S} \right) + \frac{1}{S} \frac{D\tau S}{Dt} a_{ij}$$

$$\frac{1}{S} \frac{D\tau S}{Dt} = \frac{\tau}{S} \frac{DS}{Dt} + (C_{\epsilon_2} - 1) + (C_{\epsilon_1} - 1) \frac{P}{\epsilon}$$

$$\begin{aligned} \tau^2 S \frac{D}{Dt} \left( \frac{a_{ij}}{\tau S} \right) = & - \left[ C_1 + C_{\epsilon_2} - 2 + (2 - C_{\epsilon_1}) \frac{P}{\epsilon} + \frac{\tau}{S} \frac{DS}{Dt} \right] a_{ij} \\ & - \frac{8}{15} \tau S_{ij} \\ & + (1 - \alpha + \beta) \tau (a_{il} \Omega_{lj} + a_{jl} \Omega_{li}) \end{aligned}$$

$$C_\mu \sim \frac{4/15}{C_1 + C_{\epsilon_2} - 2 + (2 - C_{\epsilon_1}) \frac{P}{\epsilon} + \frac{\tau}{S} \frac{DS}{Dt}}$$



Stagnation Streamline Flow

## SOLUTION FOR THREE DIMENSIONS

Pope (1975), Spencer & Rivlin (1959, 1960)

For  $a_{ij} = a_{ij}(S_{ij}, \Omega_{ij}) = \mathbf{a}(\mathbf{S}, \mathbf{\Omega})$

There are Ten Independent Symmetric Tensors.

$$\mathbf{T}^{(1)} = \mathbf{S}$$

$$\mathbf{T}^{(2)} = \mathbf{S}\mathbf{\Omega} - \mathbf{\Omega}\mathbf{S}$$

$$\mathbf{T}^{(3)} = \mathbf{S}^2 - \mathbf{I}\{\mathbf{S}^2\}/3$$

$$\mathbf{T}^{(4)} = \mathbf{\Omega}^2 - \mathbf{I}\{\mathbf{\Omega}^2\}/3$$

$$\mathbf{T}^{(5)} = \mathbf{\Omega}\mathbf{S}^2 - \mathbf{S}^2\mathbf{\Omega}$$

$$\mathbf{T}^{(6)} = \mathbf{\Omega}^2\mathbf{S} - \mathbf{S}\mathbf{\Omega}^2 - 2\mathbf{I}\{\mathbf{S}\mathbf{\Omega}^2\}/3$$

$$\mathbf{T}^{(7)} = \mathbf{\Omega}\mathbf{S}\mathbf{\Omega}^2 - \mathbf{\Omega}^2\mathbf{S}\mathbf{\Omega}$$

$$\mathbf{T}^{(8)} = \mathbf{S}\mathbf{\Omega}\mathbf{S}^2 - \mathbf{S}^2\mathbf{\Omega}\mathbf{S}$$

$$\mathbf{T}^{(9)} = \mathbf{\Omega}^2\mathbf{S}^2 + \mathbf{S}^2\mathbf{\Omega}^2 - 2\mathbf{I}\{\mathbf{S}^2\mathbf{\Omega}^2\}/3$$

$$\mathbf{T}^{(10)} = \mathbf{\Omega}\mathbf{S}^2\mathbf{\Omega}^2 - \mathbf{\Omega}^2\mathbf{S}^2\mathbf{\Omega}$$

$$\text{Then : } a_{ij} = \sum_{n=1}^{10} G^{(n)} T_{ij}^{(n)}$$

Where  $G^{(n)}$  can be functions of the invariants :

$$\{\mathbf{S}^2\}, \{\mathbf{\Omega}^2\}, \{\mathbf{S}^3\}, \{\mathbf{\Omega}^2\mathbf{S}\}, \{\mathbf{\Omega}^2\mathbf{S}^2\}$$

### 3-D SOLUTION Continued

$$\begin{aligned}
 [C_1 + C_{\epsilon_2} - 2 + (2 - C_{\epsilon_1})\frac{P}{\epsilon}]a_{ij} &= -\frac{8}{15}\frac{k}{\epsilon}S_{ij} \\
 &+ (1 - \alpha + \beta)\frac{k}{\epsilon}(a_{il}\Omega_{lj} + a_{jl}\Omega_{li}) \\
 &- (1 - \alpha - \beta)\frac{k}{\epsilon}(a_{il}S_{lj} + a_{jl}S_{li} - \frac{2}{3}a_{kl}S_{lk}\delta_{ij})
 \end{aligned}$$

$$1 - \alpha + \beta = (1 + 7C'_2)/11$$

$$1 - \alpha - \beta = (5 - 9C'_2)/11 \rightarrow \text{small, neglect term}$$

$$\begin{aligned}
 a_{ij} &= \sum_{n=1}^{10} G^{(n)} T_{ij}^{(n)} \\
 &= -2C_\mu \frac{k}{\epsilon} S_{ij} - \alpha_2 g^2 \frac{k^2}{\epsilon^2} (S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}) \\
 &\quad - \alpha_6 g^3 \frac{k^3}{\epsilon^3} (\Omega_{ik}\Omega_{kl}S_{lj} + S_{ik}\Omega_{kl}\Omega_{lj} - \frac{2}{3}S\Omega^2\delta_{ij}) \\
 &\quad - \alpha_7 g^4 \frac{k^4}{\epsilon^4} (\Omega_{ik}S_{kl}\Omega_{lm}\Omega_{mj} - \Omega_{ik}\Omega_{kl}S_{lm}\Omega_{mj})
 \end{aligned}$$

$$C_\mu = \frac{4g}{15} \frac{1 - \frac{1}{2} [(1 - \alpha + \beta)g\frac{k}{\epsilon}]^2 \Omega^2}{1 + \frac{1}{2} [(1 - \alpha + \beta)g\frac{k}{\epsilon}]^2 \Omega^2 + [(1 - \alpha + \beta)g\frac{k}{\epsilon}]^4 (\Omega^2)^2}$$

$$\alpha_2 = \frac{8}{15} \frac{(1 - \alpha + \beta)\{1 - 2 [(1 - \alpha + \beta)g\frac{k}{\epsilon}]^2 \Omega^2\}}{1 + \frac{1}{2} [(1 - \alpha + \beta)g\frac{k}{\epsilon}]^2 \Omega^2 + [(1 - \alpha + \beta)g\frac{k}{\epsilon}]^4 (\Omega^2)^2}$$

$$g = [C_1 + 2C_{\epsilon_2} - 2 + (2 - C_{\epsilon_1})P/\epsilon]^{-1}$$

## CONCLUDING REMARKS

For simplicity and numerical solution purposes, explicit forms are highly desirable.

An Algebraic Reynolds Stress Model and its corresponding Nonlinear Stress Model can be formulated which closely reproduces the Reynolds Stress Model solution as long as :

- 1) The mean velocity field is not rapidly changing.
- 2) There are no boundary conditions or imposed flow conditions which give rise to strong non-local effects.

Coefficients in the stress-strain models are not in general constant but depend on the strain field and the time parameter of the turbulence.

A good explicit stress-strain relation which represents the anisotropy of the turbulence should replace the linear relation now used in the  $k - \epsilon$  model for practical applications.

THE PRESENT STATUS AND  
FUTURE DIRECTION OF ALGEBRAIC  
REYNOLDS STRESS MODELS  
—COMMENT ON—

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# INTRODUCTION

- AS EXPLAINED IN THE POSITION PAPER OF TAULBEE, ALGEBRAIC STRESS MODELS (ASM) ARE "QUICK" FIXES FOR DEFICIENCIES IN BOUSSINESQ EDDY VISCOSITY MODEL (BEVM).
- BEVM WAS PERHAPS NEVER INTENDED AS A GENERAL CONSTITUTIVE MODEL FOR COMPLEX TURBULENT FLOWS
- RIVLIN (1957) SUGGESTED MORE APPROPRIATE ANALOGY
  - TURBULENT "NEWTONIAN FLUID" FLOW  $\sim$  NON-NEWTONIAN FLUID
  - .NOT. TURBULENT FLUCTUATION  $\simeq$  MOLECULAR VIBRATION
- LUMLEY (1970) & POPE (1975) DISCUSSED POSSIBILITIES FOR PROPER CONSTITUTIVE RELATIONS — BUT NO PRACTICAL ANSWER FOR COMPLEX/ENGINEERING FLOWS
- 0-, 1-, & 2-EQ. MODELS CONTINUE TO USE BEVM  $\rightarrow k - \epsilon$  MODELS BECAME VERY POPULAR
- DEFICIENCIES IN THE PRESENCE OF
  - CURVATURE
  - ROTATION/SWIRL
  - GRAVITATION/BODY FORCES
  - STREAMWISE CORNERS



- ☐ REYNOLDS STRESS MODELS(RSM) APPEARED NOT TO SUFFER THESE DEFICIENCIES
- ☐ ASM TO THE RESCUE — CONTAIN ESSENTIAL ELEMENTS OF RSM
- ☐ LIMITED GOAL OF ASM — ASPIRE TO RSM RESULTS WITHIN 2 EQ. FRAMEWORK
- ☐ RELATIVELY SUCCESSFUL IN MANY CASES — HELPED TO BUILD REPUTATION OF MODELS
- ☐ TO COMPLEMENT TAULBEE'S PRESENTATION SUCCESS AND LIMITATIONS IN APPLICATION TO COMPLEX FLOWS ARE ADDRESSED

# TYPES OF ASM

## □ DESIRABLE:-

- $\text{EVM} \rightarrow \frac{\overline{u_i u_j}}{k} = \frac{2}{3} \partial_{ij} - 2 C_\mu \tau S_{ij} + O(\tau^2, S^2, \Omega^2, \dots)$   
where  $\tau = \frac{k}{\epsilon}$

## □ STANDARD:-

- $\text{BEVM} \rightarrow O(\tau^2, S^2, \Omega^2, \dots) = 0$
- $k - \epsilon \rightarrow C_\mu = 0.09 = \text{constant}$
- $\text{EDDY DIFFUSIVITY} \rightarrow \frac{\overline{u_i \phi}}{k} = -\frac{C_\mu}{\sigma_\phi} \tau \Phi_{,i}$   
where Prandtl/Schmidt No.  $\sigma_\phi = \text{constant}$

## □ CORRECTIONS

- (i)  $C_\mu \neq \text{constant}$ 
  - Easiest form to apply
  - $f^n$  of  $P/\epsilon$ ,  $Ri$ , Curvature, Rotation, etc.
  - Multiple scale  $k - \epsilon$  models fall somewhat in this category by making  $C_\mu = f^n \tau_s / \tau$  ( $\tau_s = \text{time scale of large eddies}$ )
- (ii)  $\sigma_\phi \neq \text{constant}$ ;  $\sigma_{\phi_*} \neq \sigma_\phi$
- (iii)  $O(\tau^2, S^2, \Omega^2, \dots) \neq 0$ 
  - Most Primitive form of ASM

Workshop on Engineering Turbulence Modeling,  
NASA LERC Cleveland, OH, Aug. 1991

- Applicable to 3D Flows
- Explicit Relations difficult for 3D
- Non-linear Stress-Strain Relations (Speziale 1987, Yoshizawa 1984, Barton and Rubinstein 1990) fall in this category

## SOME SUCCESSES

- ☐ CASE(I)
  - LESCHZNER & RODI (1981)
- ☐ CASE(II)
  - HOSSAIN & RODI (1982)
  - DEMUREN, SCHOENUNG & RODI (1987)
- ☐ CASE(III)
  - GIBSON & LAUNDER(1976), GIBSON(1979)
  - DEMUREN & RODI (1984,1987)

# TYPE I

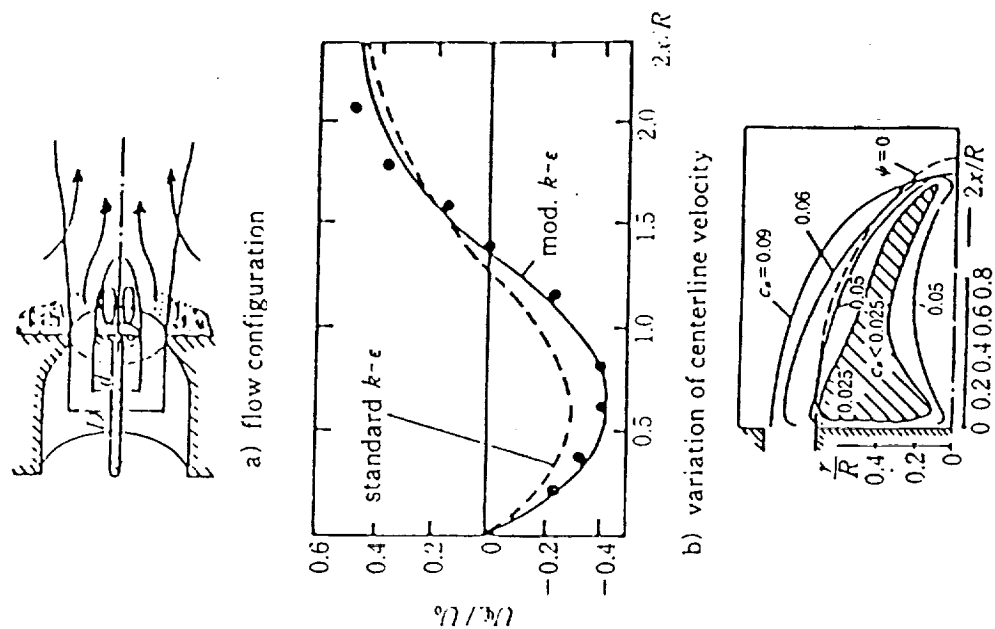
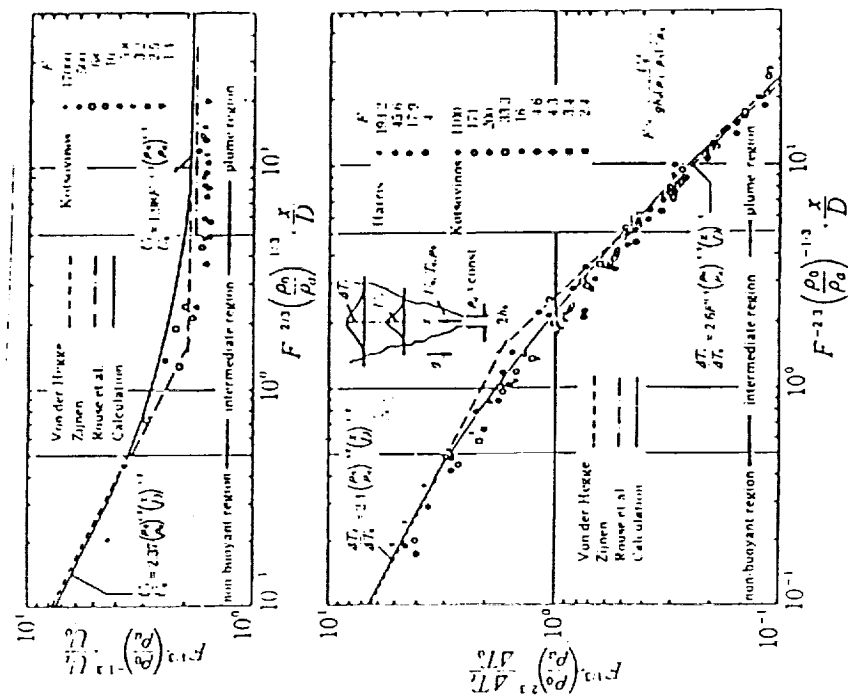
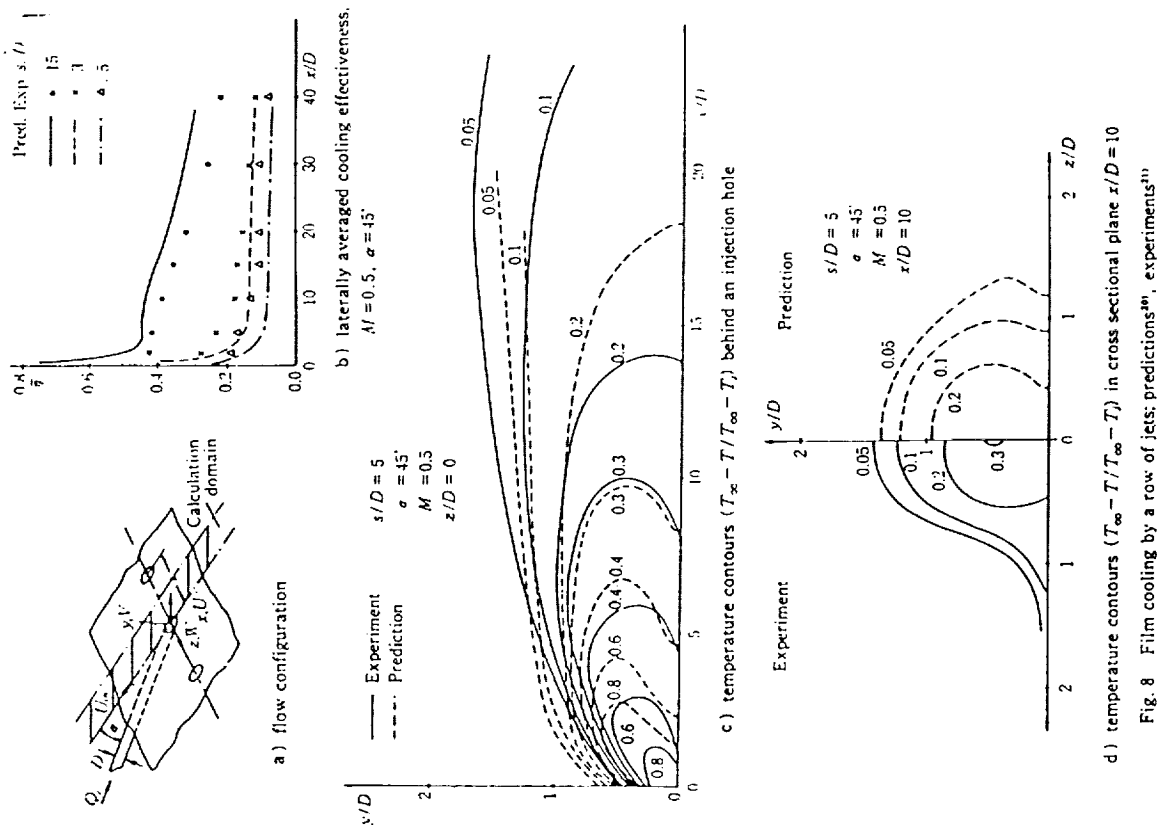
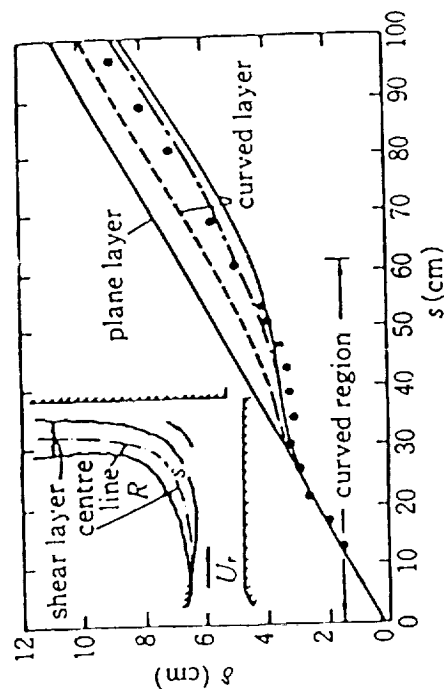


Fig. 7 Annular jet with wake behind disk  
—, ... calculations<sup>(15)</sup>, ● data<sup>(16)</sup>

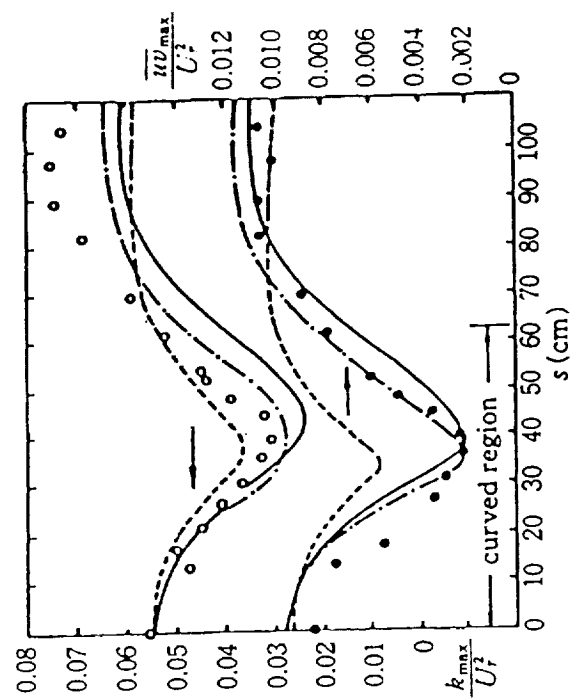
# TYPE II



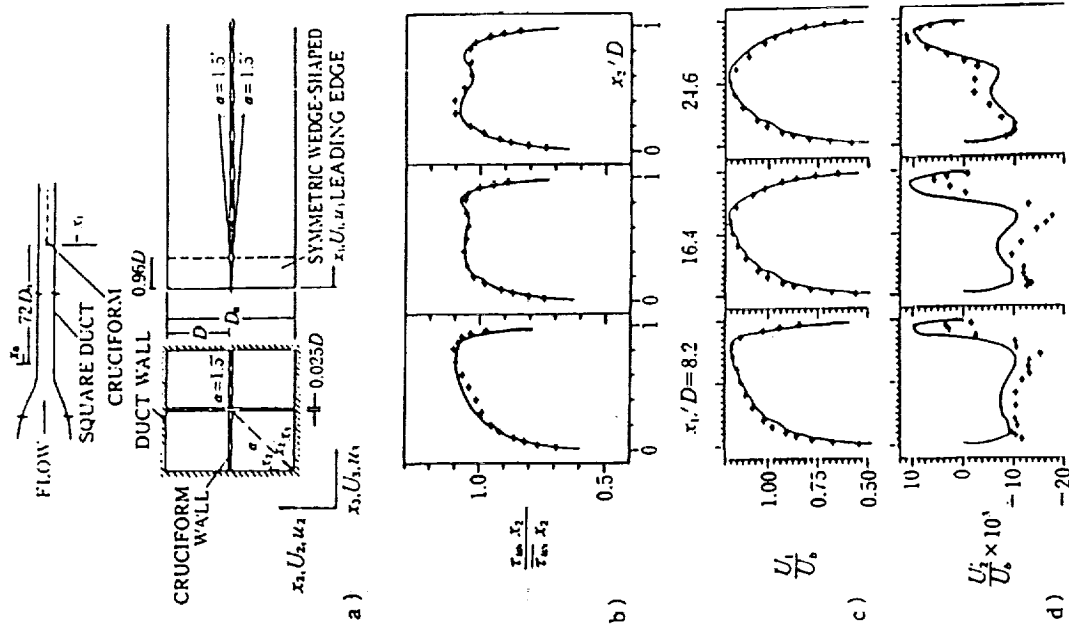
# TYPE III



a) growth of layer width  $\delta$



b) variation of maximum  $k$  and  $uv$   
 Curved mixing layer; — stress-equation model<sup>10)</sup>, ..... standard  $k-\epsilon$  model<sup>10)</sup>, --- algebraic stress model<sup>11)</sup>,  $\circ$  ● experiments<sup>12)</sup>



**Fig. 1** Developing flow in a square duct with non-uniform inlet conditions (from 6) 7))

- a) flow configuration
- b) shear stress along cruciform wall
- c) longitudinal velocity along corner bisector
- d) secondary velocity along corner bisector

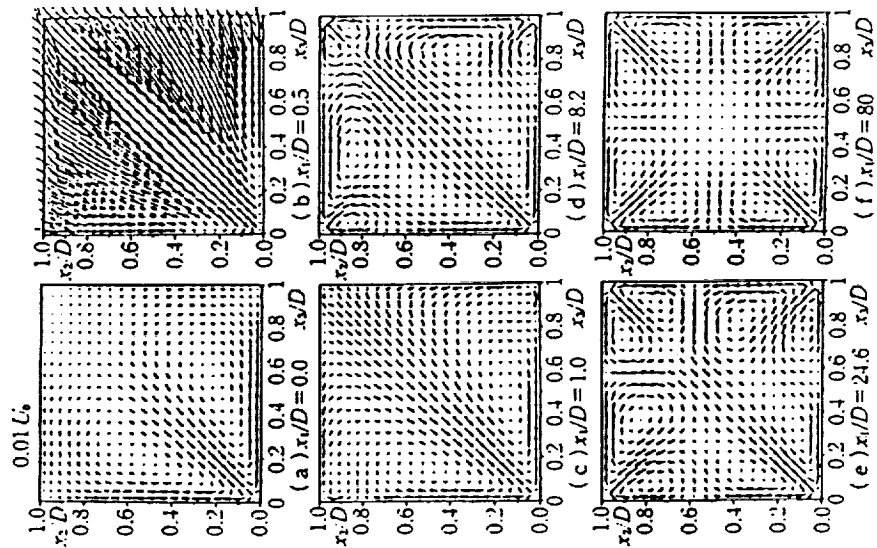


Fig. 1 e) development of secondary velocity



# LIMITATIONS

□ ASM SUFFERS FROM AT LEAST ANY SHORTCOMINGS OF CORRESPOND-

ING RSM

□ AS SHOWN IN THE POSITION PAPER

$$\frac{D u_i u_j}{D t} \neq \frac{u_i u_j}{k} \frac{D k}{D t} \text{ in Homogeneous Shear Flows}$$

$$\text{Ditto } C \frac{u_i u_j}{u_i u_j} - D \frac{u_i u_j}{u_i u_j} \neq \frac{u_i u_j}{k} (C_k - D_k)$$

- Will certainly not be true in Complex Flows.
- e.g. Round Jet with or without Swirl

□ PROBLEMS WITH CORIOLIS/CENTRIFUGAL TERMS

□ TRANSPORT OF  $\overline{u_i u_j}$  IMPORTANT IN COMPLEX FLOWS

□ ASM CANNOT PREDICT COUNTER-GRADIENT DIFFUSION

□ RSM MORE STABLE (MORE FORGIVING) THAN ASM

□ BITE THE BULLET — USE RSM?

- 2 DIFFULTIES

→ (i) Stiff Momentum Equations.

→ (ii) Computational Cost

- REMEDIES

→ (i) Split  $\overline{u_i u_j}$  in momentum equations into Implicit/Explicit parts following non-linear stress-strain ideas.

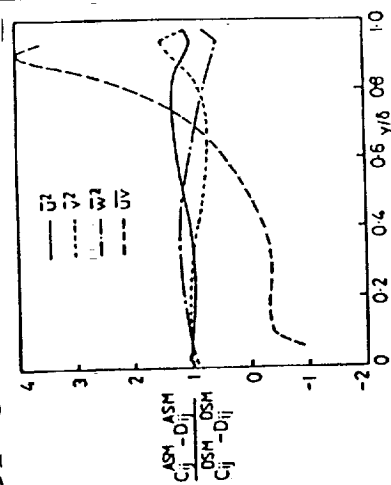
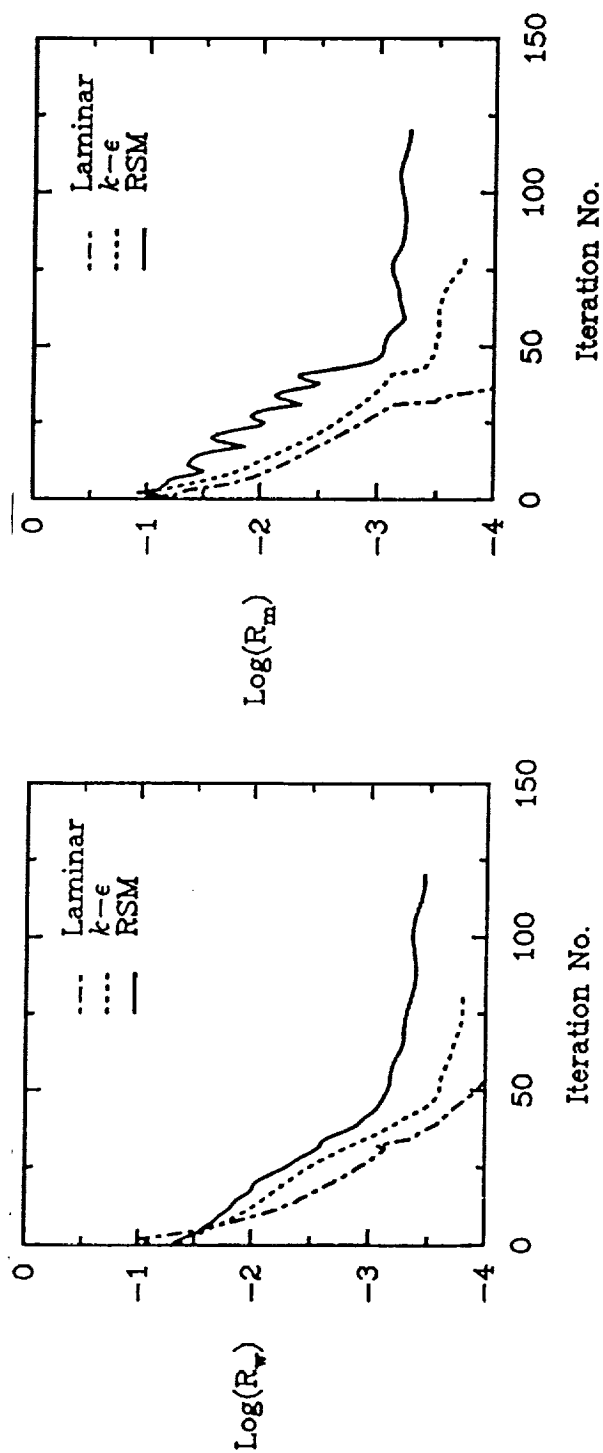
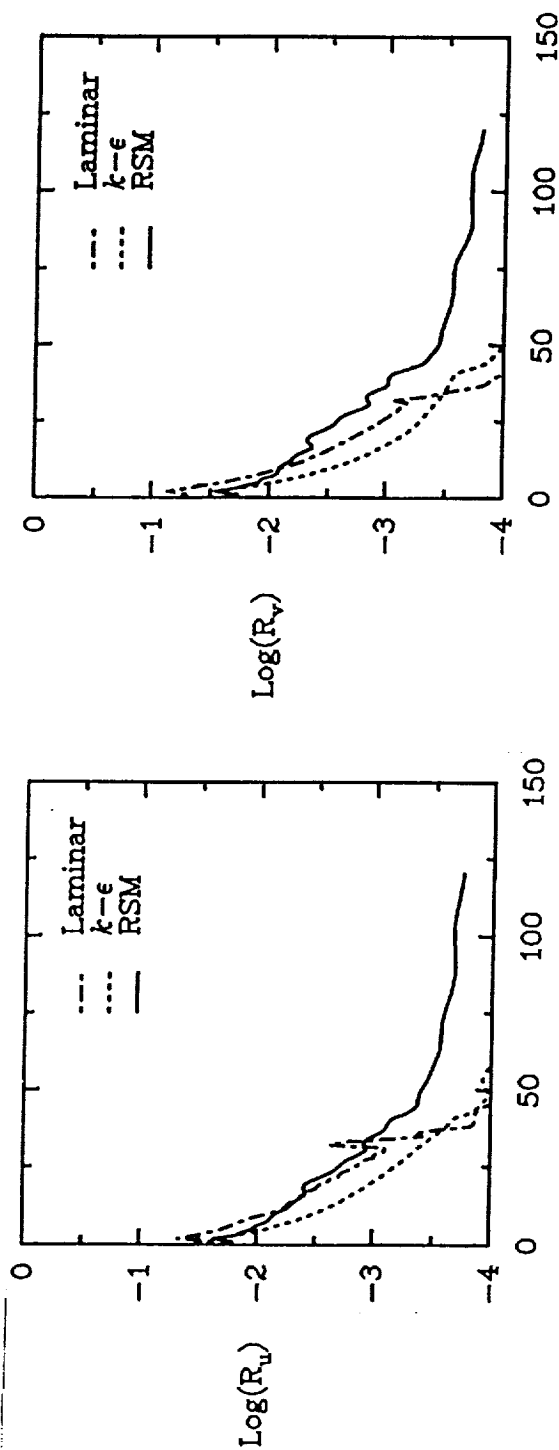


Fig. 3 Ratio of ASM-to-DSM stress transport evaluated for fully developed round jet

→ (ii) Use Multigrid Acceleration for rapid convergence  
e.g. 3D Opposed Jets in Crossflow



## DISCUSSION

A. Demuren

I just want to comment on the  $k - \epsilon$  model. It appears the reason it performs so poorly is the value of epsilon at the wall. A very simple fix is to eliminate epsilon at the wall and use a simple mixing length. This works very well, and gives the right behavior in adverse pressure gradient, back facing step and separated flow, etc. It is quite an easy fix for the  $k - \epsilon$  model and yields decent results.

Ronald So

A comment about the compressible calculation with a  $k - \epsilon$  model. What we have found is that if you do the analysis correctly, you can actually predict compressible flow very well up to Mach 10. What Dave has shown up there about  $C_f$  vs. Mach number is not quite correct. You can get the prediction of the adiabatic and cool wall cases very well. We have used the baseline model and Sarkar's correction.

D.C. Wilcox (to B.E. Launder)

When you gave your talk this morning, you said that ASM suffered "frame-invariance". Could you comment on this?

B.E. Launder (reply)

It depends on what hypothesis you use to relate the convective transport of stress to the convective transport of strain. Work attributed to Rodi shows that you get a different answer if your frame of reference is at rest or rotating at a constant angular velocity. You can devise a scheme, Ahmadi and Speziale have done so, that is frame invariant and Dale Taulbee was talking about these things. At the end of it though, you aren't going to get

a better model out of it.

D.C. Wilcox (to B.E. Launder)

Are you saying that if I have flow over a curved wall, if you forget to include the Coriolis and centrifugal forces as you go over it, that this is what messes it up?

B.E. Launder (reply)

Yes.

Something that Dale said towards the end of his talk he just slipped in there. You guessed that if you have important diffusive transport, then ASM won't work. There are many free flows where diffusive transport is very important. I just don't know of a good algebraic representation of it. My feeling is that if you haven't already got the software in place for ASM, then you should look beyond ASM for better answers.

D.B. Taulbee (reply)

How about all the people who have  $k - \epsilon$  programs sitting there. You can easily upgrade them by changing the explicit stress-strain relations. Not everyone has access to RSM's. They just can't buy them because they're too expensive. G. Huang (comment to D. Taulbee) ASM's are just as complicated to code as RSM's.

D. Bushnell

Brian Launder said it very well: if you have a situation that the physics is such that this ASM is fine then it may work. Under the NASP contract, we had a similar workshop about turbulence modelling about two and a half

years ago. What we asked was, "do you want the wrong answer very easily or the right answer?" In the NASP project, inside the scramjet combustors the flows are such that we need to go to RSM's to get the proper physics.

W. K. George

Since Dennis was free to paraphrase Brian, I feel free to paraphrase my colleague Dale Taulbee. If, for some reason, you don't have the resources to go to RSM and the physics is bad for ASM, then things will be a hell of a lot worse for a  $k - \epsilon$  model and you shouldn't be using that either.

Also let me add that there is a lot of beating to death about the difference between 0.98 and 0.95 for the spreading rate of jets. It is probably absolutely impossible to determine this experimentally.

Let me comment on the emphasis on getting the constants in the Milliken formulation. If one goes back and looks at the original data and the compromises made in putting those constants there, numbers like 5.1 are an average of numbers that go from (0.5-20.0)! In fact the experiments just aren't that good. And the theory used to interpret them is not that good either.

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## Session II

### Second Order Closure and PDF Method

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THE PRESENT STATUS AND THE FUTURE DIRECTION OF  
SECOND ORDER CLOSURE MODELS FOR  
INCOMPRESSIBLE FLOWS

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Center for Modeling of Turbulence and Transition

ICOMP, NASA Lewis Research Center

59-34  
86-290  
p. 55  
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## OUTLINE

- Basic equations and model terms
- Model constraints and assumptions
- Present state of various closure models
- Future direction

## Basic Equations and Model Terms for Incompressible Flows

Equations for the mean quantities:

$$U_{i,i} = 0$$

$$\frac{D}{Dt}U_i = -\frac{1}{\rho}P_{,i} - (\overline{u_i u_j})_{,j} + \beta_i \Theta + \nu U_{i,jj}$$

$$\frac{D}{Dt}\Theta = -(\overline{\theta u_j})_{,j} + \gamma \Theta_{,jj}$$

We need models for  $\overline{u_i u_j}$ ,  $\overline{\theta u_i}$  — second moments.

Equations for the second moments:

Reynolds stress equation:

$$\frac{D}{Dt} \overline{u_i u_j} = D_{ij} + P_{ij} + F_{ij} + T_{ij} + \Pi_{ij} - \varepsilon_{ij}$$

$$D_{ij} = \nu \overline{u_i u_j}_{,kk}$$

$$P_{ij} = -\overline{u_i u_k} U_{j,k} - \overline{u_j u_k} U_{i,k}$$

$$F_{ij} = \beta_i \overline{\theta u_j} + \beta_j \overline{\theta u_i}$$

$$T_{ij} = -[\overline{u_i u_j u_k} + \frac{1}{\rho} (\overline{u_i p} \delta_{jk} + \overline{u_j p} \delta_{ik})],_k$$

$$\Pi_{ij} = \frac{1}{\rho} \overline{p(u_{i,j} + u_{j,i})}$$

$$\varepsilon_{ij} = 2\nu \overline{u_{i,k} u_{j,k}}$$

$T_{ij}$ ,  $\Pi_{ij}$  and  $\varepsilon_{ij}$  must be modeled.

Heat flux equation:

$$\frac{D}{Dt}\overline{\theta u_i} = D_{i\theta} + P_{i\theta} + F_{i\theta} + T_{i\theta} + \Pi_{i\theta} - \varepsilon_{i\theta}$$

$$D_{i\theta} = \gamma(\overline{u_i \theta})_{,k} + \nu(\overline{u_{i,k} \theta})_{,k}$$

$$P_{i\theta} = -\overline{\theta u_k} U_{i,k} - \overline{u_i u_k} \Theta_{,k}$$

$$F_{i\theta} = \beta_i \overline{\theta^2}$$

$$T_{i\theta} = -[\overline{\theta u_i u_k} + \frac{1}{\rho} \overline{p \theta} \delta_{ik}]_{,k}$$

$$\Pi_{i\theta} = \frac{1}{\rho} \overline{p \theta_{,i}}$$

$$\varepsilon_{i\theta} = (\nu + \gamma) \overline{\theta_{,k} u_{i,k}}$$

$T_{i\theta}$ ,  $\Pi_{i\theta}$  and  $\varepsilon_{i\theta}$  must be modeled.

$$\frac{D}{Dt}\overline{\theta^2} = \gamma(\overline{\theta^2})_{,kk} - 2\overline{\theta u_k} \Theta_{,k} - (\overline{\theta^2 u_k})_{,k} - 2\gamma \overline{\theta_{,k} \theta_{,k}}$$

## Model Constraints

1. Invariant — independent of coordinate rotation.
  2. Symmetry in various indices.
  3. Correct behavior at large and small Reynolds numbers and incompressibility.
- 
4. Realizability (Schumann 1977, Lumley, 1978):
    - Non-negative variance of all turbulent quantities (Realizability).
    - Schwarz's inequality for all turbulent quantities (Joint Realizability).
- 
5. Material frame indifference at 2D-2C turbulence.
  6. Linearity principle of passive scalar turbulence.
  7. ....

## Assumptions

Turbulence can be characterized by energy containing scales.

.....

## The Present State of Various Closure Models

Pressure strain (and temperature gradient) correlation terms:

$$\Pi_{ij}, \Pi_{i\theta}$$

Molecular dissipation terms:

$$\varepsilon_{ij}, \varepsilon_{i\theta}$$

Turbulent diffusion terms (triple correlation plus pressure transport):

$$-[\overline{u_i u_j u_k} + \frac{1}{\rho}(\overline{u_i p} \delta_{jk} + \overline{u_j p} \delta_{ik})],_k$$

$$-[\overline{\theta u_i u_k} + \frac{1}{\rho} \overline{p \theta} \delta_{ik}],_k$$

$$\overline{\theta^2 u_i}$$

Mechanical and scalar dissipation rates:  $\varepsilon, \varepsilon_\theta$



Modeling of pressure correlation terms:

$$-\frac{1}{\rho}p_{,jj} = 2U_{i,j}u_{j,i} + u_{i,j}u_{j,i} - \beta_i\theta_{,i} - \overline{u_{i,j}u_{j,i}}$$

Based on the linearity of  $p$ ,

$$-\frac{1}{\rho}p_{,jj}^{(1)} = 2U_{i,j}u_{j,i} \quad \text{rapid part}$$

$$-\frac{1}{\rho}p_{,jj}^{(2)} = u_{i,j}u_{j,i} \quad \text{slow part}$$

$$-\frac{1}{\rho}p_{,jj}^{(3)} = -\beta_i\theta_{,i} \quad \text{buoyancy part}$$

one writes

$$\Pi_{ij} = \Pi_{ij}^{(1)} + \Pi_{ij}^{(2)} + \Pi_{ij}^{(3)}$$

$$\Pi_{i\theta} = \Pi_{i\theta}^{(1)} + \Pi_{i\theta}^{(2)} + \Pi_{i\theta}^{(3)}$$

Using the solutions of above equations, we obtain (for homo. flows)  
for the rapid terms:

$$\begin{aligned}\Pi_{ij}^{(1)} &= -2U_{p,q} \frac{1}{4\pi} \int_V [(\overline{u_q(r)u_i(r')})_{,pj} + (\overline{u_q(r)u_j(r')})_{,pi}] \frac{dv}{|r-r'|} \\ &= 2U_{p,q} (X_{pjqi} + X_{piqj})\end{aligned}$$

$$\begin{aligned}\Pi_{i\theta}^{(1)} &= -2U_{j,k} \frac{1}{4\pi} \int_V [\overline{u_k(r)f(r')}]_{,ij} \frac{dv}{|r-r'|} \\ &= 2U_{j,k} X_{ijk}\end{aligned}$$

for buoyancy terms:

$$\begin{aligned}\Pi_{ij}^{(3)} &= \beta_k \frac{1}{4\pi} \int_V [(\overline{\theta(r)u_i(r')})_{,kj} + (\overline{\theta(r)u_j(r')})_{,ki}] \frac{dv}{|r-r'|} \\ &= -\beta_k (Y_{kij} + Y_{kji})\end{aligned}$$

$$\begin{aligned}\Pi_{i\theta}^{(3)} &= \beta_k \frac{1}{4\pi} \int_V [\overline{\theta(r)\theta(r')}]_{,ik} \frac{dv}{|r-r'|} \\ &= -\beta_k Y_{ik}\end{aligned}$$

For slow term, following Lumley (1978):

$$\begin{aligned}
 -\Phi_{ij}\varepsilon &= \Pi_{ij}^{(2)} - \varepsilon_{ij} + \frac{2}{3}\varepsilon\delta_{ij} \\
 -\Phi_{i\theta}\frac{\varepsilon}{q^2} &= \Pi_{i\theta}^{(2)} - \varepsilon_{i\theta}
 \end{aligned}$$

To model the pressure related terms, we need to model the following tensors:

$$\begin{array}{ll}
 X_{pjqi}, & X_{ijk} \\
 Y_{kij}, & Y_{ik} \\
 \Phi_{ij}, & \Phi_i
 \end{array}$$

Note that the dissipation tensors  $\varepsilon_{ij}$  and  $\varepsilon_{i\theta}$  have been combined into the slow terms.

# Models for the rapid terms:

$$\Pi_{ij}^{(1)} = \frac{1}{\rho} \overline{p^{(1)}(u_{i,j} + u_{j,i})}$$

Based on  $\overline{u_i u_j}$  only:

Naot, Shavit & Wolfshtein (1970)

Launder, Reece & Rodi (1975)

Speziale, Sarkar and Gatski (1990)

Realizability

Shih & Lumley (1985)

Fu & Launder (1987)

Ristorcelli & Lumley (1988)

Reynolds (1988)

Based on  $\overline{u_i u_j}$  & eddy structure tensors  $D_{ij}$ :

Reynolds (1990)

$$\Pi_{i0}^{(1)} = \frac{1}{\rho} \overline{p^{(1)} \theta_{,i}}$$

Based on  $\overline{u_i u_j}$  and  $\overline{\theta u_i}$ :

Launder (1975)

Zeman and Lumley (1976)

Lumley (1978)

Shih & Lumley (1985)

Ristorcelli & Lumley (1988)

Craft and Launder(1989)

## Reynolds eddy structure model:

Basic idea:

$$\Pi_{iq}^{(1)} = 2U_{p,j}(X_{ijpq} + X_{qjpi})$$

$$X_{ijpq} = \int_K \frac{k_p k_q}{k^2} E_{ij}(\mathbf{k}) d^3 \mathbf{k}$$

$$X_{ijpp} = \overline{u_i u_j}$$

$$X_{iipq} = \int_K \frac{k_p k_q}{k^2} \bar{E}_{ii}(\mathbf{k}) d^3 \mathbf{k} = D_{pq} \quad \text{orientation tensor}$$

$$\Pi_{ij}^{(1)} = F(\overline{u_i u_j}, D_{ij})$$

In addition to Reynolds stress equations, we need to solve six extra modeled equations for  $D_{ij}$ .



Shih & Lumley:

$$\begin{aligned}\Pi_{i\theta}^{(1)} &= 0.8\overline{\theta u_k U_{i,k}} - 0.2\overline{\theta u_k U_{k,i}} + 2C_{D1}\overline{\theta u_k b_{ij} U_{j,k}} \\ &\quad + 2C_{D2}(\overline{\theta u_j b_{ik}} + \overline{\theta u_i b_{jk}})U_{j,k} + 2C_{D3}\overline{\theta u_l b_{kl} U_{i,k}}\end{aligned}$$

Craft & Launder:

$$\begin{aligned}\Pi_{i\theta}^{(1)} &= 0.8\overline{\theta u_k U_{i,k}} - 0.2\overline{\theta u_k U_{k,j}} + \frac{1}{6}\frac{\varepsilon}{k}\overline{\theta u_i} \frac{P_{kk}}{\varepsilon} \\ &\quad - 1.6\overline{\theta u_k b_{il} S_{kl}} + 0.8\overline{\theta u_k b_{ik} b_{ml} S_{ml}} - 0.2\overline{\theta u_k (b_{im} P_{mk} + 2b_{mk} P_{im})} \\ &\quad + 1.2b_{mk} S_{kl}(\overline{b_{mk} \theta u_i} - \overline{b_{mi} \theta u_k}) \\ &\quad - 0.2b_{ml}[7\overline{b_{mk} (\theta u_i U_{k,l} + \theta u_k U_{i,l})} - \overline{\theta u_k (b_{ml} U_{i,k} + b_{mk} U_{i,l})}]\end{aligned}$$



### Difference between models of LRR and SSG:

- Model coefficients.
- If appropriate model constraints imposed, LRR and SSG should be exactly the same.

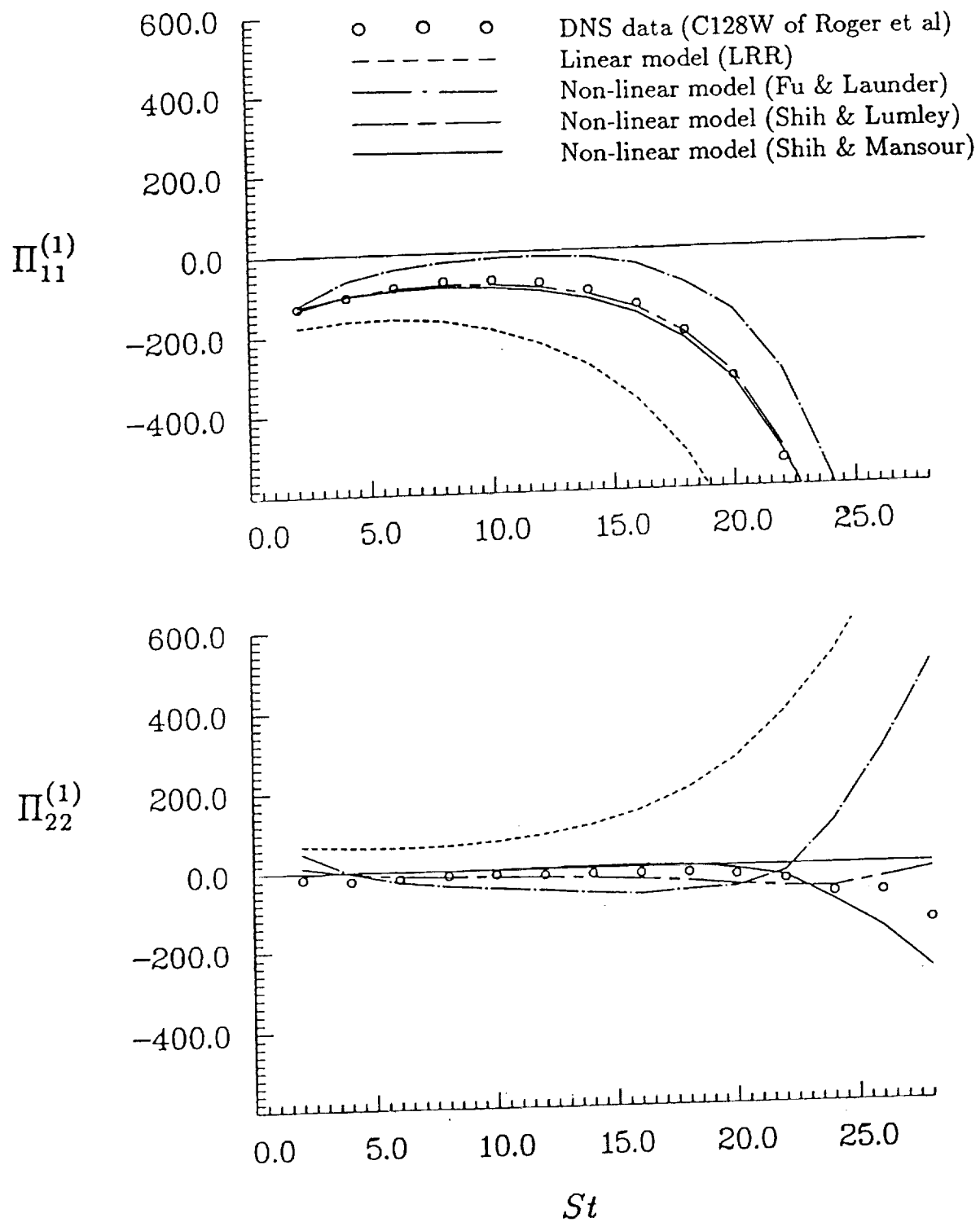
### Difference between models of UMIST and Cornell:

- UMIST: Realizability on  $\overline{u_i u_j}$ .
- Cornell: Both Realizability and Joint Realizability on  $\overline{u_i u_j}$ ,  $\overline{\theta u_i}$ .

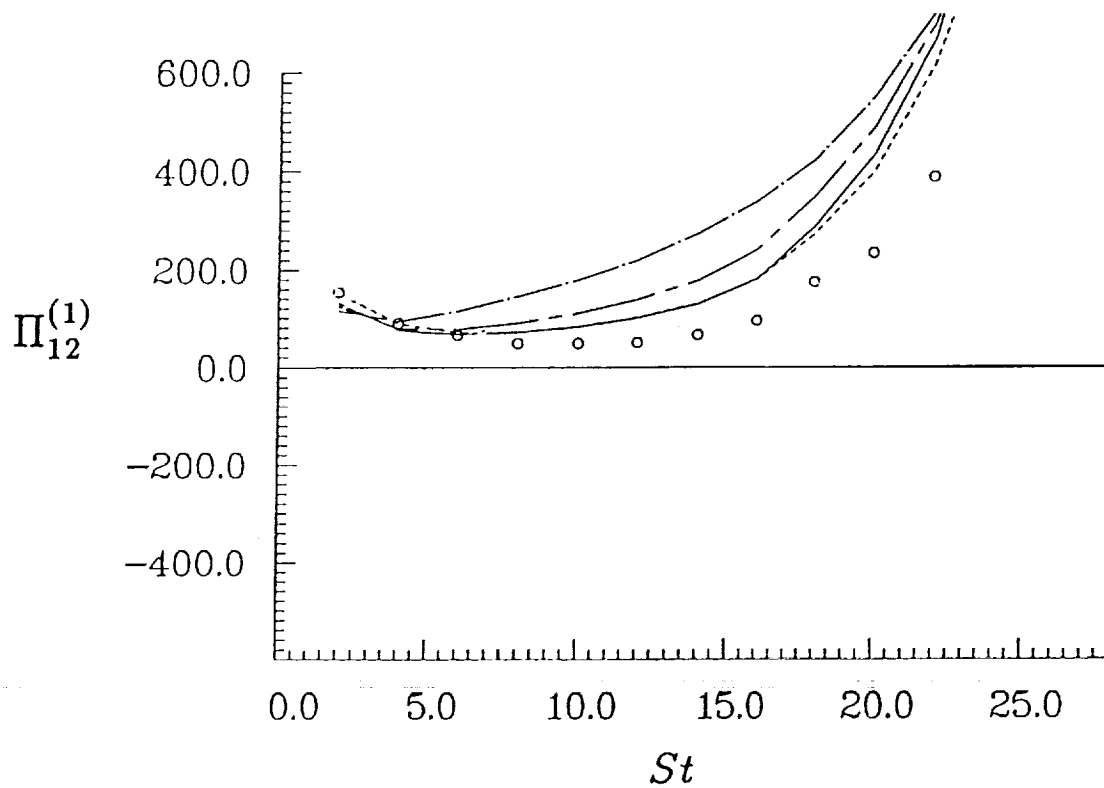
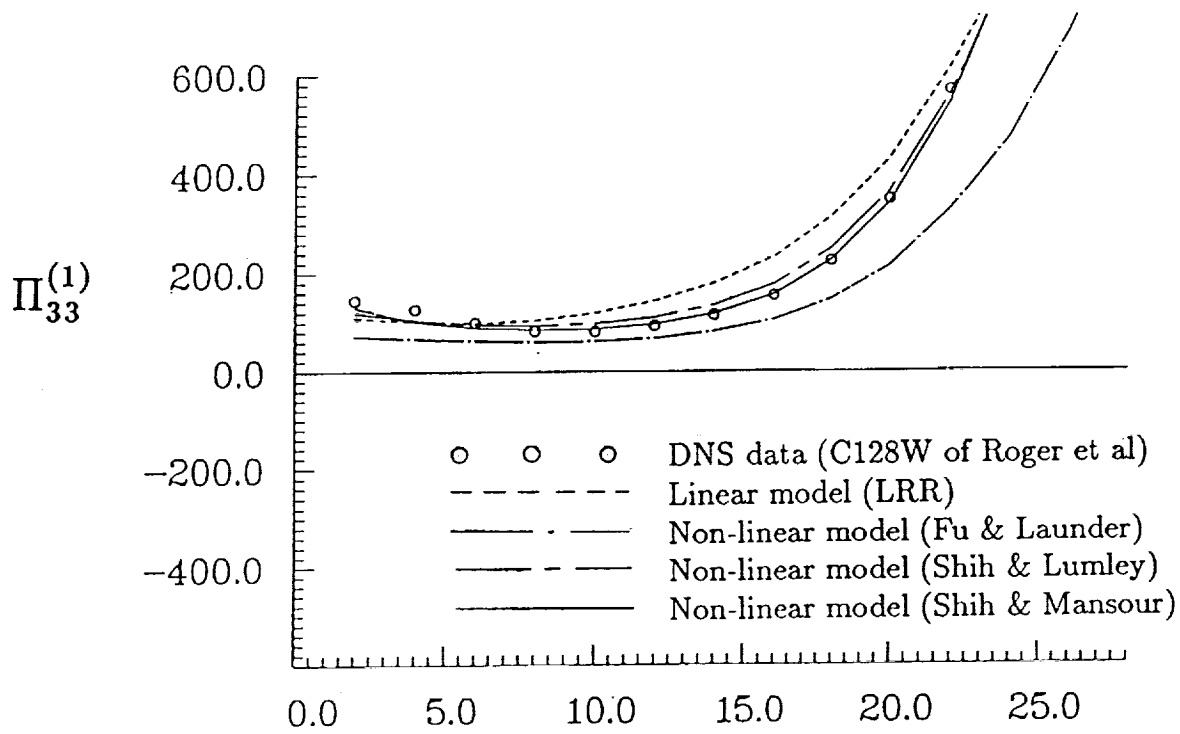
Imposing joint realizability results in a simpler rapid pressure models (Lumley, 1983, Shih & Lumley, 1985).

If joint realizability is imposed, we expect that UMIST model will reduce to Cornell model.

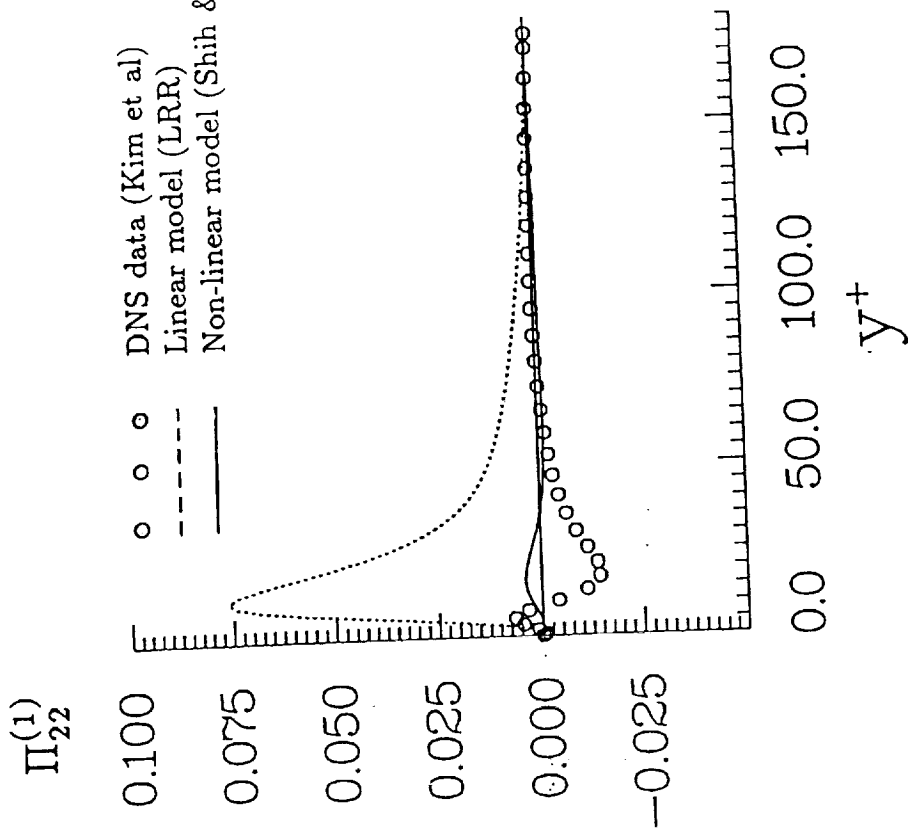
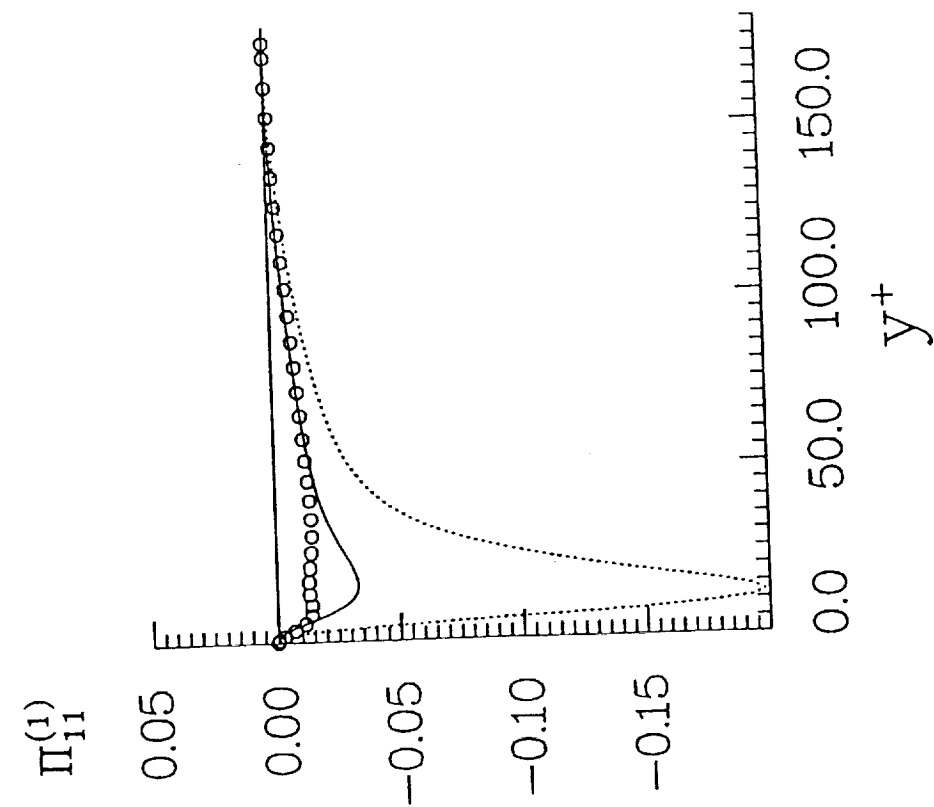
## Validation of the models:



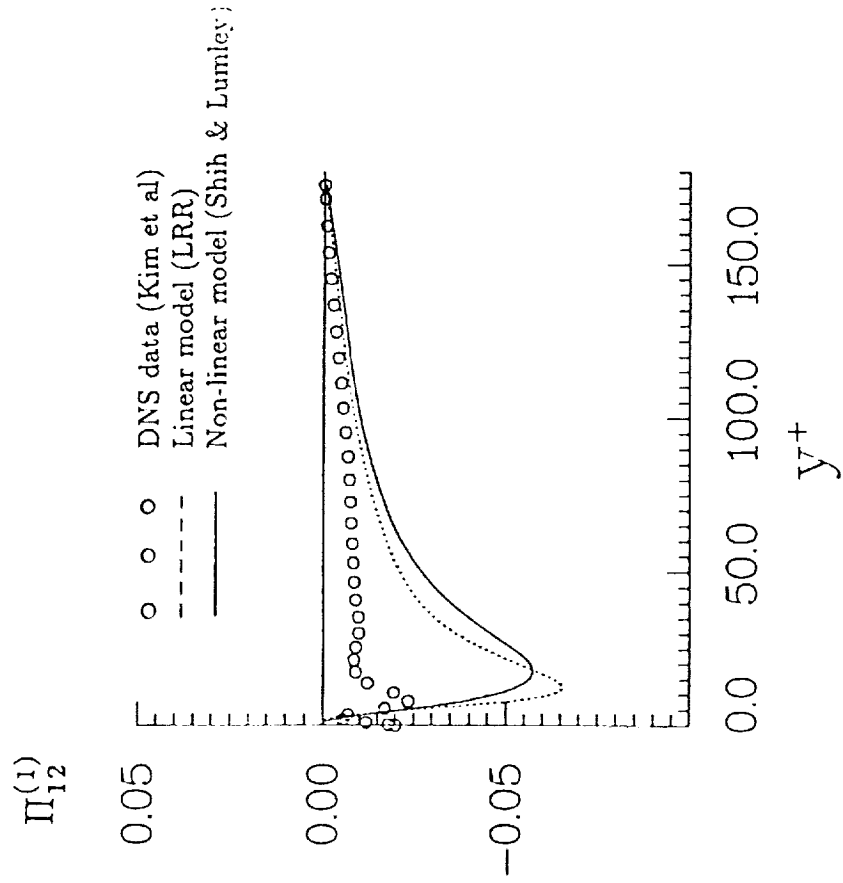
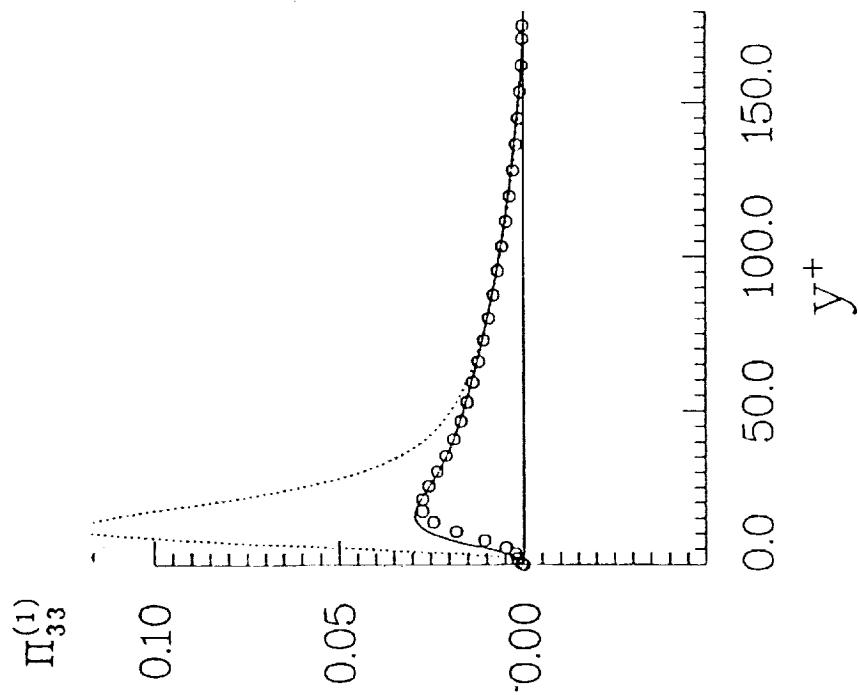
Homogeneous Shear flow: Shear rate  $S = 56.57$



Homogeneous Shear flow: Shear rate  $S = 56.57$



Two-Dimensional Channel Flow:  $Re_\tau = 180$



Two-Dimensional Channel Flow:  $Re_\tau = 180$

## Models for the slow terms:

$$\Pi_{ij}^{(2)} = \frac{1}{\rho} \overline{p^{(2)}(u_{i,j} + u_{j,i})}$$

Based on  $\overline{u_i u_j}$  only:

Rotta (1951)

Lumley (1978)

Shih and Mansour (1988)

Sakar and Speziale (1989)

Craft and Launder (1989)

Based on two-point correlation:

Weinstock (1987)

Based on  $\overline{u_i u_j}$  and eddy structure tensors:

Reynolds (1990)

$$\Pi_{i\theta}^{(2)} = \frac{1}{\rho} \overline{p^{(2)}\theta_i}$$

Based on  $\overline{\theta u_i}$  only:

Monin (1965)

Lumley (1978)

Shih and Lumley (1985)

Shih and Shabbir (1990)

Gibson et al (1987)

Based on  $\overline{\theta u_i}$  and  $\overline{u_i u_j}$ :

Craft and Launder (1989)

Realizability  $\rightarrow$

Lumley (1978):

$$\Pi_{ij}^{(2)} = -\varepsilon[\beta b_{ij} + \gamma(b_{ij}^2 + 2II\delta_{ij}/3)]$$

$$\beta = \beta(II, III, Re), \quad \gamma = \gamma(II, III, Re), \quad Re = \frac{q^2}{9\varepsilon\nu}$$

Weinstock (1981):

$$\Pi_{ij}^{(2)} = -\varepsilon C_{ij} b_{ij} \quad (\text{no summation on } i \text{ and } j)$$

$$C_{ij} = F_{ij}(\text{Int. of } E_{ij}(\mathbf{k}))$$

Reynolds (1990):

$$\Pi_{ij}^{(2)} = -0.3(d_{ik}\overline{u_k u_j} + d_{jk}\overline{u_k u_i} - 2d_{nm}R_{nm}d_{ij})/\tau$$



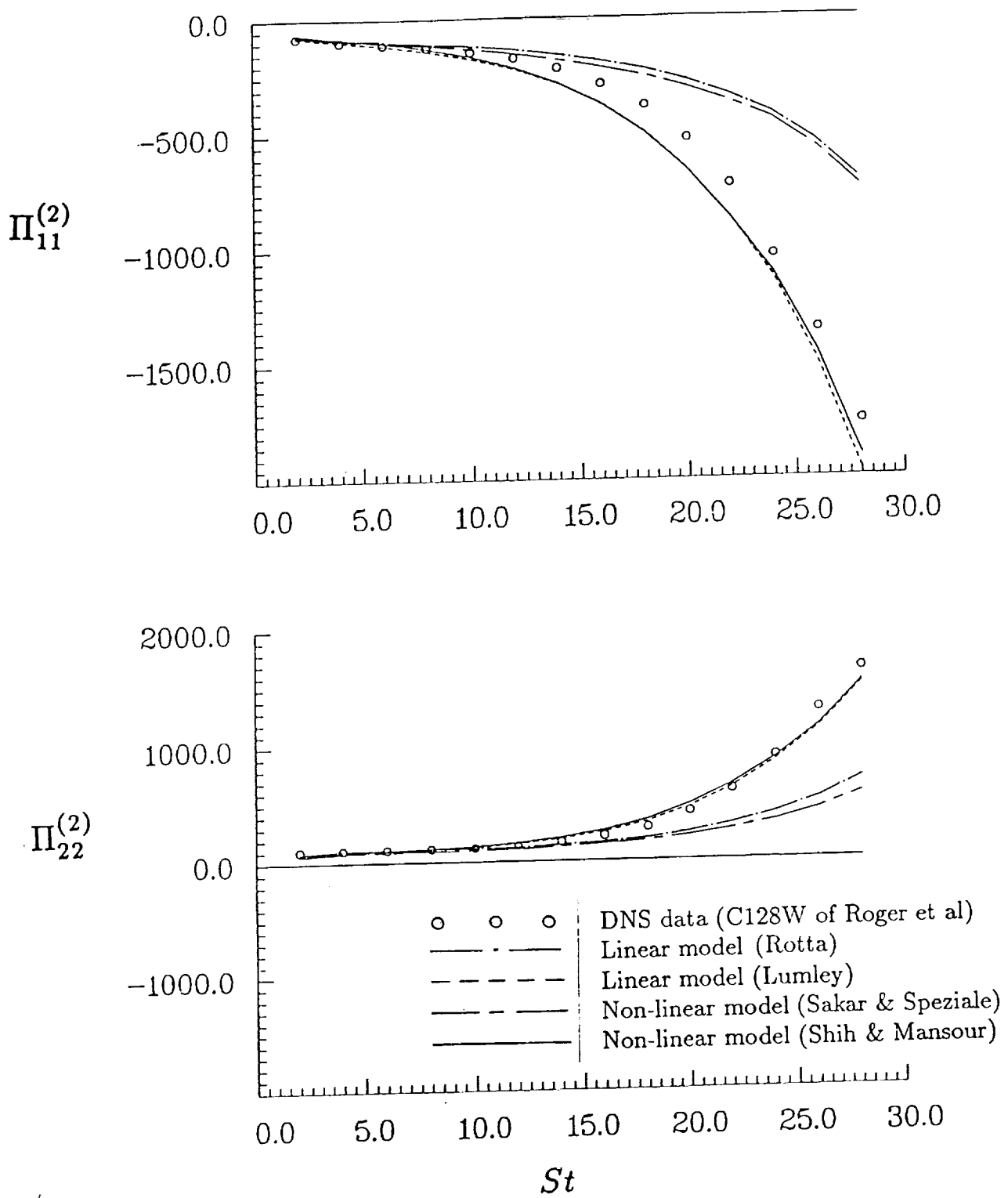
Shih and Lumley (1985):

$$\Pi_{i\theta}^{(2)} = -\frac{\varepsilon}{2k}(\phi_1 \overline{\theta u_i} + \phi_2 \overline{b_{ik} \theta u_k})$$

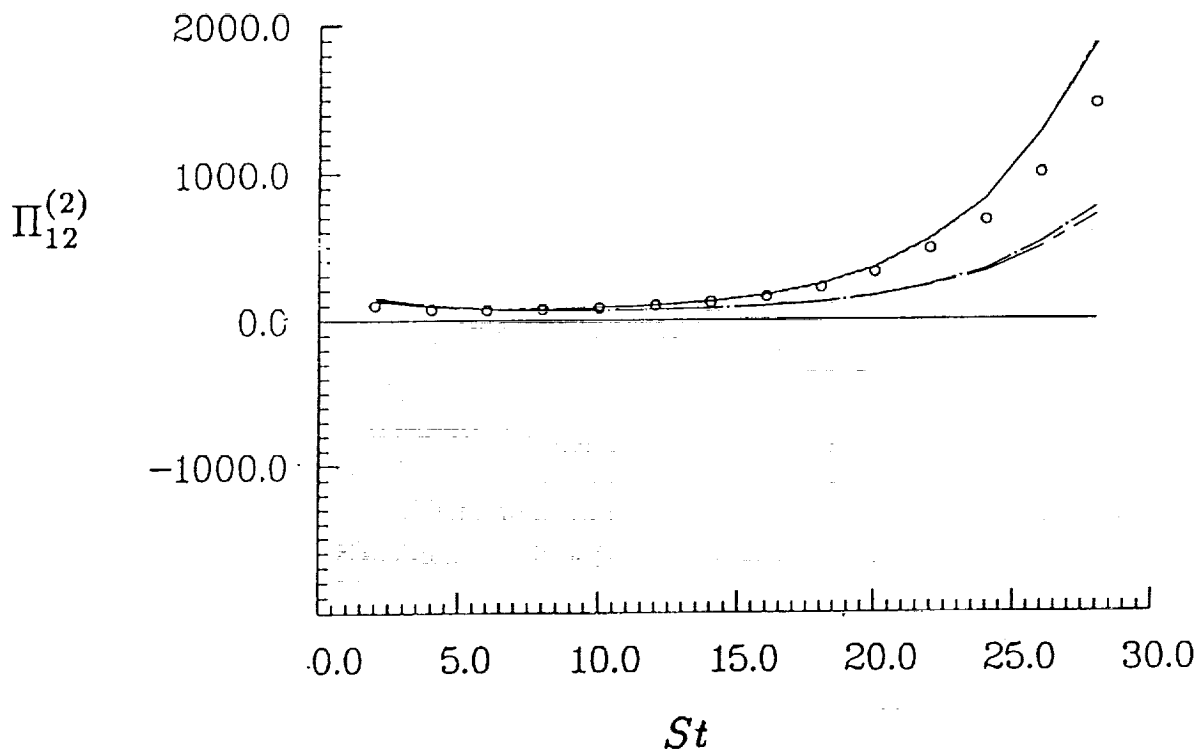
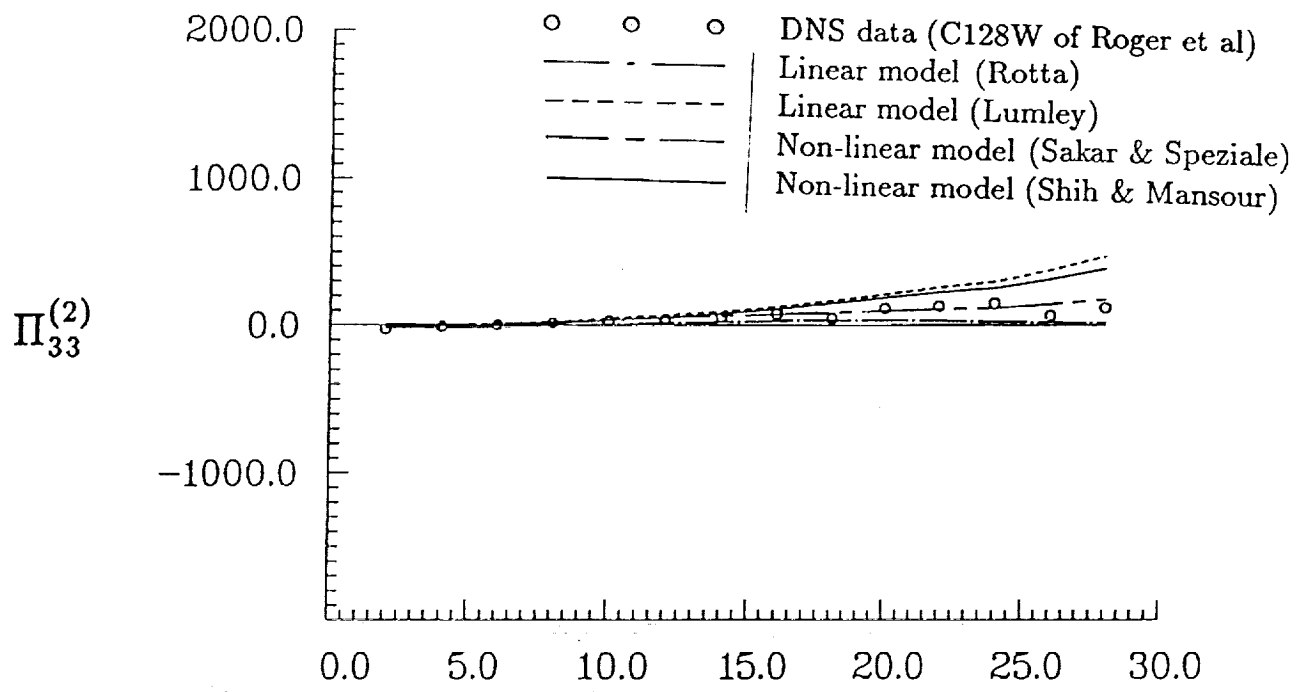
Craft and Launder (1989):

$$\Pi_{i\theta}^{(2)} = F(\overline{\theta u_i}, \overline{b_{ik} \theta u_k}, \overline{b_{ik} b_{kj} \theta u_j}, \overline{b_{ij} \Theta_{,j}})$$

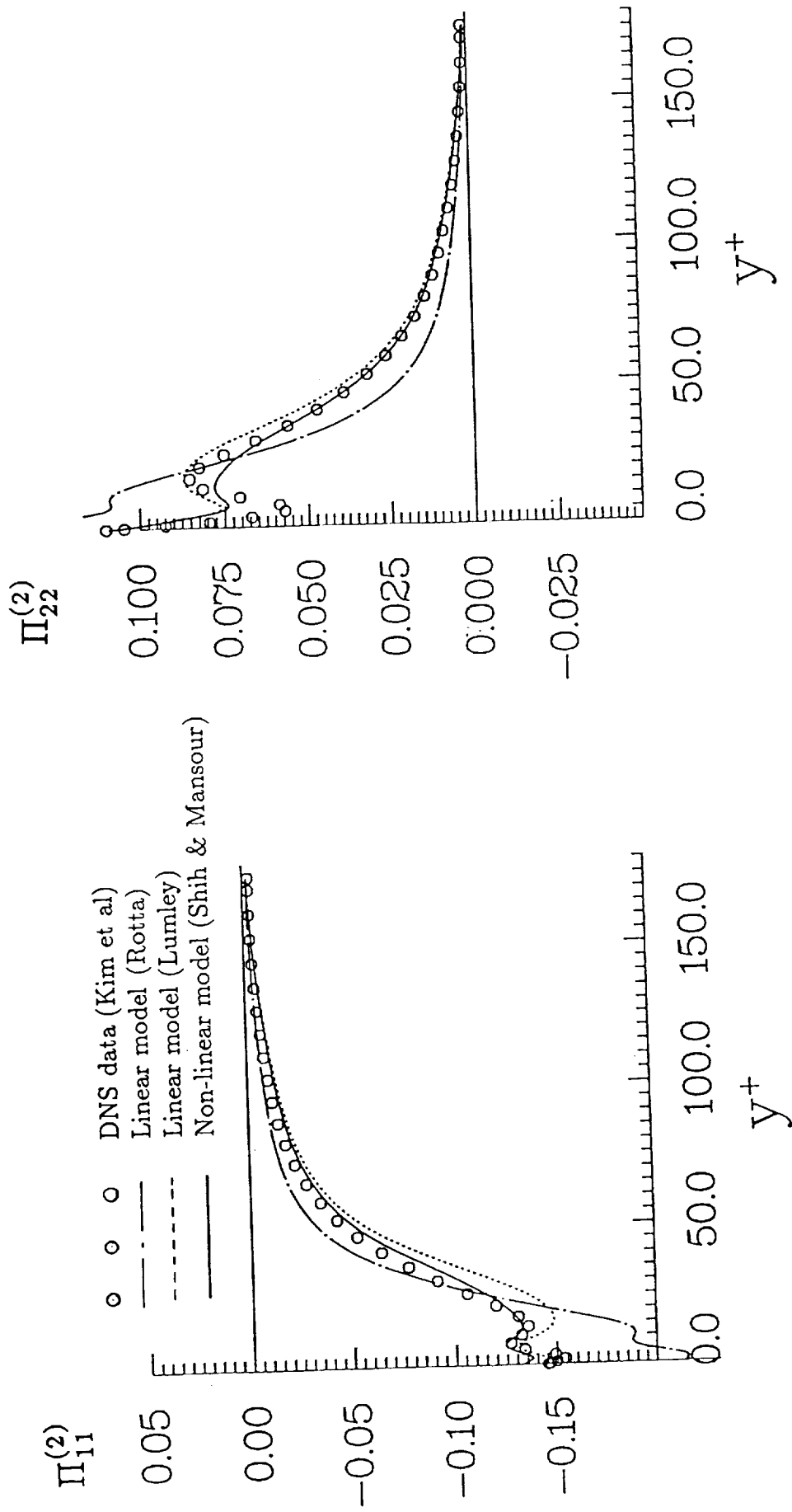
## Validation of the models:



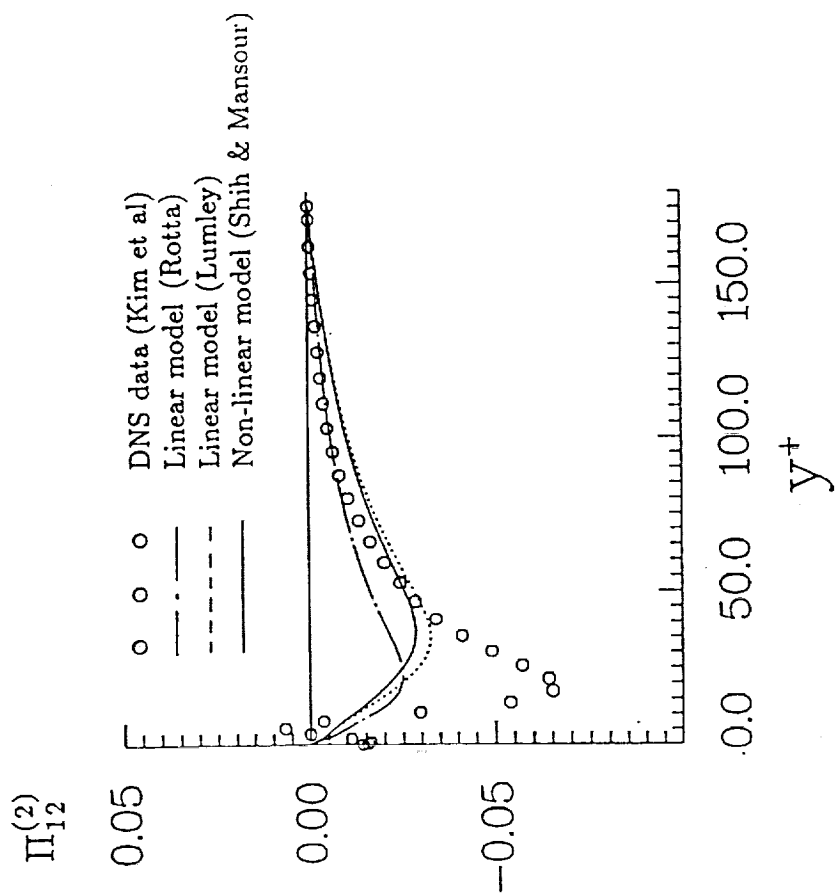
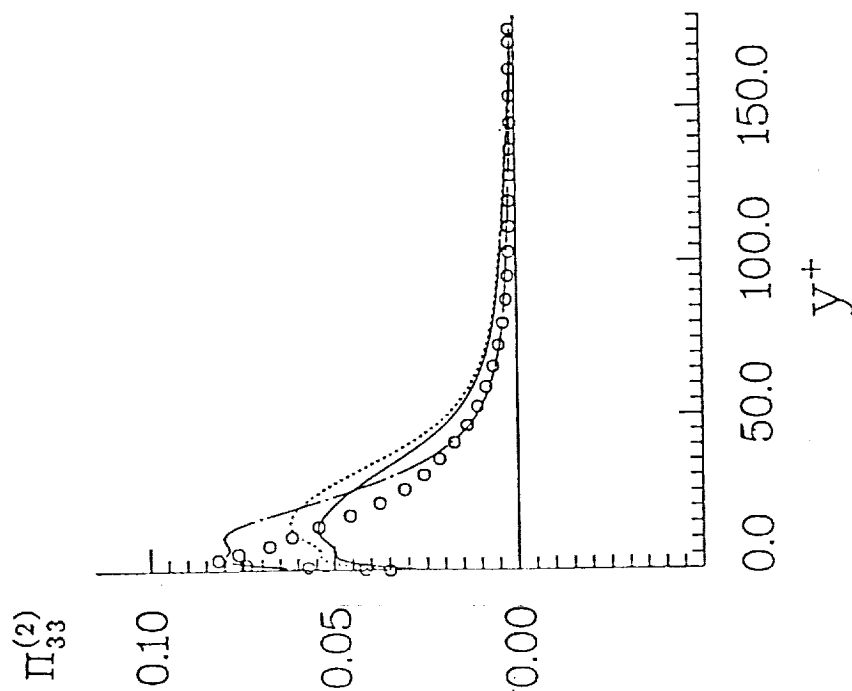
**Homogeneous Shear flow: Shear rate  $S = 56.57$**



Homogeneous Shear flow: Shear rate  $S = 56.57$



Two-Dimensional Channel Flow:  $R_{e\tau} = 180$



Two-Dimensional Channel Flow:  $R_{e\tau} = 180$

Models for the buoyancy terms:

$$\Pi_{ij}^{(3)} = \frac{1}{\rho} \overline{p^{(3)}(u_{i,j} + u_{j,i})}$$

Launder (1975)

$$\Pi_{i\theta}^{(3)} = \frac{1}{\rho} \overline{p^{(3)}\theta_i}$$

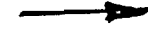
Launder (1975)

Zeman and Lumley (1976)

Zeman and Lumley (1976)

Realizability

Shih and Lumley (1985)



Shih and Lumley (1985)

Craft and Launder (1989)

Craft and Launder (1989)

$$\Pi_{ij}^{(3)} = -\beta_k(Y_{kij} + Y_{kji})$$

$$\begin{aligned} Y_{ikj} = & \beta_1 \delta_{ik} \overline{\theta u_j} + \beta_2 (\delta_{ij} \overline{\theta u_k} + \delta_{jk} \overline{\theta u_i}) + \beta_3 b_{ik} \overline{\theta u_j} \\ & + \beta_4 (b_{ij} \overline{\theta u_k} + b_{jk} \overline{\theta u_i}) + \beta_5 (\delta_{ij} b_{kp} + \delta_{kj} b_{ip}) \overline{\theta u_p} \\ & + \beta_6 \delta_{ik} b_{jp} \overline{\theta u_p} + \beta_7 b_{ik} b_{jp} \overline{\theta u_p} + \beta_8 (b_{ij} b_{kp} + b_{kj} b_{ip}) \overline{\theta u_p} \\ & + \beta_9 b_{ik}^2 \overline{\theta u_j} + \beta_{10} (b_{ij}^2 \overline{\theta u_k} + b_{jk}^2 \overline{\theta u_i}) + \beta_{11} \delta_{ik} b_{jp}^2 \overline{\theta u_p} \\ & + \beta_{12} (\delta_{ij} b_{kp}^2 + \delta_{jk} b_{ip}^2) \overline{\theta u_p} \end{aligned}$$

Craft et al (1991):

$$\begin{aligned} \Pi_{ij}^{(3)} = & -\left(\frac{4}{10} + \frac{3A_2}{80}\right) (G_{ij} - \frac{1}{3} \delta_{ij} G_{kk}) + \frac{1}{4} a_{ij} G_{kk} \\ & + \frac{3}{20} \left( \beta_i \frac{\overline{u_m u_j}}{k} + \beta_j \frac{\overline{u_m u_i}}{k} \right) \overline{u_m \theta} - \frac{1}{10} \delta_{ij} \beta_k \frac{\overline{u_m u_k}}{k} \overline{u_m \theta} \\ & - \frac{1}{4} \beta_k \left( \frac{\overline{u_k u_i}}{k} \overline{u_j \theta} + \frac{\overline{u_k u_j}}{k} \overline{u_i \theta} \right) + \frac{1}{20} \delta_{ij} \beta_k \frac{\overline{u_m u_n} \overline{u_n u_k}}{k^2} \overline{u_n \theta} \\ & - \frac{1}{8} \left[ \frac{\overline{u_m u_j}}{k} \overline{u_i \theta} + \frac{\overline{u_m u_i}}{k} \overline{u_j \theta} \right] \frac{\overline{u_m u_k}}{k} \beta_k + \frac{1}{8} \left[ \frac{\overline{u_k u_i} \overline{u_m u_j}}{k^2} + \frac{\overline{u_k u_j} \overline{u_m u_i}}{k^2} \right] \beta_k \overline{u_m \theta} \\ & - \frac{3}{40} \left( \beta_i \frac{\overline{u_m u_j}}{k} + \beta_j \frac{\overline{u_m u_i}}{k} \right) \frac{\overline{u_m u_n}}{k} \overline{u_n \theta} + \frac{1}{4} \beta_k \frac{\overline{u_m u_k} \overline{u_i u_j}}{k^2} \overline{u_m \theta}. \end{aligned}$$



Shih and Lumley (1985):

$$\Pi_{i\theta}^{(3)} = -\beta_k Y_{ik}$$

$$\begin{aligned} Y_{ik} = & \gamma_1 \overline{\theta^2} \delta_{ik} + \gamma_2 \overline{\theta^2} b_{ik} + \gamma_3 \overline{\theta u_i} \overline{\theta u_k} \\ & + \gamma_4 (\overline{\theta u_k b_{ip} \theta u_p} + \overline{\theta u_i b_{kp} \theta u_p}) \\ & + \gamma_5 \overline{\theta^2 b_{ik}^2} + \gamma_6 b_{ip} b_{kq} \overline{\theta u_p} \overline{\theta u_q} \\ & + \gamma_7 (b_{ip}^2 \overline{\theta u_p} \overline{\theta u_K} + b_{kp}^2 \overline{\theta u_p} \overline{\theta u_i}) \end{aligned}$$

Craft et al (1991):

$$\Pi_{i\theta}^{(3)} = \frac{1}{3} \overline{\theta^2} \beta_i - 2 \overline{\theta^2} b_{ik} \beta_k$$

## Validation of the models:

# Total pressure - temp. gradient in buoyant plume flow

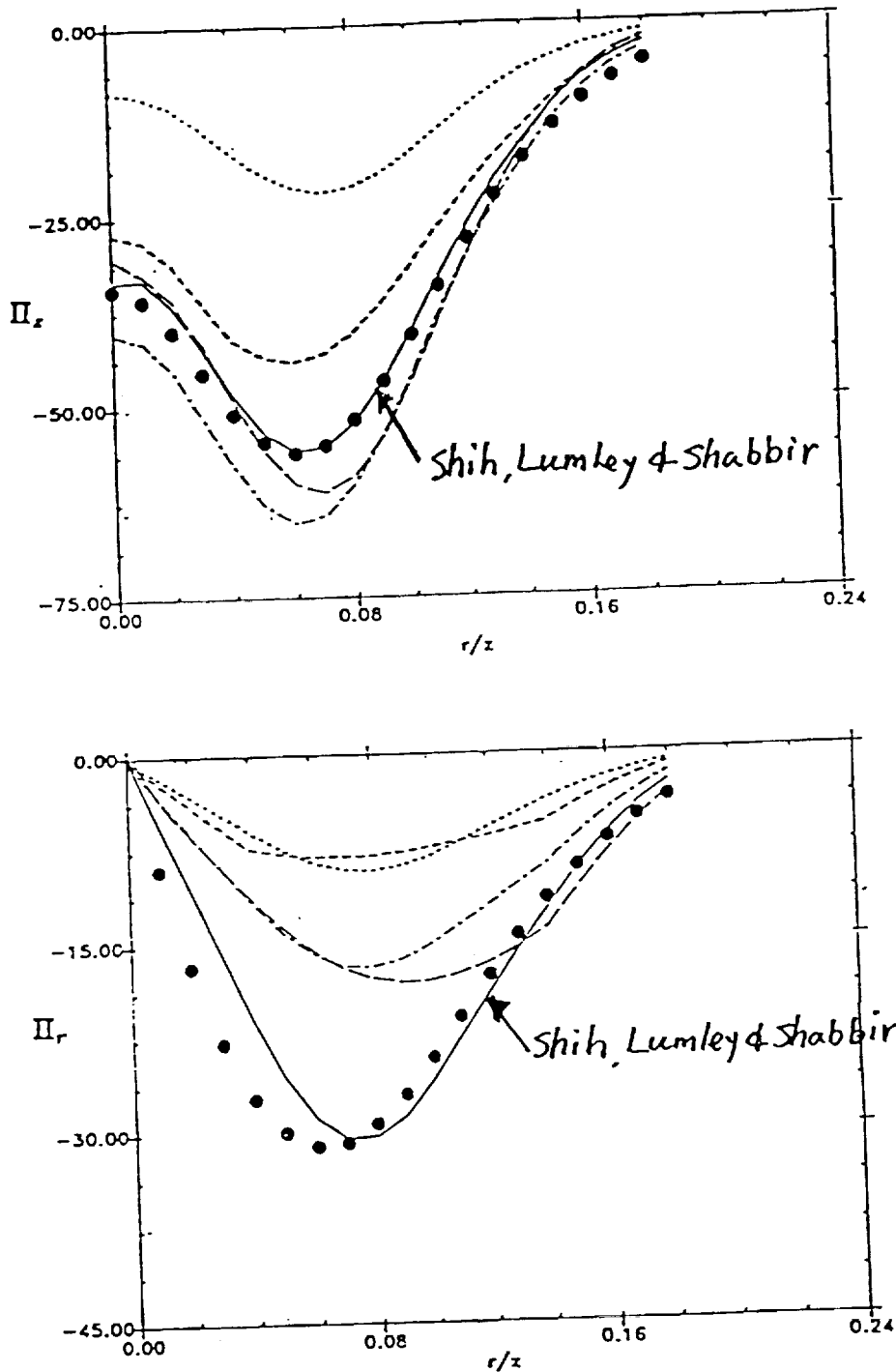


Figure 7. Comparison between models for the pressure temperature-gradient correlation and the buoyant plume experiment:..... Launder (1975) model;--- Zeman and Lumley model (1976);-.-.- Shih and Lumley (1985) model;— Craft et. al. (1990) model; ——— present model;● experiment of Shabbir and George (1990)

## Models for turbulent diffusion terms:

$$\overline{u_i u_j u_k}, \quad \overline{\theta u_i u_j}, \quad \overline{\theta^2 u_i}$$

### Algebraic models:

Daly and Harlow (1970)

Hanjalic and Launder (1972)

Lumley (1978)

Dekeyser and Launder (1983)

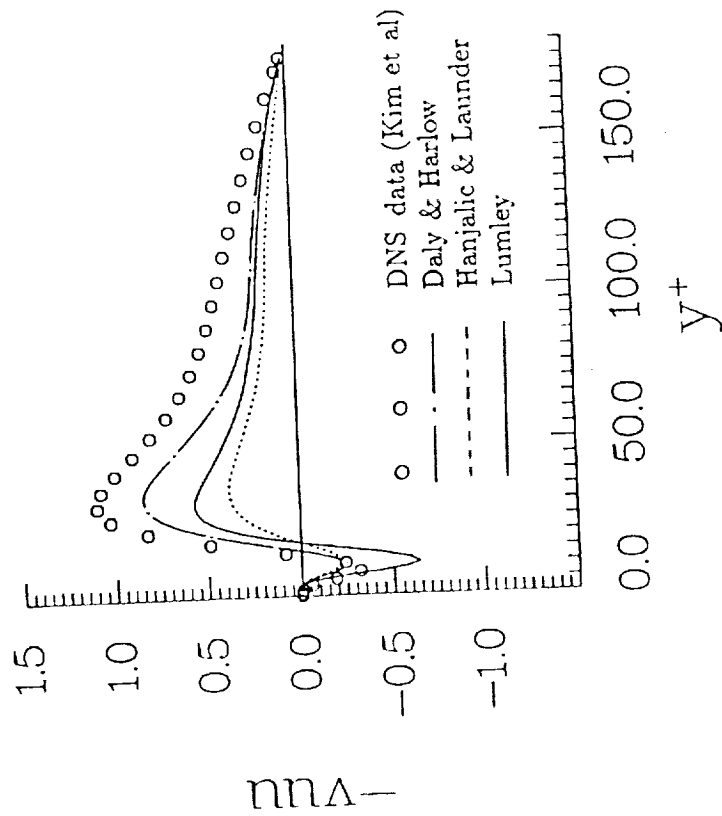
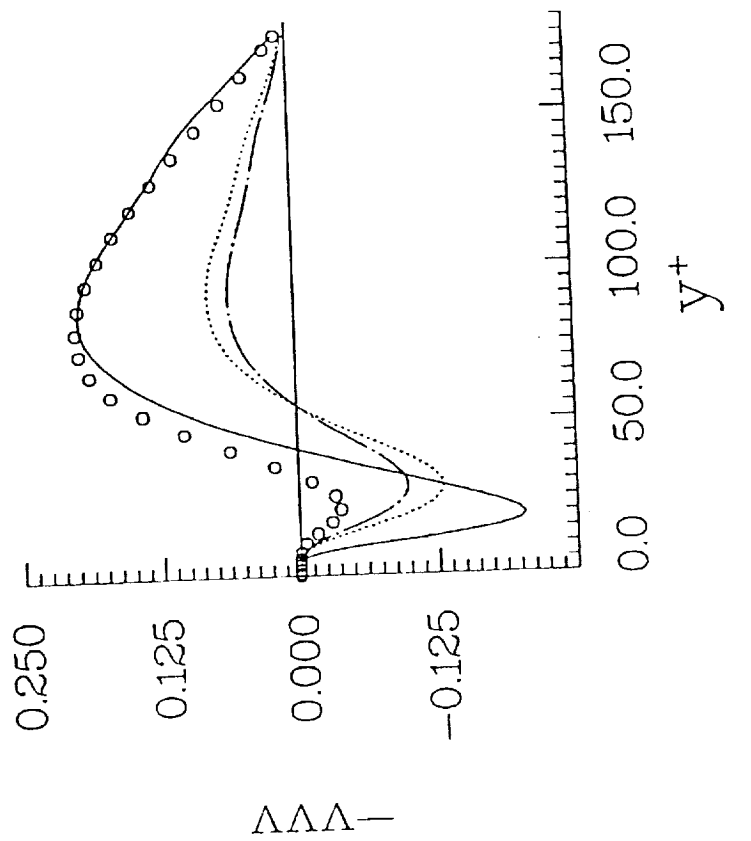
### Structural model:

Nagano and Tagawa (1990)

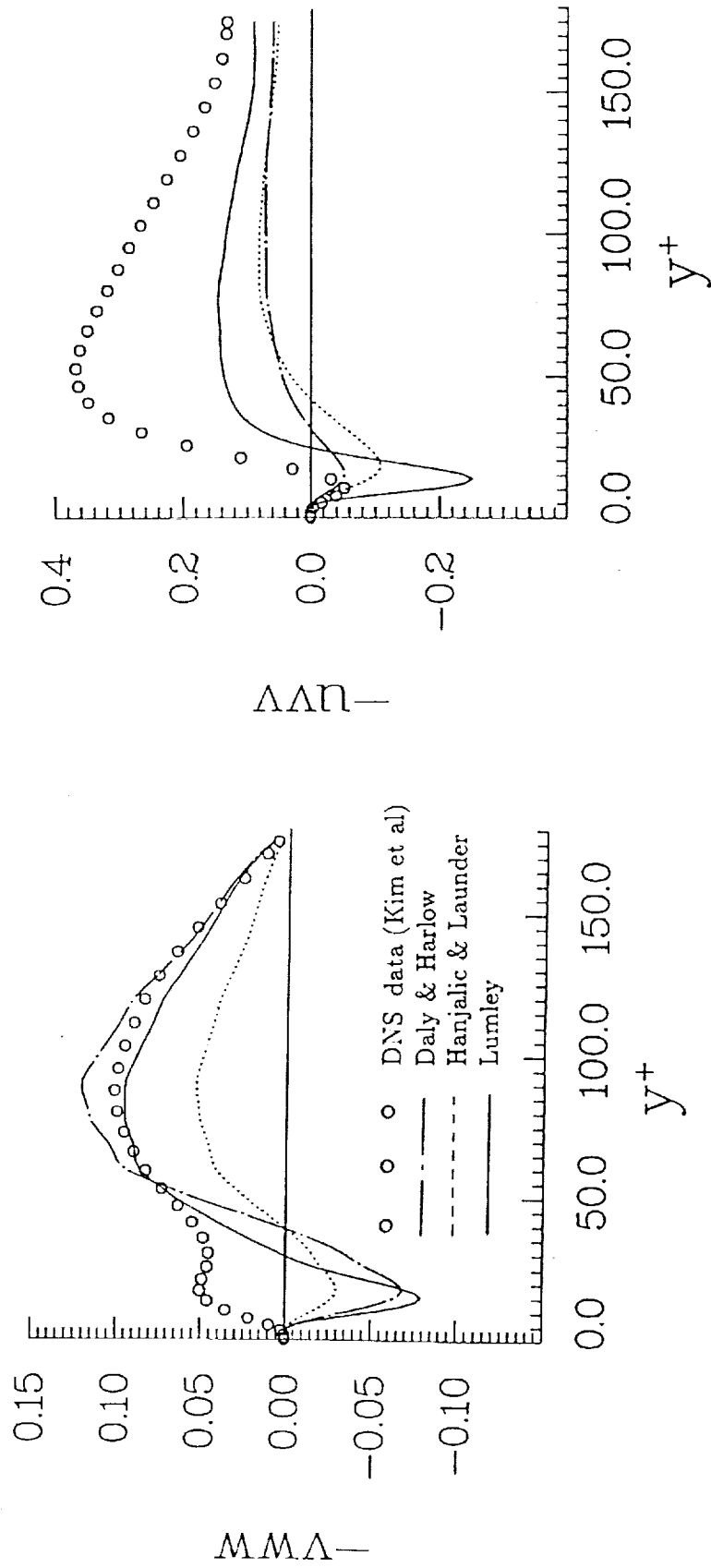
Lumley (1978):

$$\begin{aligned}
\overline{u_i u_j u_k} &= -\frac{1}{3\beta} \frac{q^2}{\bar{\epsilon}} \left[ \overline{u_k u_p (u_i u_j)}_{,p} \right. \\
&\quad + \overline{u_j u_p (u_i u_k)}_{,p} + \overline{u_i u_p (u_j u_k)}_{,p} \left. \right] \\
&\quad + \frac{\beta - 2}{9\beta} \left[ \overline{\delta_{ij} q^2 u_k} + \overline{\delta_{ik} q^2 u_j} + \overline{\delta_{jk} q^2 u_i} \right] \\
\overline{q^2 u_k} &= -\frac{3}{4\beta + 10} \frac{q^2}{\bar{\epsilon}} \left[ \overline{u_k u_p q^2}_{,p} + \overline{2 u_p u_q u_k u_{q,p}} \right] \\
\overline{\theta u_i u_j} &= -\frac{q^2}{(\beta + 2\phi_1)\epsilon} \left[ \overline{(\theta u_i)_{,k} u_j u_k} \right. \\
&\quad + \overline{(\theta u_j)_{,k} u_i u_k} + \overline{(u_i u_j)_{,k} \theta u_k} \left. \right] \\
&\quad + \frac{\beta - 2}{3(\beta + 2\phi_1)} \overline{\delta_{ij} q^2 \theta} \\
\overline{q^2 \theta} &= -\frac{q^2}{(2 + 2\phi_1)\epsilon} \left[ \overline{2 u_p u_q (\theta u_{p,q} + \theta u_q q^2_{,q})} \right] \\
\overline{\theta^2 u_k} &= -\frac{\theta^2}{(2 + \phi_1)\epsilon_\theta} \left[ \overline{u_k u_p \theta^2}_{,p} + \overline{\theta u_p \theta u_{k,p}} \right]
\end{aligned}$$

## Validation of the models:

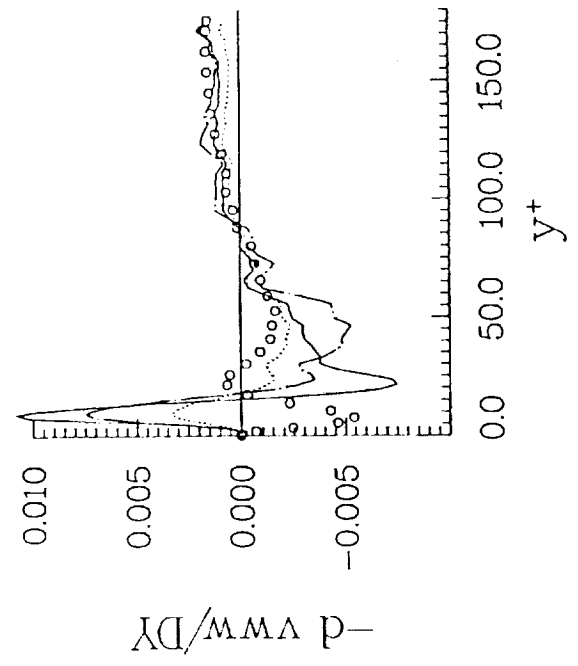
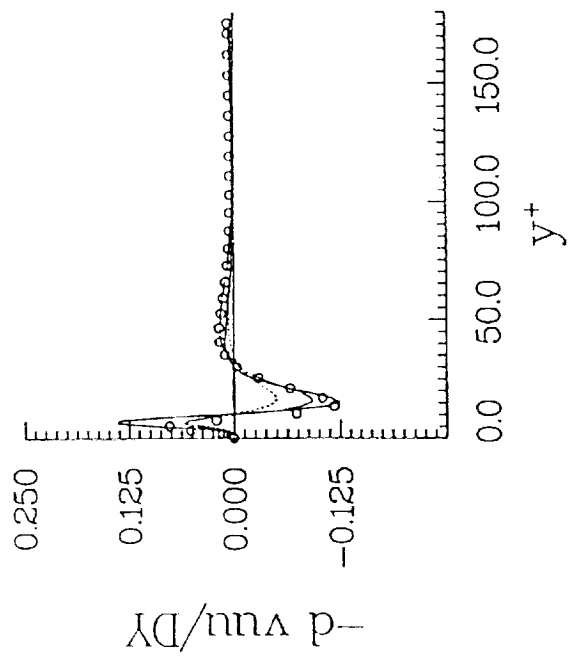
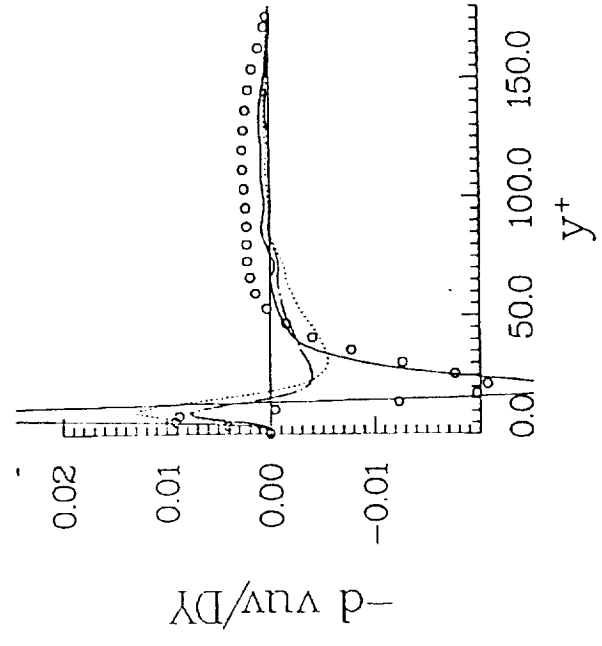
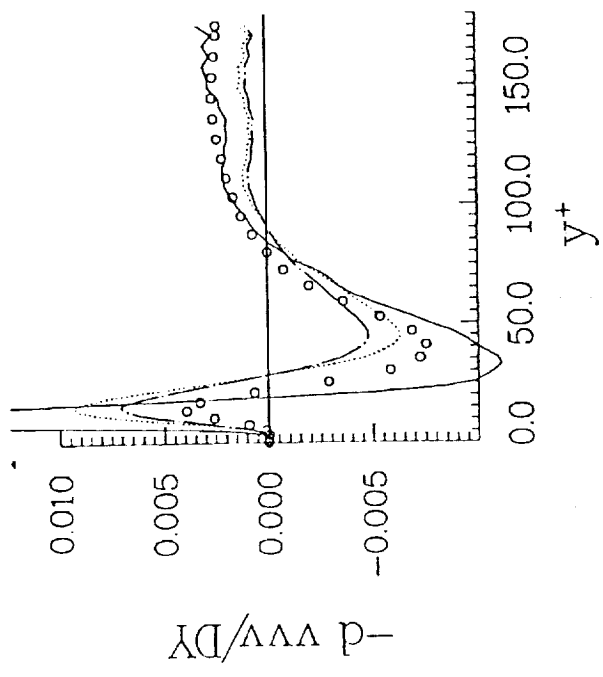


Two-Dimensional Channel Flow:  $Re_\tau = 180$



Two-Dimensional Channel Flow:  $Re_\tau = 180$





Two-Dimensional Channel Flow:  $R_{e\tau} = 180$

# Nagano and Tagawa (1990):

$$\overline{vu}v = \frac{1}{3[(\pi/2)^2 - 1]} [S(u) + \sigma_{uv} \frac{1}{2} \pi S(v)]$$

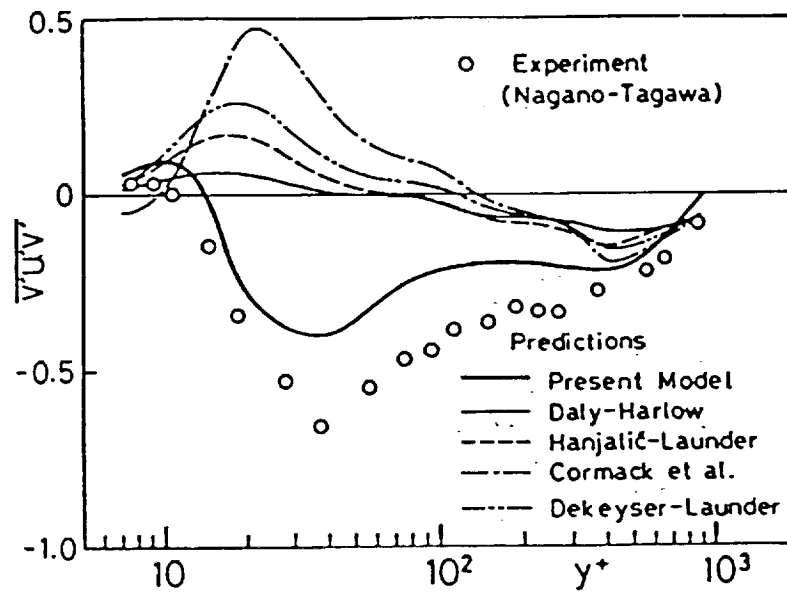
$$\overline{vuu} = \frac{1}{3[(\pi/2)^2 - 1]} [S(v) + \sigma_{uv} \frac{1}{2} \pi S(u)]$$

$$\overline{vv\theta} = \frac{1}{3[(\pi/2)^2 - 1]} [S(\theta) + \sigma_{uv} \frac{1}{2} \pi S(v)]$$

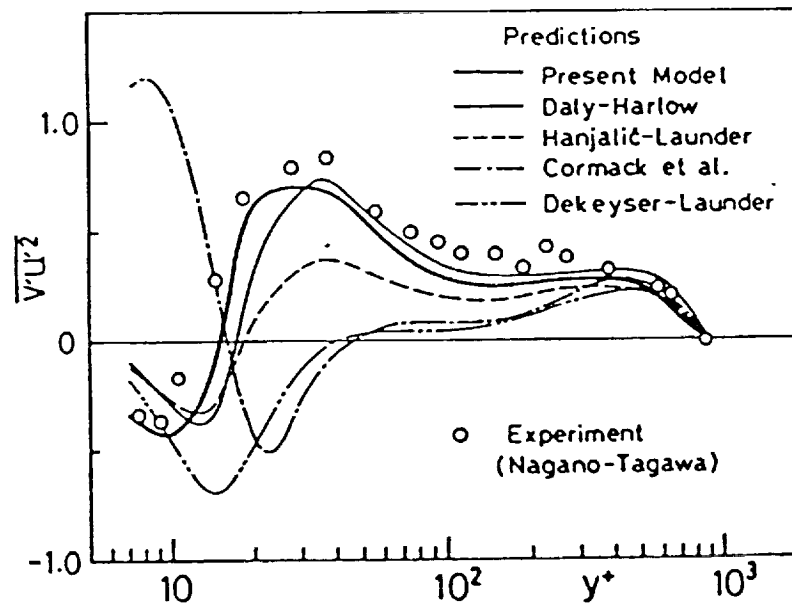
$$\overline{v\theta^2} = \frac{1}{3[(\pi/2)^2 - 1]} [S(v) + \sigma_{uv} \frac{1}{2} \pi S(\theta)]$$

$$\overline{vu\theta} = \sigma_{uv} \frac{1}{4} \pi v v \theta$$

$$\sigma_x = 1 \quad x \geq 0, \quad \sigma_x = -1 \quad x \leq 0.$$



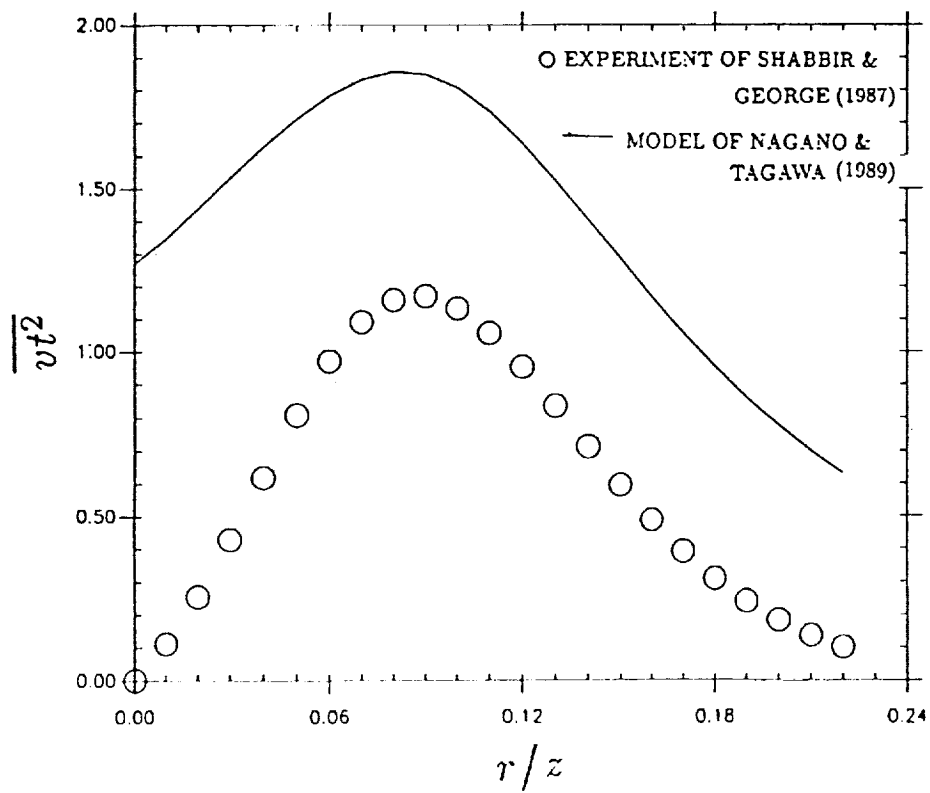
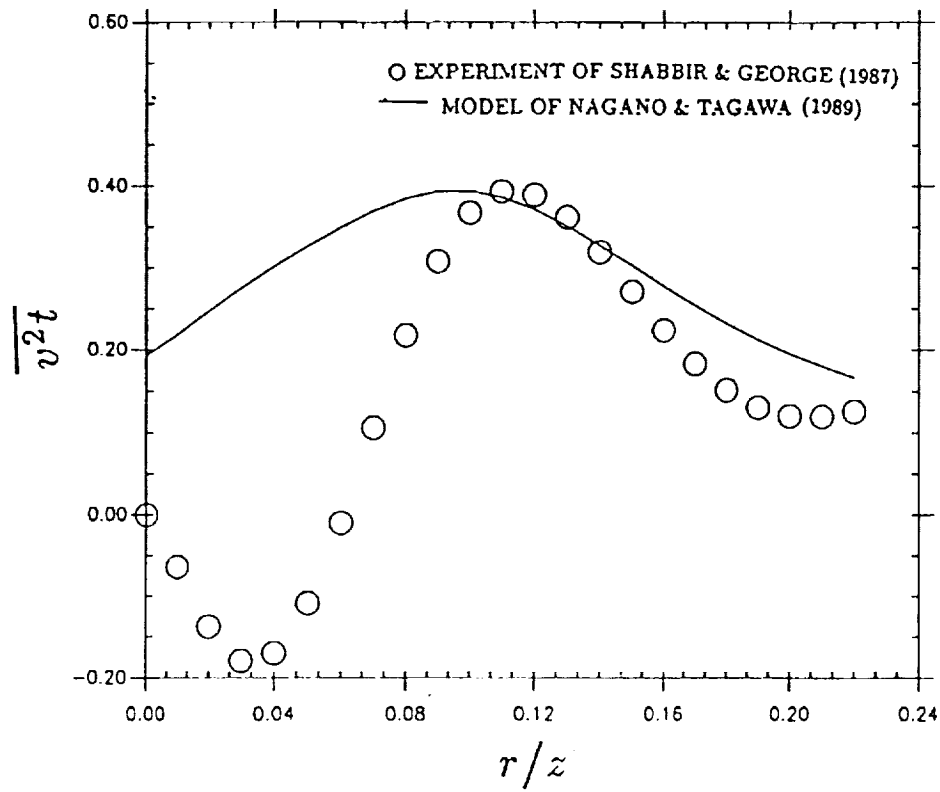
(a)  $\overline{v'u'v'}$



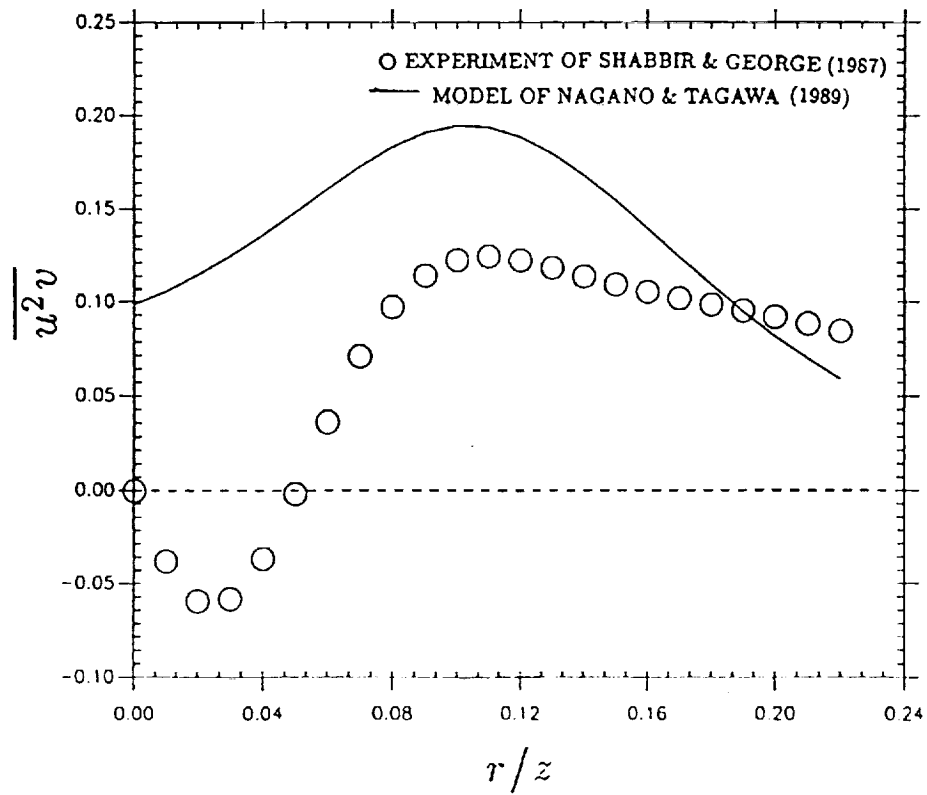
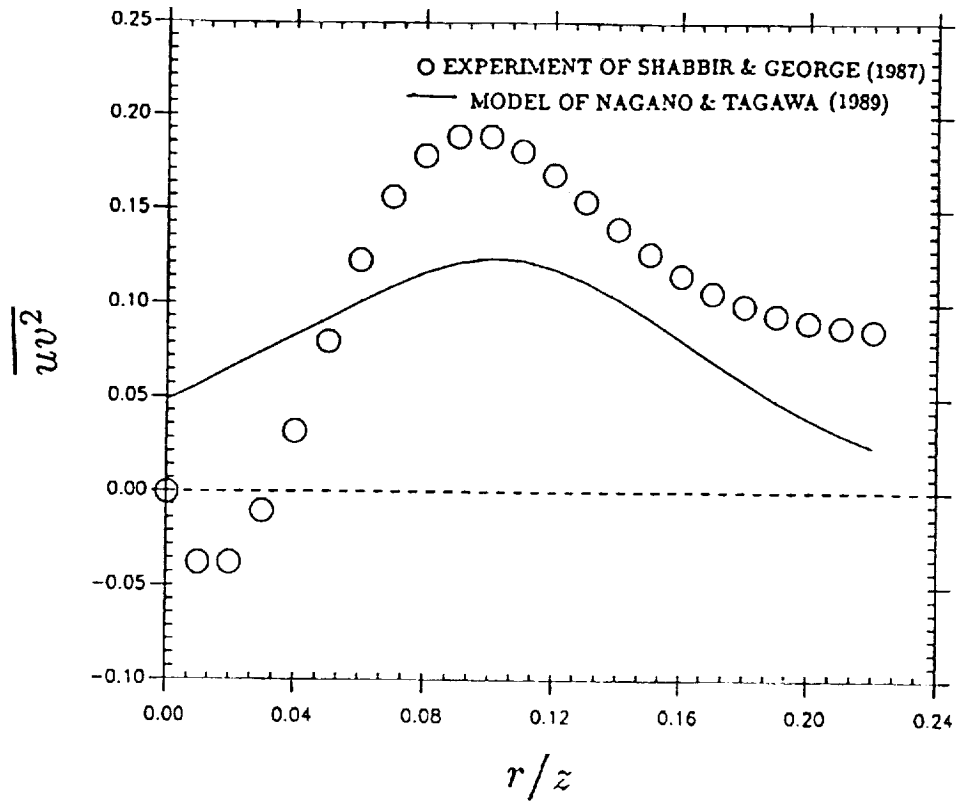
(b)  $\overline{v'u'^2}$

Fig. 5 Comparison of model results for triple velocity correlations in wall turbulence.

# BUOYANT PLUME



# BUOYANT PLUME



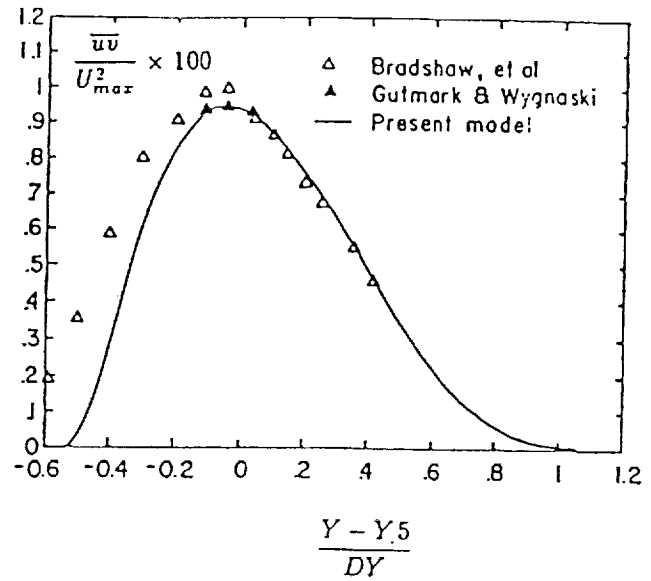


Fig. 3 Shear stress profiles in 2-D mixing layer.  $\Delta$ : Bradshaw, et al<sup>[19]</sup>,  $\blacktriangle$ : Gutmark & Wygnanski<sup>[21]</sup>, —: Present model.

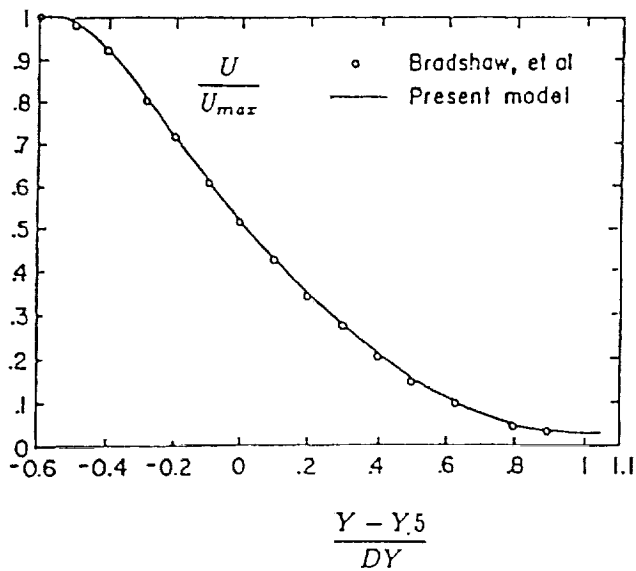


Fig. 2 Mean velocity profile in 2-D mixing layer.  $U_{max}$ : the free stream velocity,  $Y_5$ : the position where  $U = \frac{1}{2}U_{max}$ .  $\circ$ : Bradshaw, et al<sup>[19]</sup>, —: Present model.

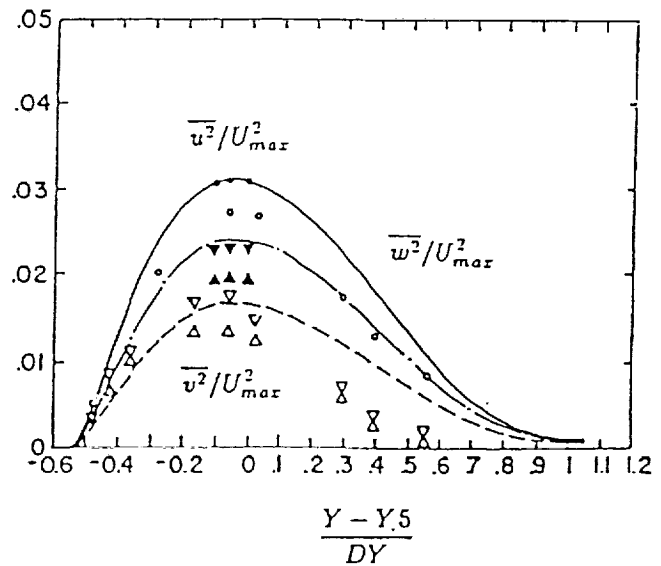


Fig. 4 Normal stress profiles in 2-D mixing layer.  $\circ, \Delta, \nabla$  Castro<sup>[20]</sup>,  $\bullet, \blacktriangle, \blacktriangledown$ : Gutmark & Wygnanski<sup>[21]</sup>, The lines represent the present model.

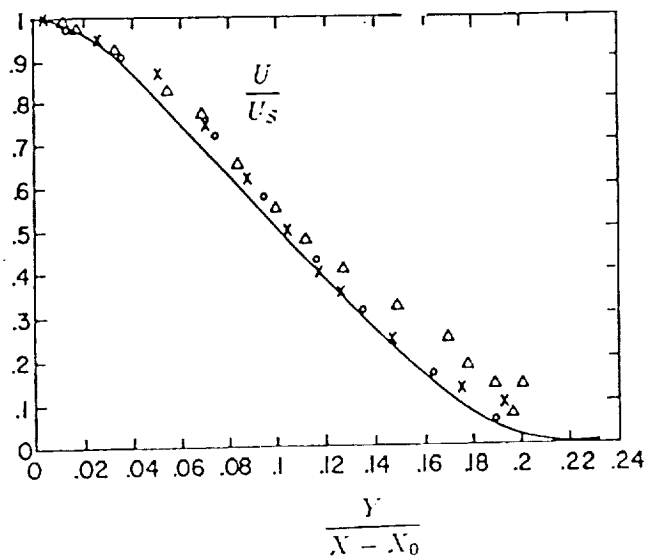


Fig. 5 Mean velocity profile in planar jet.  $U_s$ : the center line mean velocity.  $\circ$ : Bradbury<sup>[22]</sup>,  $\Delta$ : Heskestad<sup>[23]</sup>,  $\times$ : Gutmark & Wygnanski<sup>[25]</sup>, —: Present model.

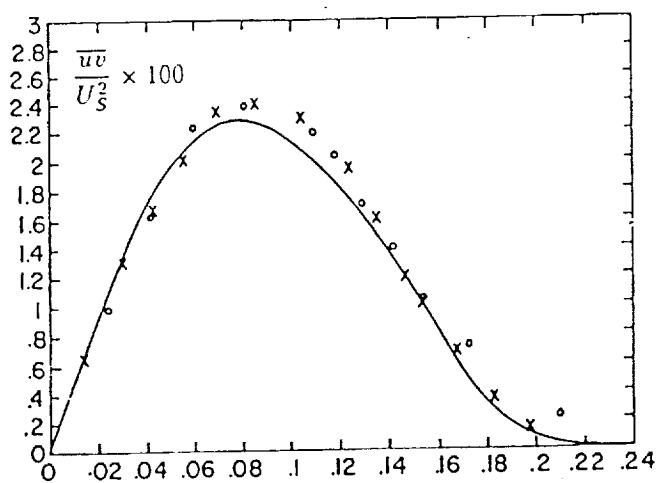


Fig. 6 Shear stress profile in planar jet. Legend as in Fig. 5.

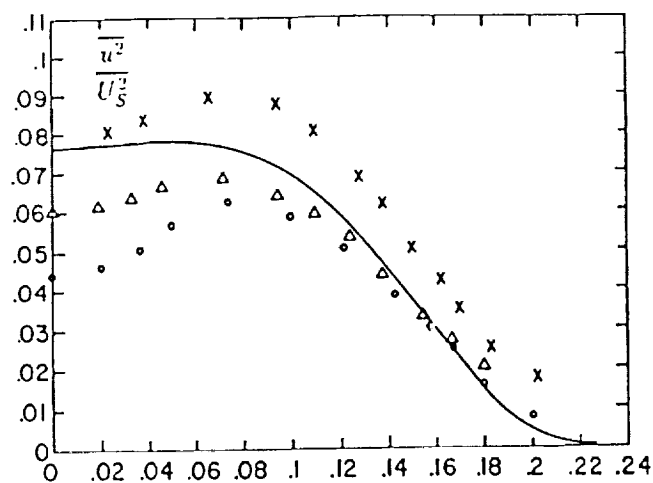


Fig. 7 Normal stress  $\overline{u^2}/U_s^2$  profile in planar jet. Legend as in Fig. 5.

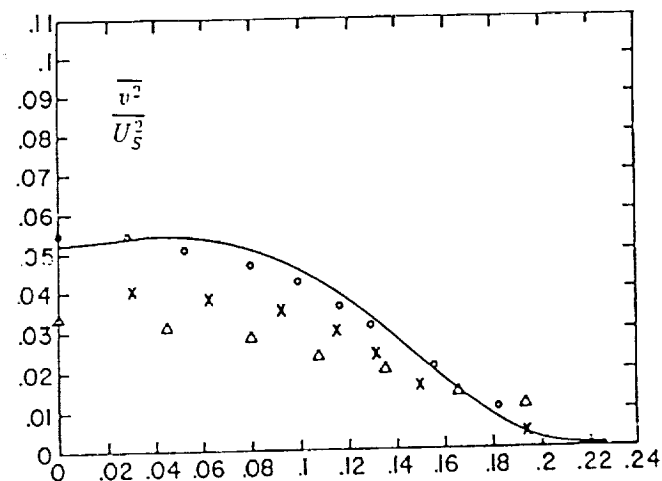


Fig. 8 Normal stress  $\overline{v^2}/U_s^2$  profile in planar jet. Legend as in Fig. 5.

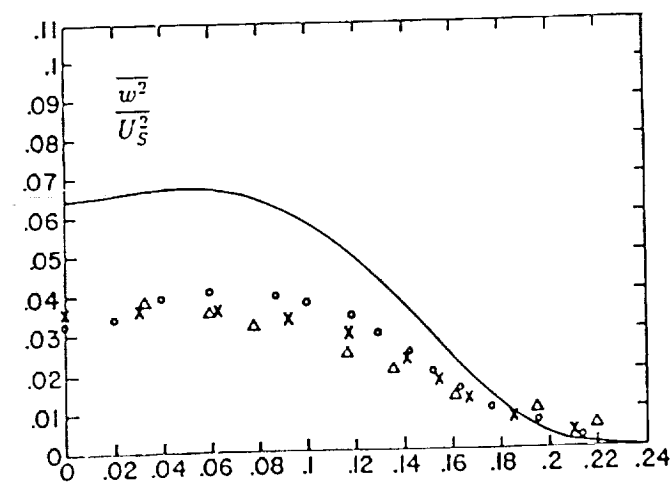


Fig. 9 Normal stress  $\overline{w^2}/U_s^2$  profile in planar jet. Legend as in Fig. 5.

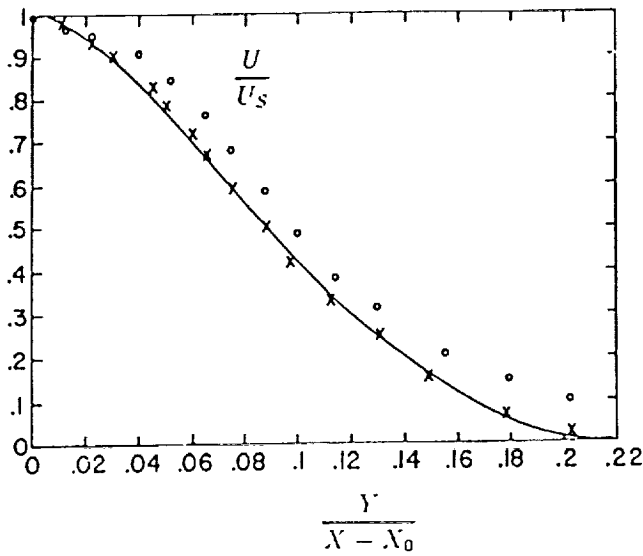


Fig. 10 Mean velocity  $U/U_s$  profile in axisymmetric jet.  $U_s$ : the centerline mean velocity.  $\circ$ : Abbiss et al.<sup>[24]</sup>,  $\times$ : Wygnanski & Fiedler<sup>[25]</sup>, —: Present model.

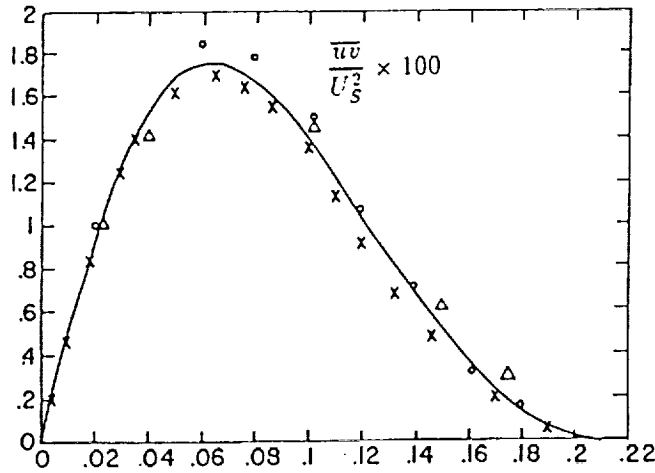


Fig. 11 Shear stress  $\overline{uv}/U_s^2$  profile in axisymmetric jet.  $\circ$ : Rodi<sup>[26]</sup>,  $\times$ : Wygnanski & Fiedler<sup>[25]</sup>,  $\Delta$ : Abbiss et al., —: Present model.

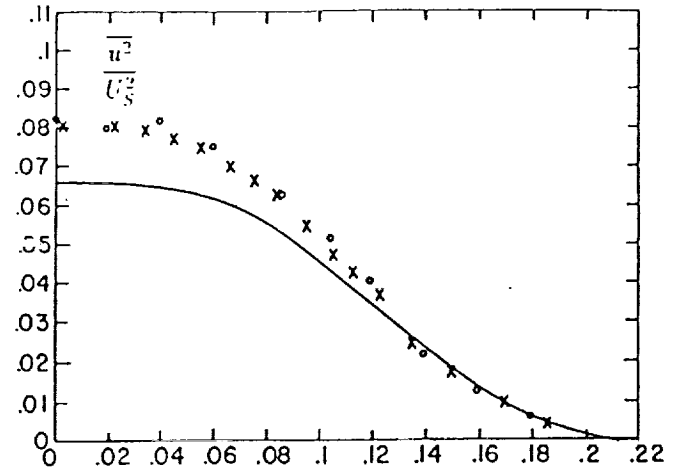


Fig. 12 Normal stress  $\overline{u^2}/U_s^2$  profile in axisymmetric jet. Legend as in Fig.11.

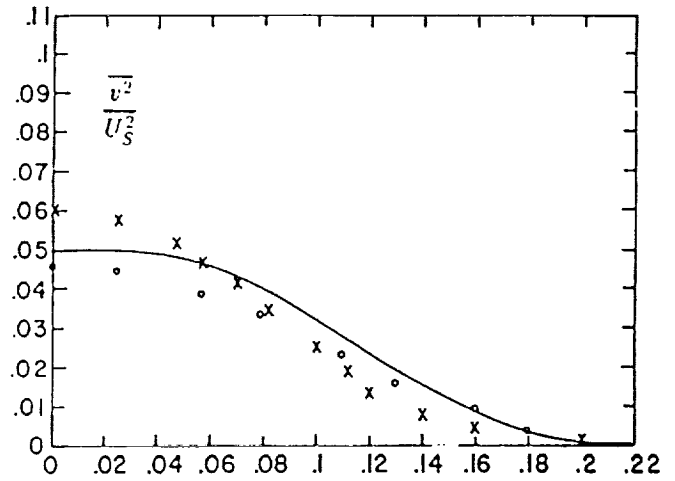


Fig. 13 Normal stress  $\overline{v^2}/U_s^2$  profile in axisymmetric jet. Legend as in Fig.11.

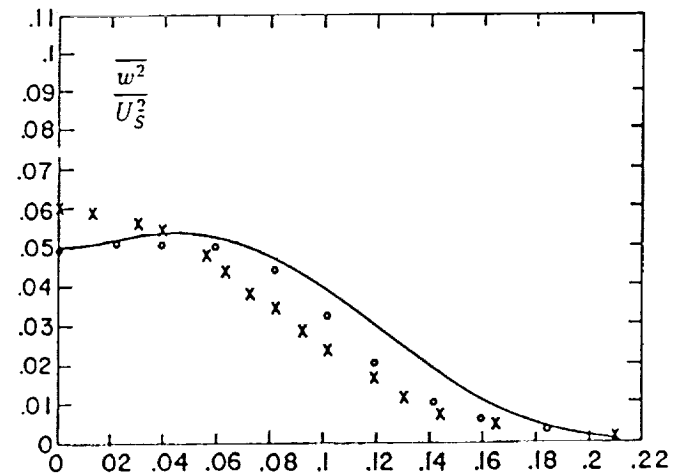


Fig. 14 Normal stress  $\overline{w^2}/U_s^2$  profile in axisymmetric jet. Legend as in Fig.11.



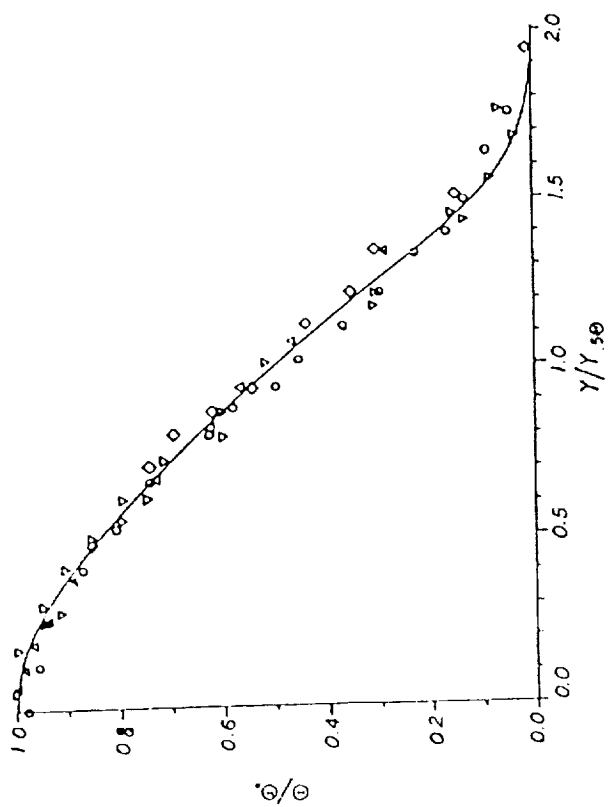
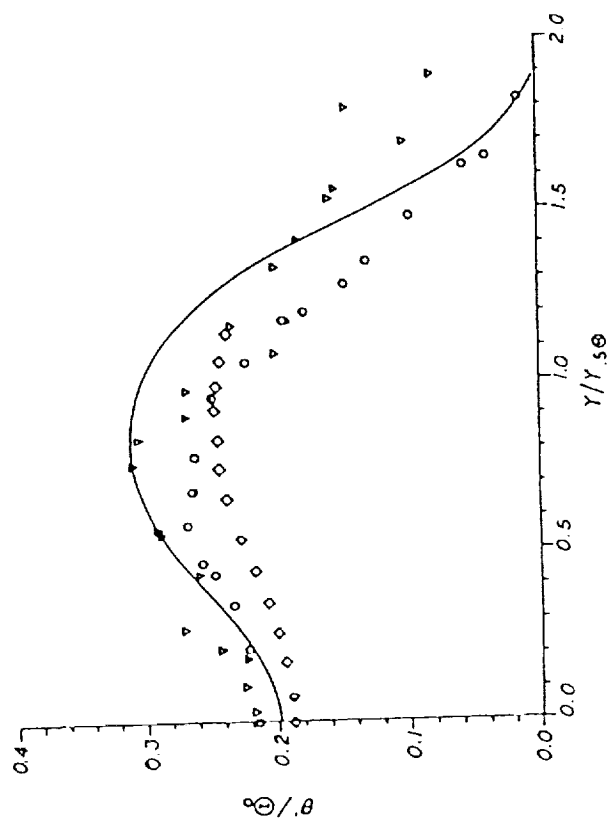
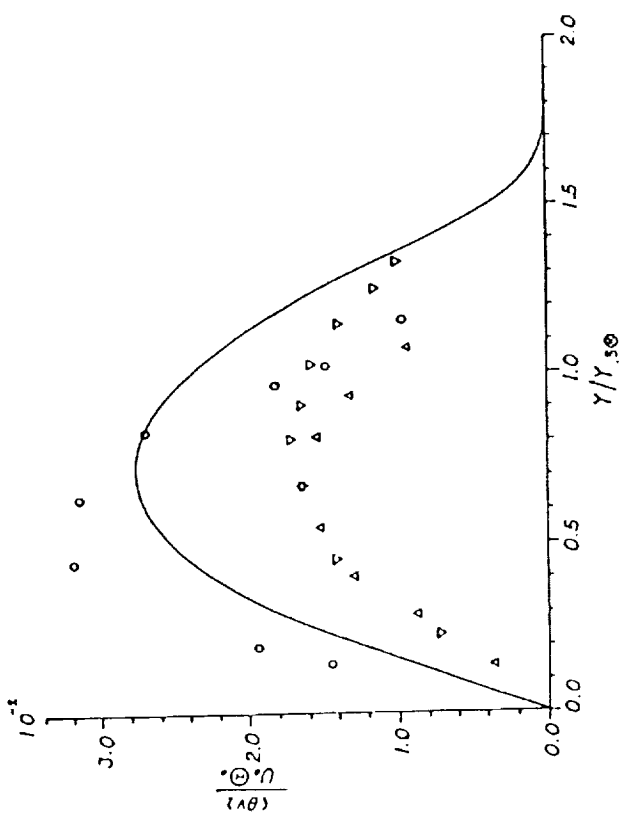


Fig. 1 Mean temperature in heated plane jet [ $\Delta$ , Browne et al. (1984);  $\nabla$ , Bashir & Uberoi (1975);  $\diamond$ , Uberoi & Singh (1975);  $\circ$ , Jenkins & Goldschmidt (1973); —, present model].

Fig. 2 Heat flux  $(\rho, v)/U_0\theta_0$  in heated plane jet [ $\diamond$ , Jenkins (1976);  $\Delta$ , Antonia (1985);  $\nabla$ , Browne et al. (1984); —, present model].

Fig. 5 Square-root of temperature variance in heated plane jet [ $\diamond$ , Antonia et al. (1983);  $\circ$ , Bashir & Uberoi (1975);  $\nabla$ , Uberoi & Singh (1975); —, present model].

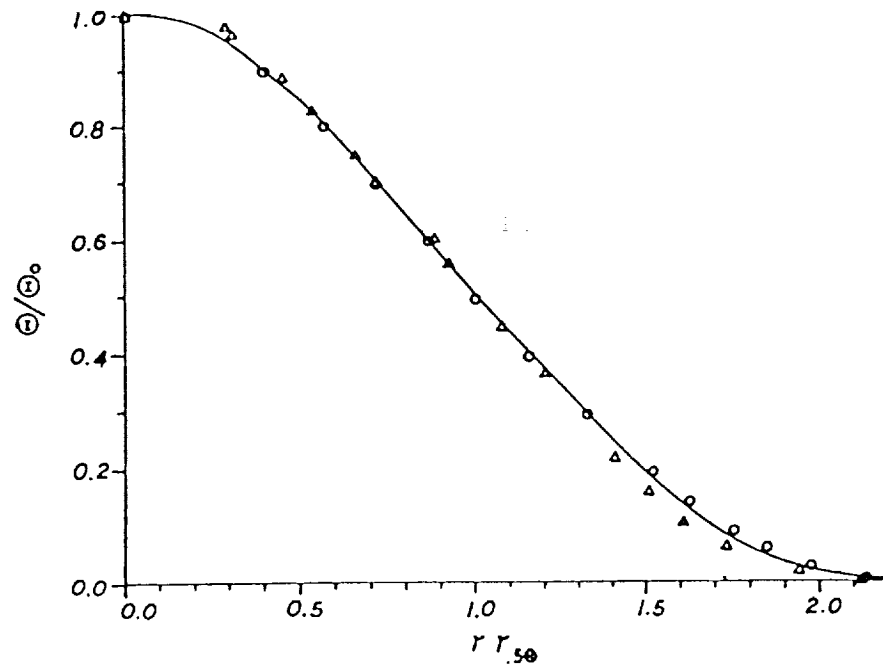


Fig. 6 Mean temperature in heated round jet [ $\Delta$ , Lockwood & Moneib (1980);  $\circ$ , Becker et al. (1967); —, present model].

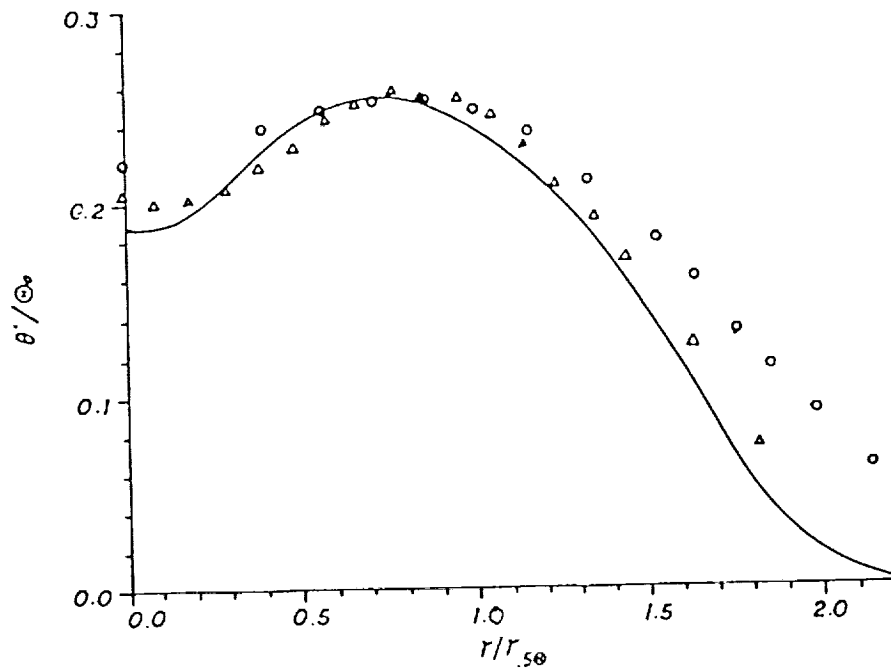


Fig. 7 Square-root of temperature variance in heated round jet [ $\Delta$ , Lockwood & Moneib (1980);  $\circ$ , Becker et al. (1967); —, present model].

## Future Direction

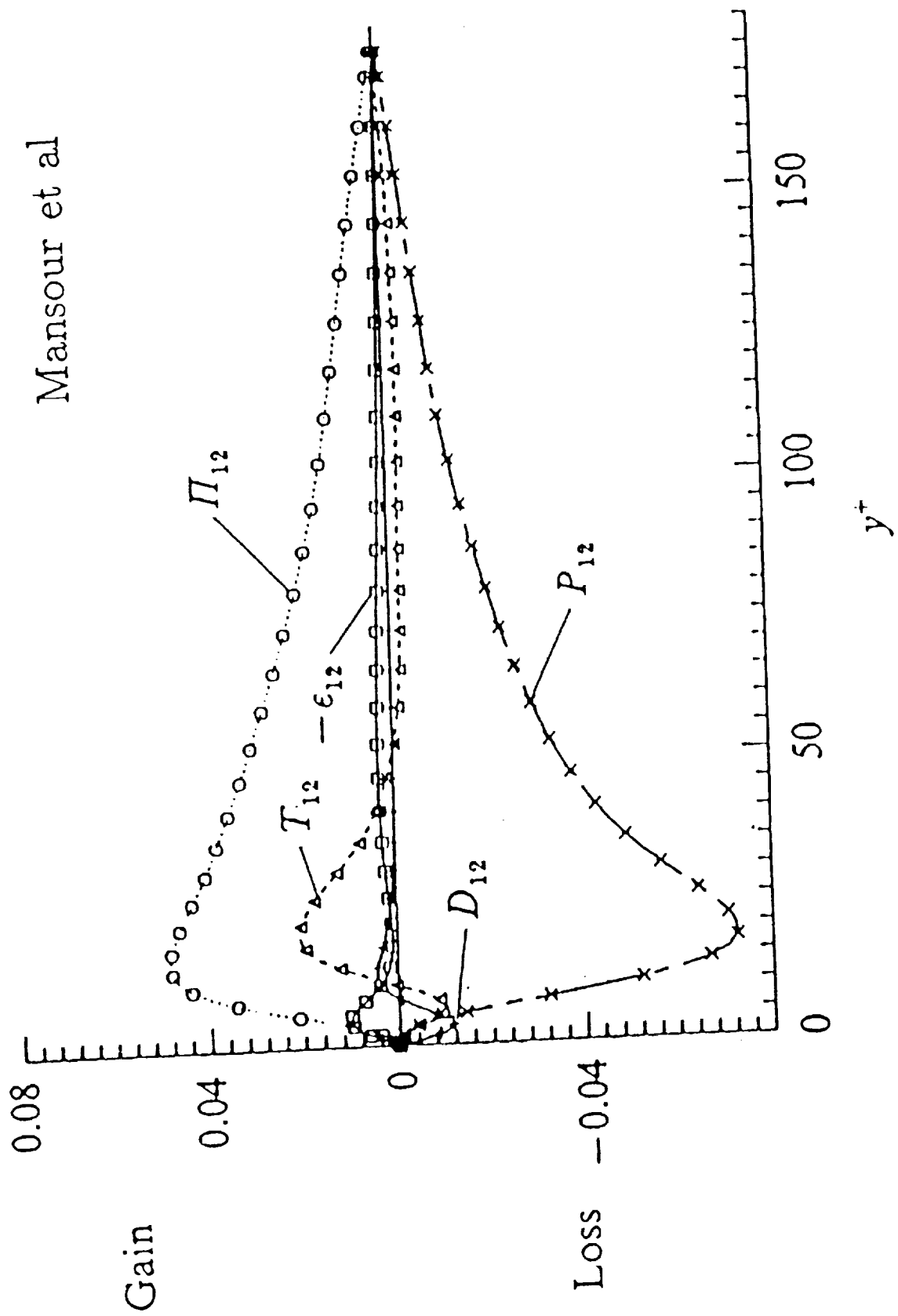
“Phenomenological Modeling: Present ... and Future?” by  
Launder (1989, Whither Turbulence? at Cornell)

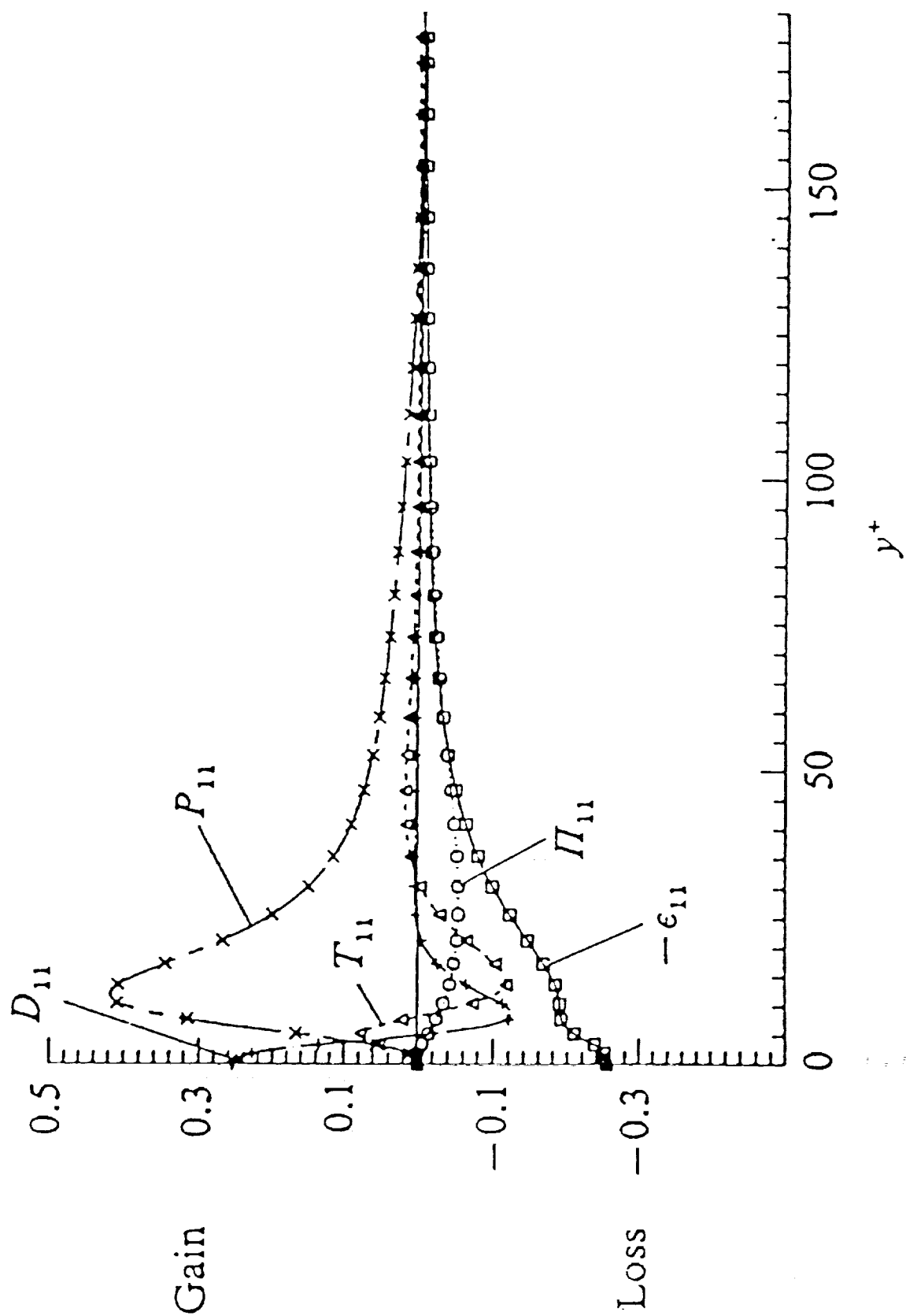
- Application and Assessment
- Extension and Refinement
- New modeling Development

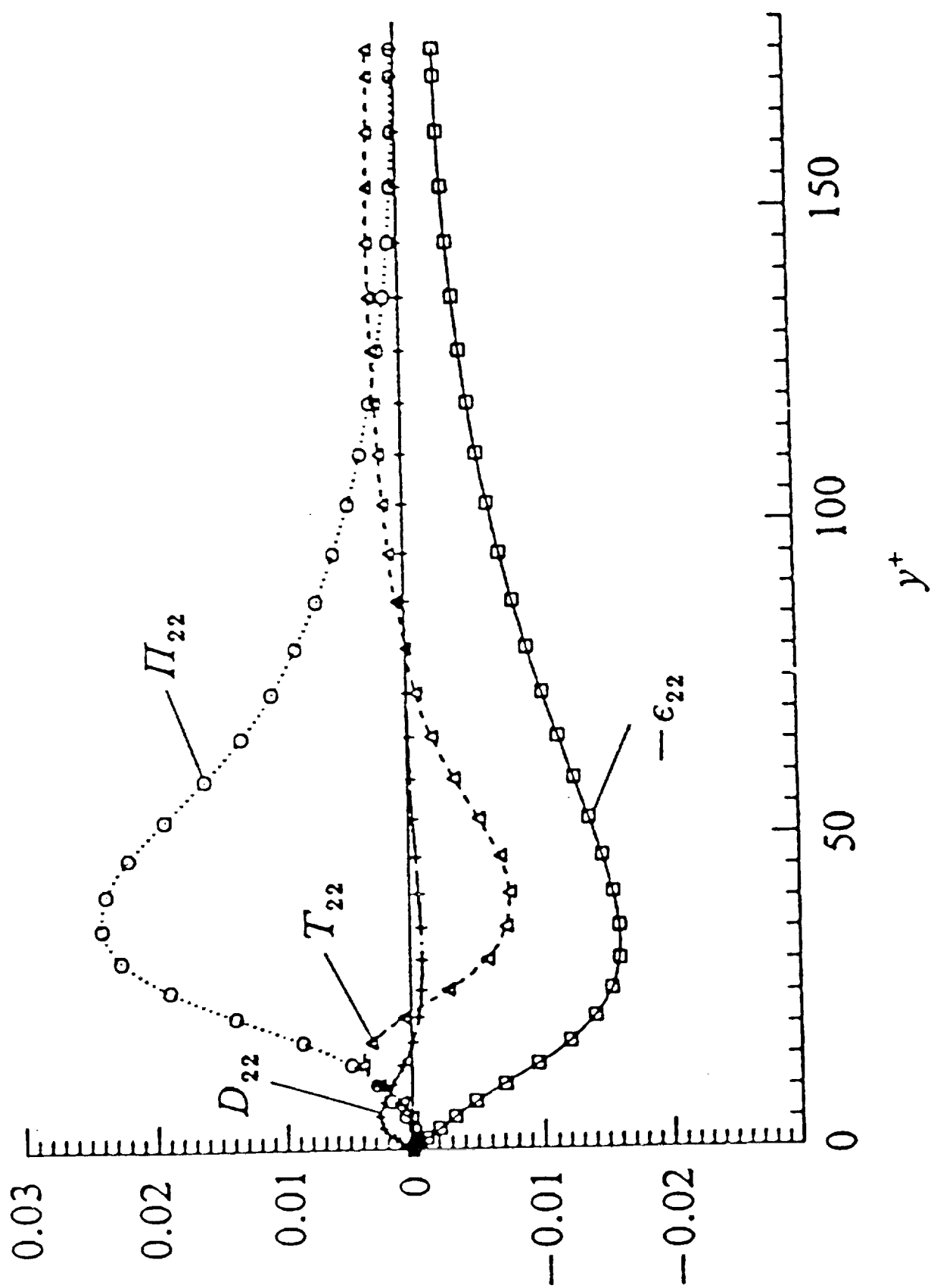
Above issues addressed in Launder’s paper are still very appropriate.

One of the weakest situations in second-order closure is Wall-bounded turbulent flows:

- General feature: Rapid variation near the wall and multiple scale problem.
- Models for the rapid pressure terms failed in modeling their near wall behavior, especially in buffer layer.
- Models for the turbulent transport terms also failed in the buffer layer.
- Models for the return terms (slow pressure + anisotropic dissipation tensor) seem ok from the comparison with the DNS data.
- The present treatment of the wall effect [ Hanjalic and Launder (1972), Launder and Tselepidakis (1991) Lai and So (1990), Shih and Mansour (1990) ] have not resolved above model deficiencies.











N92-24524

J.R. RISTORCELLI: THESIS TITLE

86891

Second order turbulence simulation of the  
rotating, buoyant, recirculating convection  
in the Czochralski crystal melt

p-10

1) PROBLEM STATEMENT

2OM for rotating turbulence

2) THE RAPID PRESSURE MODELS TESTED

LINEAR

NONLINEAR

1) The IP model

3) Launder's nonlinear

2) The SSG model

4) Shih & Lumley's model

5) The 2DMFI model

3) THE COMPUTATIONAL PROBLEM

4) SHORTCOMINGS AND FUTURE WORK

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## THE CZOCHRALSKI PROBLEM

$$Re_s = \omega_s R_s^2 / \nu \quad Re_s < 10^5 \quad Re_c = \omega_c R_c^2 / \nu \quad Re_c < 10^5$$

$$Gr = g\beta\Delta T R_c^3 / \nu^2 \quad Gr \lesssim 10^{12} \quad Re = Gr^{1/2} \quad Re < 10^6$$

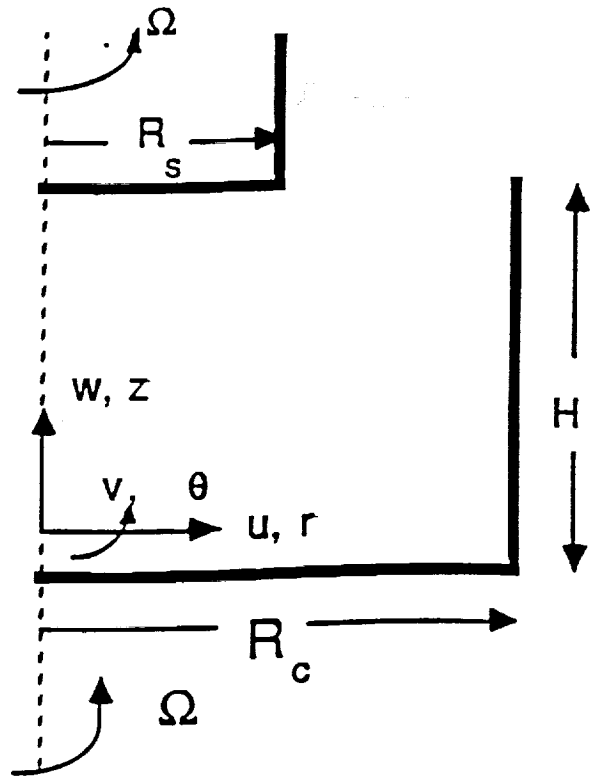
$$Pr = \nu / \alpha \quad Pr \sim .01 \quad Ar = H / R_c \quad Ar \sim 1.$$

$$Bi = \varepsilon \sigma T_m^4 R_c / k \Delta T \quad Bi \lesssim 4 \quad \Delta = \Delta T / T_m \quad \Delta \sim .05$$

$$Ma = -\sigma_T \Delta T R_s Pr / \rho \nu^2 \quad Ma < 10^5$$

$$1 \leq Ro = Re / |Re_c| \leq \infty$$

$$Sb = Re_s^2 / Gr \leq 10^2$$



## THE COMPUTATIONAL PROBLEM

### 17 NONLINEAR COUPLED PDE's

MEAN [ U, V, W, T ]

$$20M \begin{bmatrix} \langle uu \rangle & r^2 \langle uv \rangle & \langle uw \rangle \\ r^2 \langle vw \rangle & r^4 \langle vv \rangle & r^2 \langle vw \rangle \\ \langle uw \rangle & r^2 \langle vw \rangle & \langle ww \rangle \end{bmatrix} \begin{bmatrix} \langle \theta u \rangle \\ r^2 \langle \theta v \rangle \\ \langle \theta w \rangle \end{bmatrix} \langle \theta \theta \rangle$$

$$[ \varepsilon, \varepsilon_\theta ] \langle \theta \theta \theta \rangle$$

### 24 NONLINEAR ALGEBRAIC EQUATIONS

$$30M \begin{bmatrix} \langle \theta \theta \theta \rangle \\ \langle \theta \theta u_j \rangle \\ \langle \theta u_i u_j \rangle \\ \langle u_i u_j u_k \rangle \end{bmatrix}$$

### 17 INEQUALITIES (at least)

$$\langle u_i u_j \rangle, \langle \theta \theta \rangle, d_{ij} = \langle u_i u_j \rangle \langle \theta \theta \rangle - \langle \theta u_i \rangle \langle \theta u_j \rangle$$

$$\langle u_\alpha u_\gamma \rangle / (\langle u_\gamma u_\gamma \rangle^{1/2} \langle u_\alpha u_\alpha \rangle^{1/2})$$

$$\langle \theta u_\alpha \rangle / (\langle \theta \theta \rangle \langle u_\alpha u_\alpha \rangle^{1/2})$$



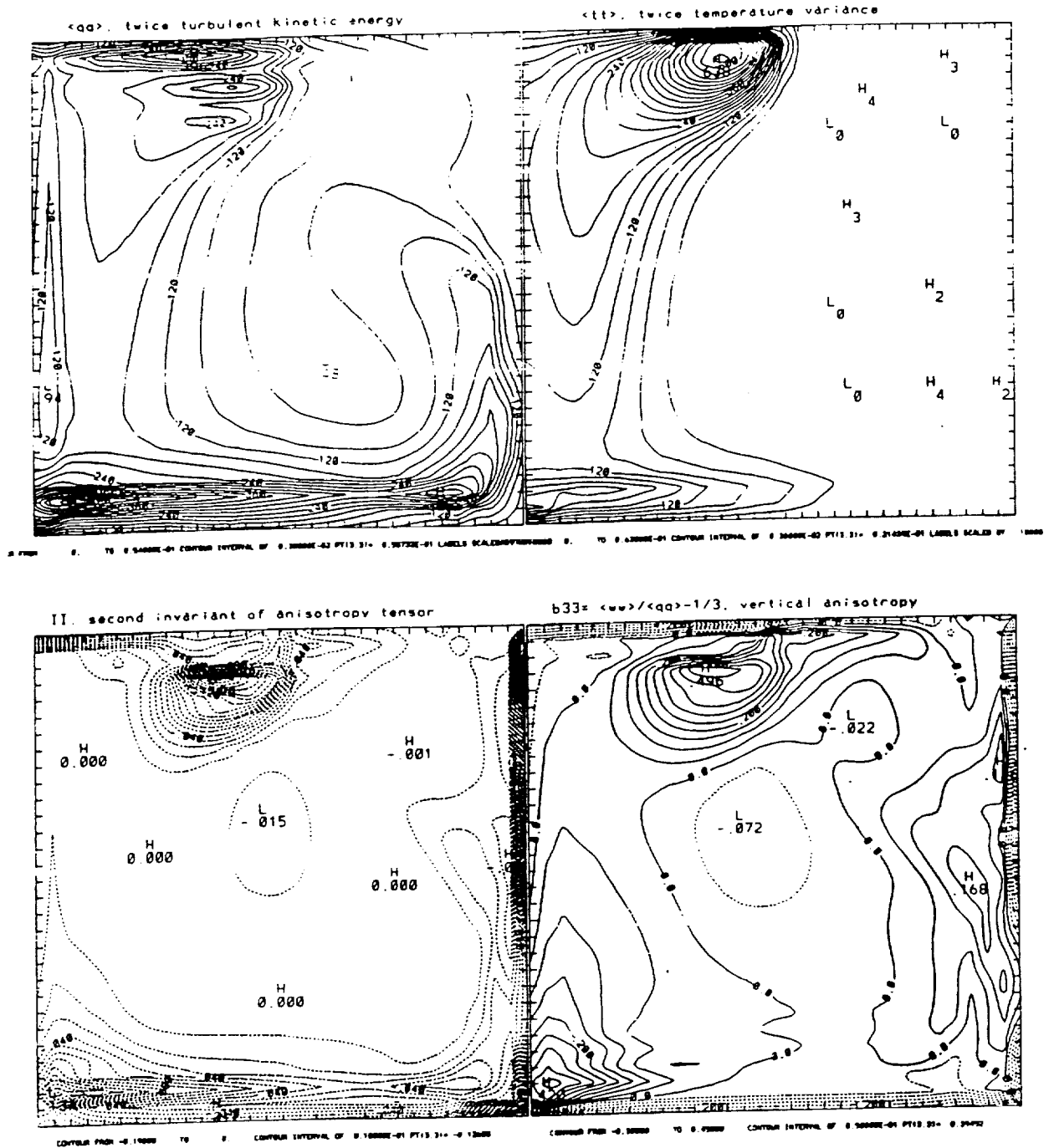


Figure 7.10: Twice turbulence energy,  $q^2$ , and scalar variance,  $\langle \theta\theta \rangle$ , second invariant,  $II$ , and  $(ij=3,3)$  component of the anisotropy tensor,  $b_{33}$ ; Solution with Ristorcelli and Lumley's rapid and buoyancy pressure models.  $Gr = 10^9$ ,  $Re = 3.16 \cdot 10^4$ ,  $Re_s = 0.0$ ,  $Re_c = 0.0$ ,  $Ro = \infty$ ,  $Ma = 10^3$ ,  $Pr = 0.01$ ,  $Bi = 2.5$ ,  $R_s/R_c = 0.5$ ,  $ar = 1.0$ ,  $T = 80$ .

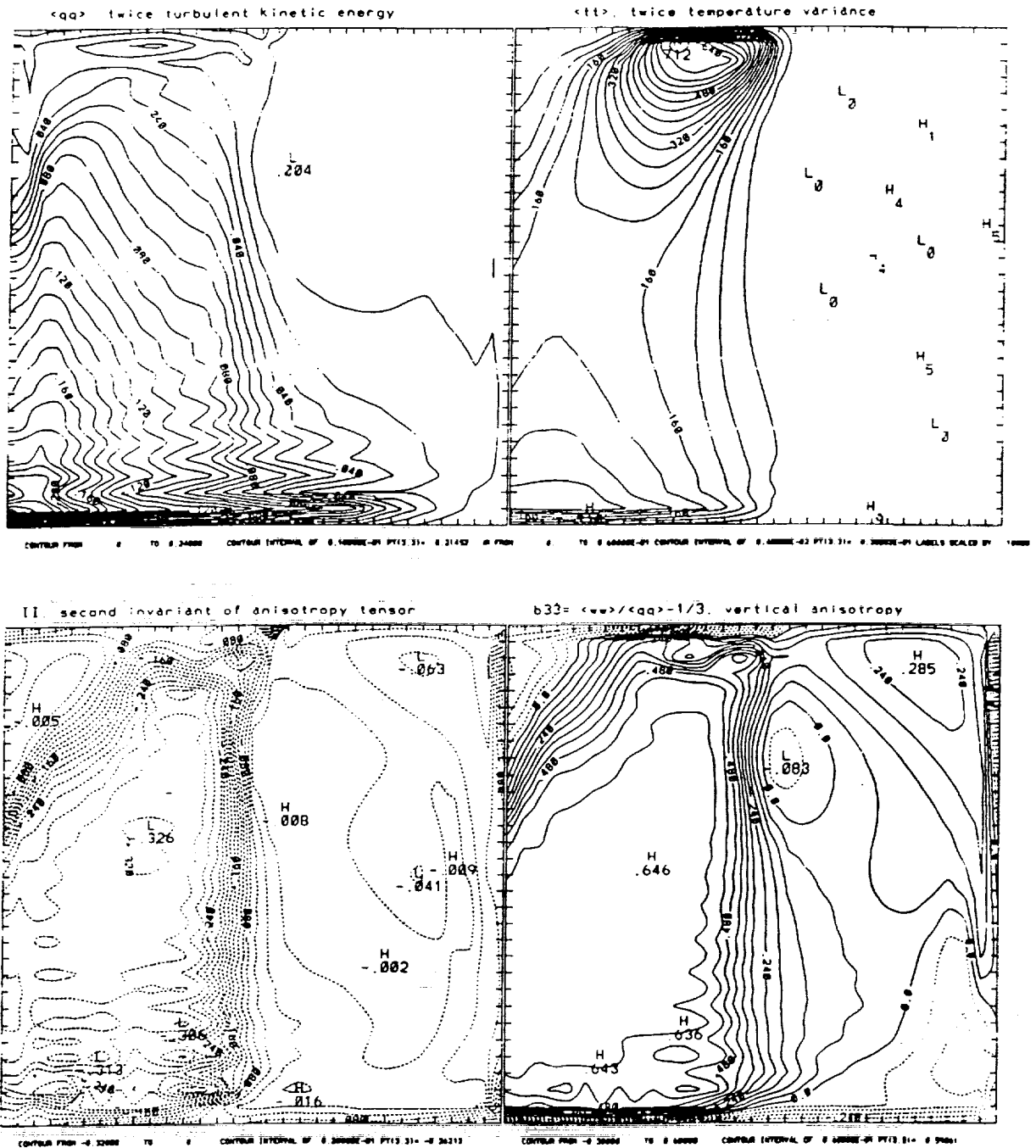
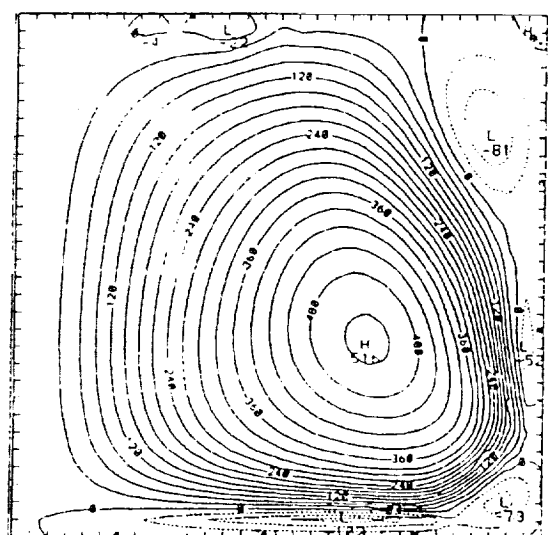
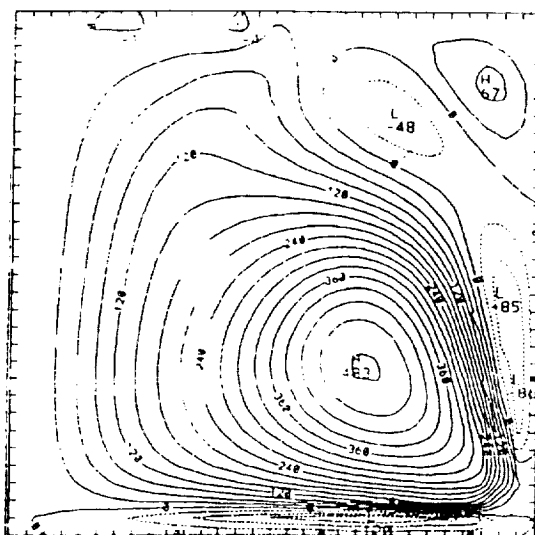
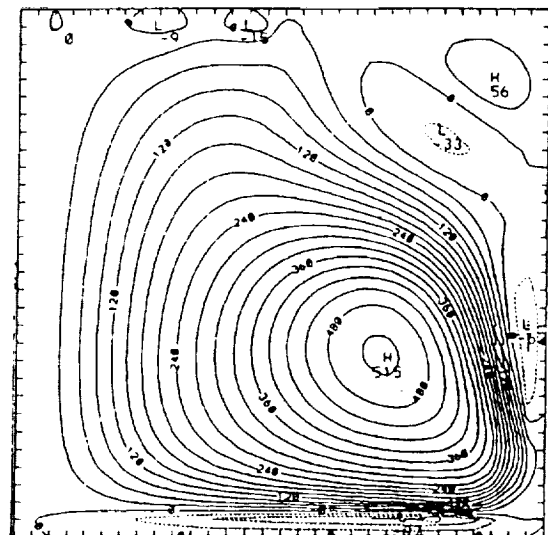
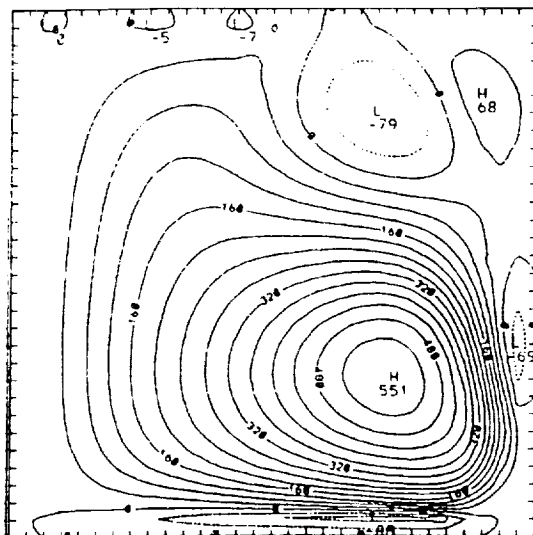
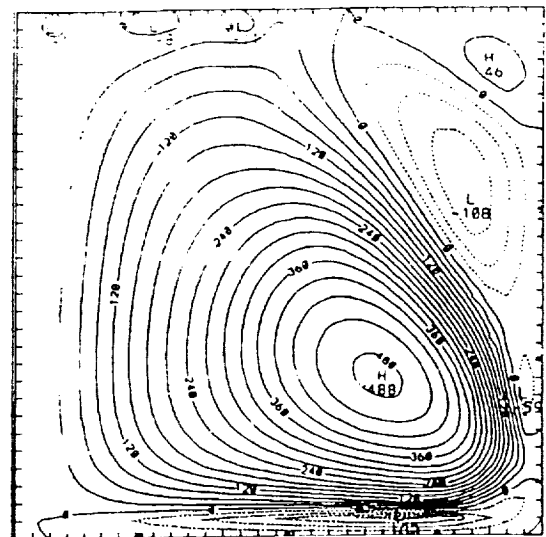
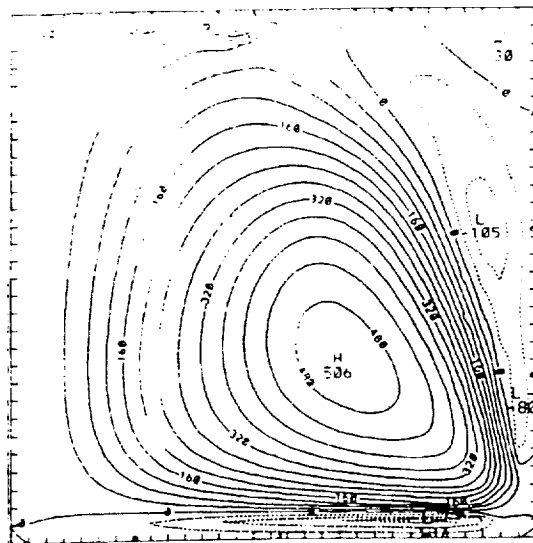


Figure 7.7: Twice turbulence energy,  $q^2$ , and scalar variance,  $\langle \theta\theta \rangle$ , second invariant,  $\Pi$ , and  $(i,j=3,3)$  component of the anisotropy tensor,  $b_{33}$ ; Solution with the SSG linear rapid pressure model and Ristorcelli and Lumley's buoyancy pressure model.  $Gr = 10^9$ ,  $Re = 3.16 \cdot 10^4$ ,  $Re_s = 0.0$ ,  $Re_c = 0.0$ ,  $Ro = \infty$ ,  $Ma = 10^3$ ,  $Pr = 0.01$ ,  $Bi = 2.5$ ,  $R_s/R_c = 0.5$ ,  $ar = 1.0$ ,  $T = 70$ .



## **FUTURE WORK / SHORTCOMINGS**

### **1) WALL FUNCTIONS**

Physics: boundary layers and turbulence

Computations: grid dependence  
resolution of flow field

Low Re 2OM Models: too many nodes

### **2) RAPID ROTATION / STABLE STRATIFICATION**

inadequate parameterization of cascade

### **3) WHAT IS THE TIME DEPENDENCE ?**

Joint Realizability couples rapid models

What does the averaging mean ?

Long time scale "coherent" structures

### **4) COMPUTATIONAL STRATEGIES**

FDAs reflecting realizability

Include mean quantities in 3OM eq's

FDAs with accurate time evolution

FDAs reflecting turbulent diffusion



## **CONTRIBUTIONS OF THE THESIS**

### **EXPECTATIONS OF 2DMFI MODELS**

- 1) Flows with strong body forces**
  - stable stratification**
  - rotation**
  - magnetic fields**
- 2) Environmental shallow water flows**
  - industrial effluents**
  - mixing between bodies of water**
- 3) Quasi two-dimensional geophysical flows**
  - large scale ocean mixing**
  - regional atmospheric modeling**
  - (mesoscale variability)**
- 4) Unsteady flows**
  - time scales > integral time scale**
  - unsteady separation**
  - large scale "coherent structures" ??**

## CONTRIBUTIONS OF THE THESIS

### 0) THE 2DMFI MODEL

Models  $X_{pri}$  &  $X_{psir}$ :  $f(b, b^2, b^3), f(b, b^2, b^3, \langle \theta u \rangle)$

Satisfies Realizability, Joint Realizability

Satisfies 2DMFI

Off-realizability corrections exact

Off-geostrophy corrections exact

Free parameters available

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N92-24525-12  
p.19

**COMMENTS ON THE PRESENT  
STATE OF SECOND-ORDER  
CLOSURE MODELS FOR  
INCOMPRESSIBLE FLOWS**

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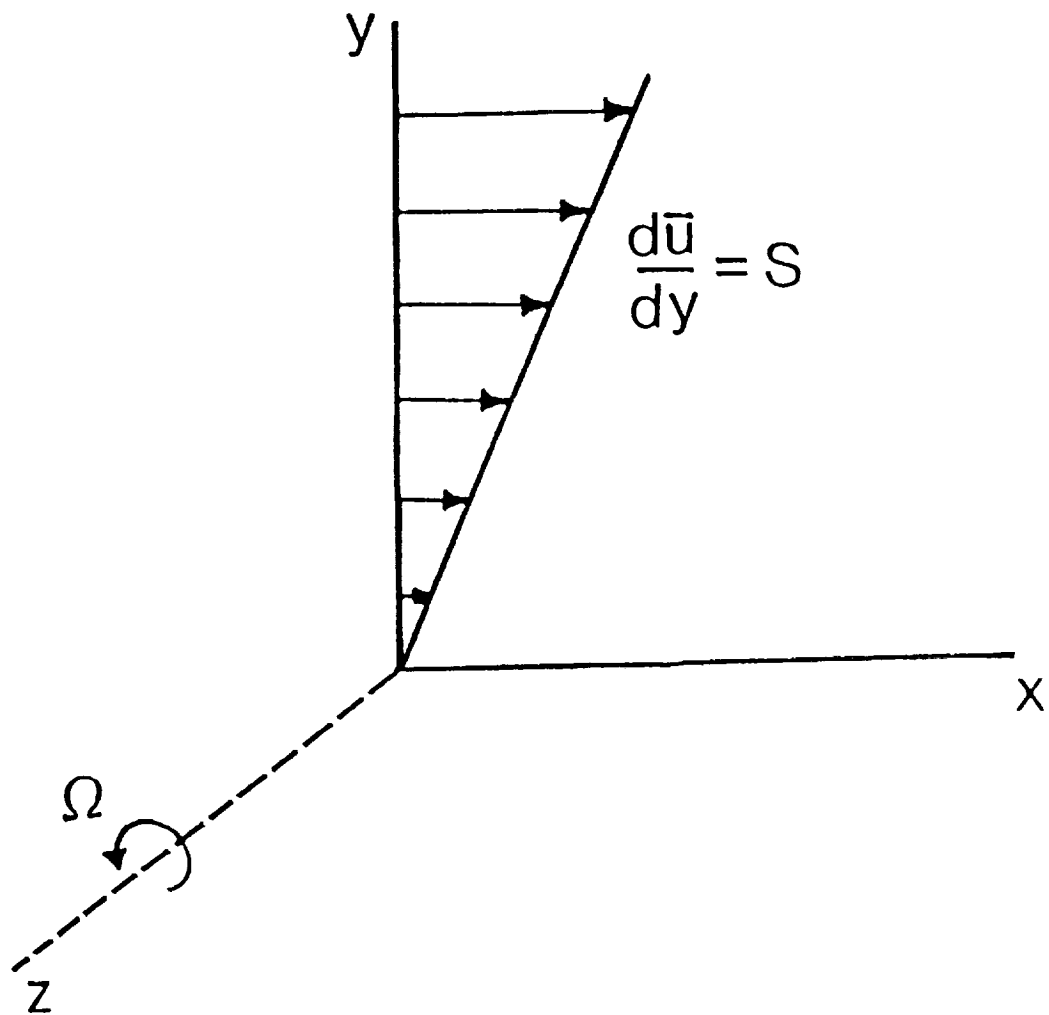
August 21 - 22, 1991

Second-order closure models account for history and nonlocal effects of the mean velocity gradients on the Reynolds stress tensor.

Turbulent flows involving:

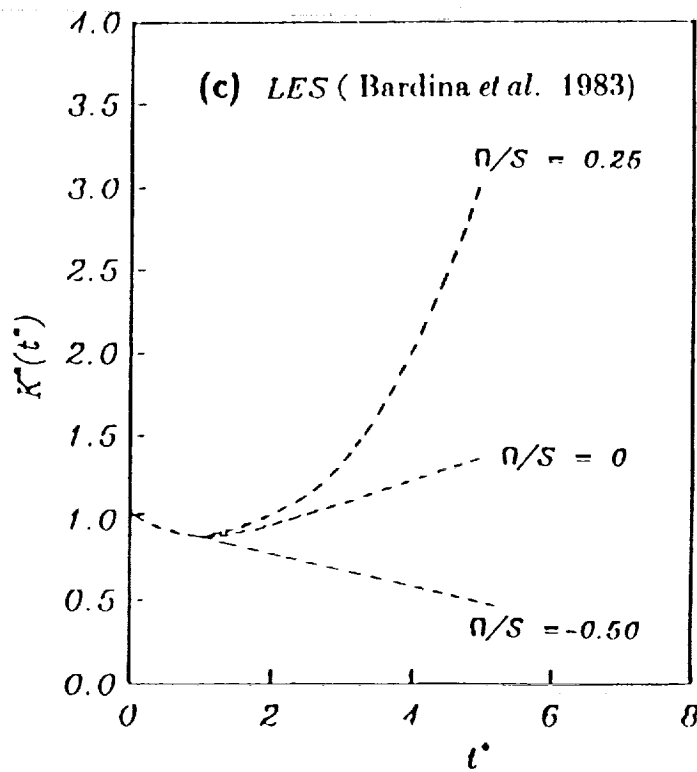
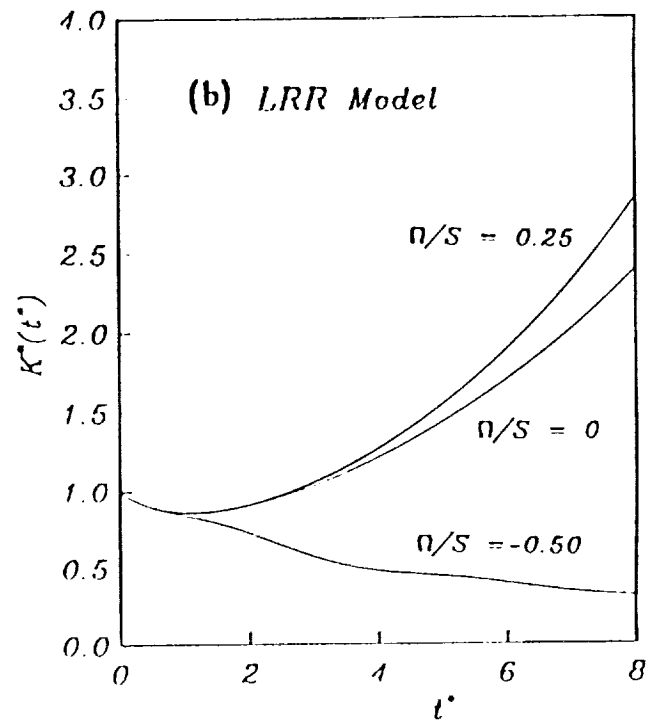
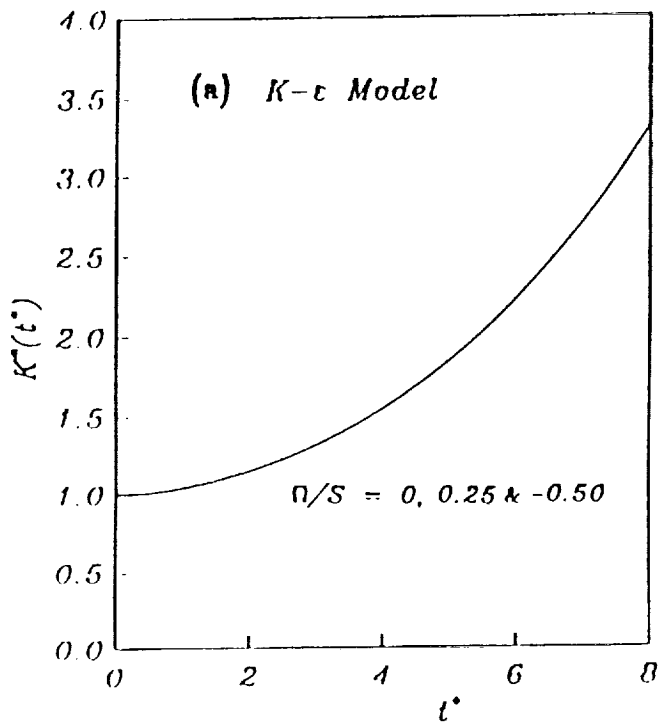
- Body forces or curvature
- Reynolds stress relaxational effects
- Counter-gradient transport

are usually better described.



**homogeneous turbulent shear flow in a rotating frame**

# Rotating Shear Flow



## REYNOLDS STRESS TRANSPORT EQUATION

$$\begin{aligned} \frac{\partial \tau_{ij}}{\partial t} + \bar{u}_k \frac{\partial \tau_{ij}}{\partial x_k} = & -\tau_{ik} \frac{\partial \bar{u}_j}{\partial x_k} - \tau_{jk} \frac{\partial \bar{u}_i}{\partial x_k} + \Pi_{ij} \\ & - \frac{2}{3} \varepsilon \delta_{ij} - \frac{\partial C_{ijk}}{\partial x_k} + \nu \nabla^2 \tau_{ij} \end{aligned}$$

where

$$\tau_{ij} = \overline{u'_i u'_j}, \quad \varepsilon = \nu \overline{\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_i}{\partial x_j}}$$

$$\Pi_{ij} = \overline{p' \left( \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right)} - \overset{\Delta}{\varepsilon}_{ij}$$

$$\overset{\Delta}{\varepsilon}_{ij} = 2\nu \overline{\frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k}} - \frac{2}{3} \varepsilon \delta_{ij}$$

$$C_{ijk} = \overline{u'_i u'_j u'_k} + \overline{p' u'_i} \delta_{jk} + \overline{p' u'_j} \delta_{ik}$$

$$K = \frac{1}{2} \tau_{ii}$$

## ISSUES IN SECOND-ORDER CLOSURE MODELING

Models for  $\Pi_{ij}$ :

Typically, it is assumed that

$$\Pi_{ij} = \varepsilon \mathcal{A}_{ij}(\mathbf{b}) + K \mathcal{M}_{ijkl}(\mathbf{b}) \frac{\partial \bar{u}_k}{\partial x_\ell}$$

where

$$b_{ij} = \frac{\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij}}{\tau_{\ell\ell}} \quad (\text{anisotropy tensor})$$

These models have deficiencies in rotating homogeneous turbulent flows (Reynolds 1989 and Speziale, Sarkar and Gatski 1990).



For the return to isotropy problem in a rotating frame (with angular velocity  $\Omega$ ), these models predict that the second and third invariants of  $b_{ij}$  are independent of  $\Omega$  in contradiction of DNS and RDT (Reynolds 1989).

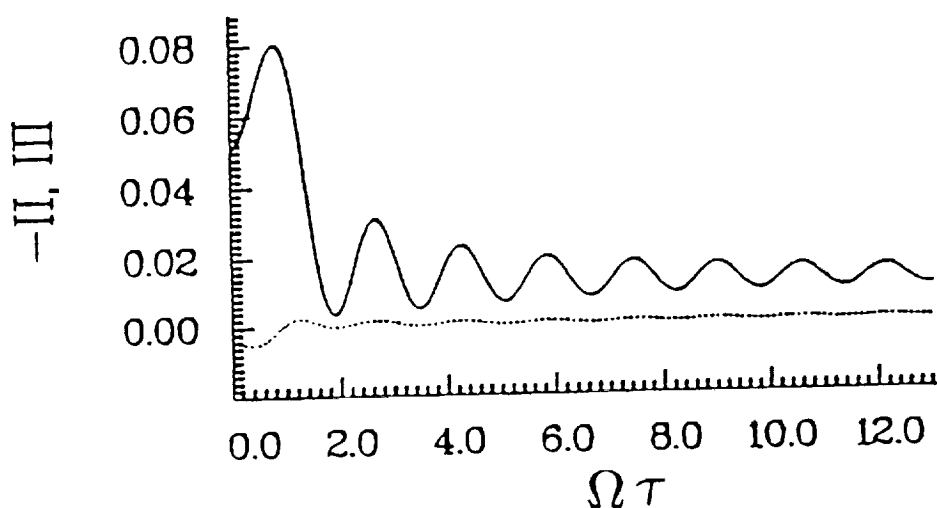
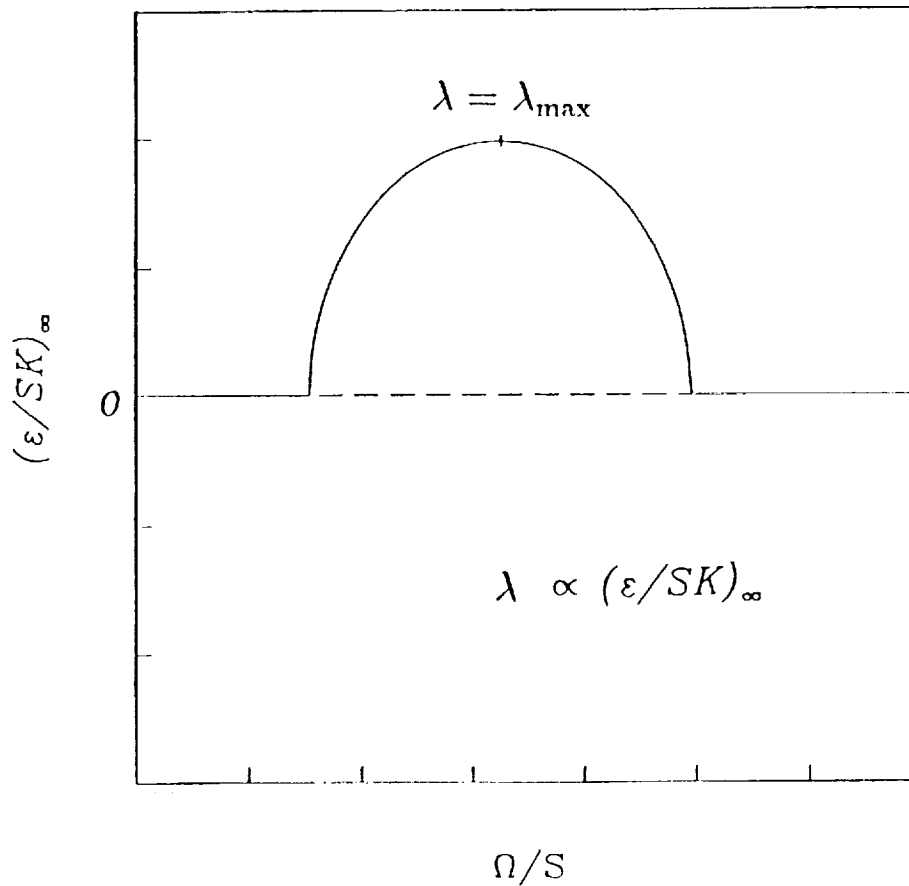


Fig. 2 Typical RDT solution for the rotation of initially anisotropic homogeneous turbulence (by T.S. Shih).

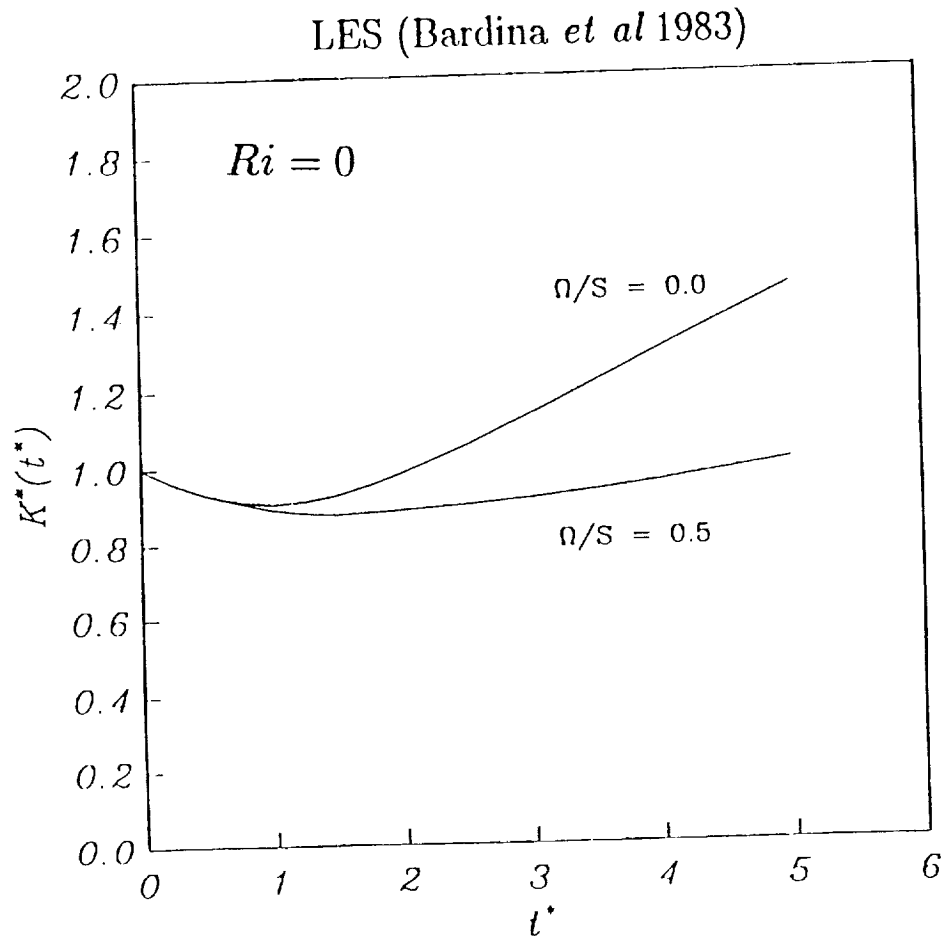
For rotating homogeneous shear flow in the unstable flow regime, these models predict that the growth rate  $\lambda$  of the flow defined by

$$K \sim e^{\lambda t}, \quad \varepsilon \sim e^{\lambda t}$$

is symmetric about its most energetic value (Speziale, Sarkar and Gatski 1990).



Consequently, if the most energetic state – where  $\lambda = \lambda_{\max}$  – is placed at  $\Omega/S = 0.25$ , the model will exhibit similarity with respect to the Richardson number  $Ri \equiv -2\Omega/S(1 - 2\Omega/S)$ . This is in violation of RDT and LES results (Speziale, Sarkar and Gatski 1990 and Speziale and Mac Giolla Mhuiris 1989).



Speziale, Sarkar and Gatski 1990 recently showed that the general model is topologically equivalent to the quadratic model

$$\begin{aligned}\Pi_{ij} = & -C_1\varepsilon b_{ij} + C_2\varepsilon (b_{ik}b_{kj} - \tfrac{1}{3}II\delta_{ij}) \\ & + C_3K\overline{S}_{ij} + C_4K(b_{ik}\overline{S}_{jk} + b_{jk}\overline{S}_{ik} \\ & - \tfrac{2}{3}b_{mn}\overline{S}_{mn}\delta_{ij}) + C_5K(b_{ik}\overline{W}_{jk} + b_{jk}\overline{W}_{ik})\end{aligned}$$

in plane homogeneous turbulent flows where

$\overline{S}_{ij} = \frac{1}{2}(\partial\overline{u}_i/\partial x_j + \partial\overline{u}_j/\partial x_i)$  and  $\overline{W}_{ij} = \frac{1}{2}(\partial\overline{u}_i/\partial x_j - \partial\overline{u}_j/\partial x_i)$ . Based on these ideas, the SSG model was developed.

The SSG model yields only modest improvements on the Launder, Reece and Rodi model. Substantial improvements will only come if  $\Pi_{ij}$  is taken to be a **nonlinear** function of the mean velocity gradients.

Two possible approaches are:

- (1) The eddy structure model of Reynolds (1990)
- (2) Tensor dissipation models (Speziale, Raj and Gatski 1990).

## NEAR WALL MODELS

We currently do not know how to properly integrate second-order closure models to a solid boundary! The major problem lies in the pressure strain correlation  $\Pi_{ij}$ . The commonly used near wall models for  $\Pi_{ij}$  have two major deficiencies:

- (1) The ad hoc dependence of  $\Pi_{ij}$  on the unit normal  $n_i$  to the wall. This does not allow for the proper treatment of wall bounded flows with corners.
- (2) Asymptotic consistency is satisfied through **singular** differential equations; for example

$$\nu \frac{\partial^2 \tau_{12}}{\partial y^2} = C_1 \frac{\varepsilon}{K} \tau_{12} + \mathcal{O}(y^2)$$

for the near-wall behavior of  $\tau_{12}$ . This can cause problems in numerically recovering an asymptotically consistent solution.

**Entirely new approaches are needed for the near wall modeling of  $\Pi_{ij}$ !**

We are at the end of the road for models of the form

$$\Pi_{ij} = \varepsilon \mathcal{A}_{ij}(\mathbf{b}) + K \mathcal{M}_{ijkl}(\mathbf{b}) \frac{\partial \bar{u}_k}{\partial x_\ell}$$

with ad hoc near wall fixes.

## NEEDED IMPROVEMENTS

- Models for  $\Pi_{ij}$  that are nonlinear in the mean velocity gradients.
- Entirely new methods for the integration of second-order closures to a solid boundary.
- Incorporation of **directional** information into the turbulence length scale (possibly via an integral tensor length scale).



## DISCUSSION

S. Sarkar (to T.-H. Shih)

I have a question to Dr. Shih about the slow term pressure strain correlation comparisons he showed. It seemed to me that Rotta and our model gave the same results. That was little surprising because the linear term coefficients were different in the two models. On top of that our model had a nonlinear term.

T.-H. Shih (reply)

The nonlinear term is very small. Linear term coefficient for LRR model is 1.5 and for your model is 1.7.

S. Sarkar (to T.-H. Shih)

We have a paper in Physics of Fluids in which we compare the two models and they are completely different.

T.-H. Shih (reply)

Your nonlinear term can also have opposite sign to linear terms thus giving results similar to LRR.

C.G. Speziale (to T.-H. Shih)

You referred to SSG model as a linear model. There is a coefficient which goes as square root of second invariant and also a term which contains a production term multiplying the anisotropic tensor. In precise mathematical terms it is a quasi-linear model.

T.-H. Shih (reply)

If the coefficient is constant the model is linear. In SSG model the coefficient is a function of second invariant and a production term.

J.L. Lumley (to C.G. Speziale)

You refer to your models as being equivalent to all the other models but only in the equilibrium situation. These flows are never in equilibrium. Would you like to comment on that.

C.G. Speziale (reply)

Question is how drastic are the departures. Then there is this issue of calibrating the coefficients. My motivation for doing SSG model was that most of the calibration we do is from homogeneous plane flows near equilibrium state. Since all the models are collapsing to this degenerate form, my feeling was to calibrate the model at this state and see the differences. It seemed to be reasonable.

J.R. Ristorcelli (to C.G. Speziale)

I have been judging these models from the point of view of computability. SSG model doesn't compute very well. It does better in rotating situations than it does in the non-rotating situations. I imagine it would do well in homogeneous shear flow situation from which it was calibrated. For me I built the principles of realizability in the computation and I can't compute the flow with SSG model.

C.G. Speziale (to J.R. Ristorcelli)

What happens?

J.R. Ristorcelli (reply)

I get correlation (coefficients) which are larger than unity or eigenvalues of matrices going to zero.

C.G. Speziale ( to J.R. Ristorcelli)

But no problems with  $k$  or  $\epsilon$ .

J.R. Ristorcelli (reply)

Well  $k$  is the sum of these eigenvalues.

C.G. Speziale (to J.R. Ristorcelli)

SSG model satisfies limited realizability. It does guarantee positive  $k$  and  $\epsilon$ .

T.B. Gatski (to J.R. Ristorcelli)

I did various calculations with homogeneous shear flows using some nonlinear models e.g. Shih-Lumley model. It was very difficult to use in homogeneous shear flow because it was very stiffening.

J.R. Ristorcelli (to T.B. Gatski)

What do you mean by stiffening?

T.B. Gatski (reply)

All the equations for these flows are ode's. You are using Runge Kutta

method and you need very very small steps. The only dilemma with making assessments of turbulence models where you have pde's is that you can not be sure unambiguously that there are no problems with the algorithm.

E. Reshotko (to J.R. Ristorcelli)

How do you know that these flows are turbulent

J.R. Ristorcelli (reply)

A lot of experiments have been done to support this e.g. at AT&T. Also the  $Gr = 10^{12}$  and  $Re = 10^6$  for these flows.

E. Reshotko (to J.R. Ristorcelli)

Will your equations with all the turbulence terms would give a laminar solution?

J.R. Ristorcelli (reply)

Turbulence would decay indicating a return toward a laminar state.

B.E. Launder (to T.-H. Shih)

Did the channel flow rapid term comparisons you showed include the inhomogeneous part of the rapid term or the wall reflection effects?

T.-H. Shih (reply)

No wall reflection or inhomogeneous effects were included. From the comparisons may be we can see how to include the inhomogeneous effects.

B.E. Launder (to J.R. Ristorcelli)

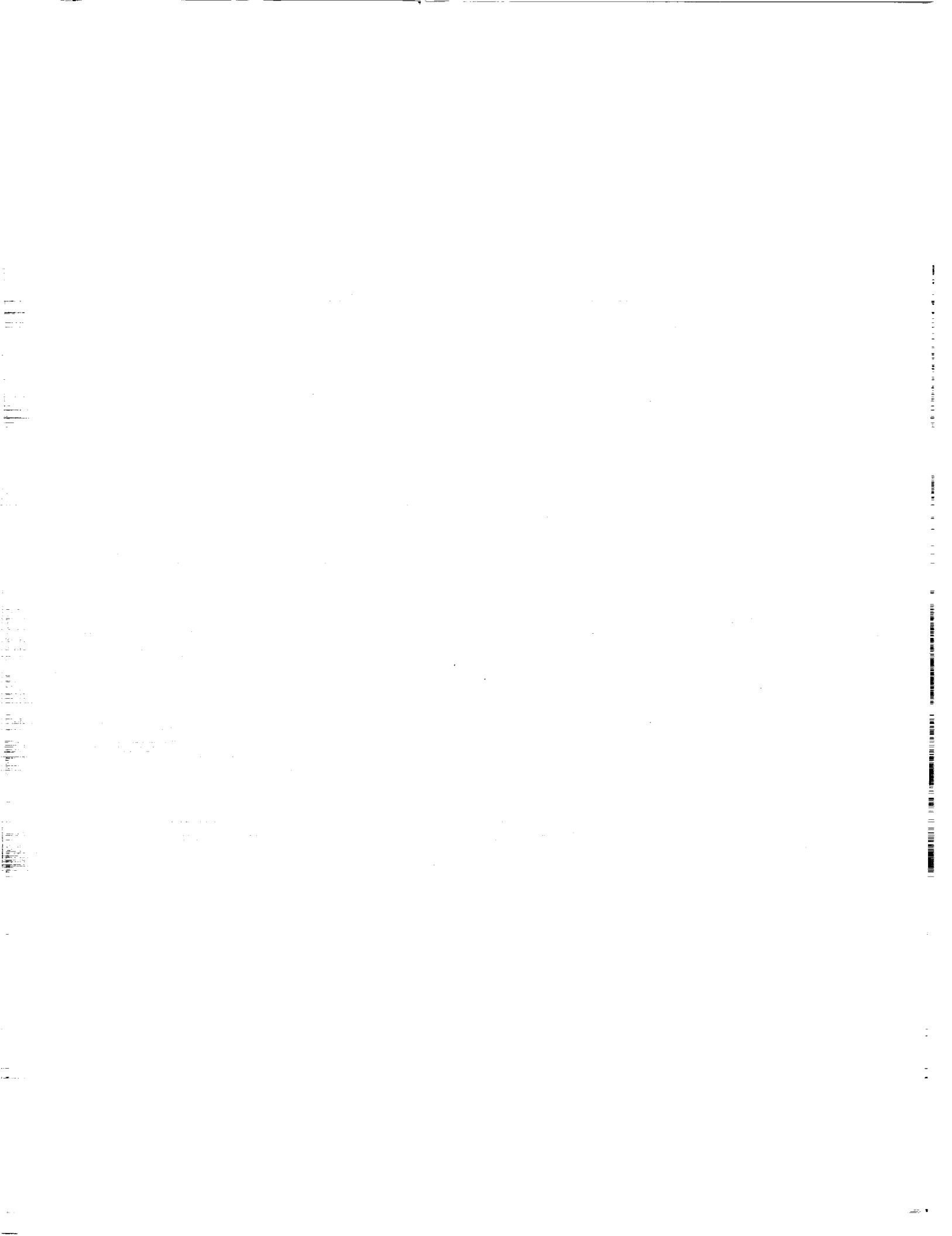
Regarding your choice of linear or nonlinear rapid term, it seems that the nature of inter-linkage between the stress and dissipation equations is crucial in determining if you get a steady state or a periodic behavior.

J.R. Ristorcelli (reply)

Everything was same and just the rapid term model was changed.

B.E. Launder (to J.R. Ristorcelli)

Since some of the  $\epsilon$  equations you are using are not the ones advocated by the model originators so what you were seeing wasn't the effect of just a change in the rapid term.



# THE PRESENT STATE AND FUTURE DIRECTION OF SECOND ORDER CLOSURE MODELS FOR COMPRESSIBLE FLOWS

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## Reynolds Stress Closure Models

- The complexity of the turbulent flow fields associated with advanced aerodynamic vehicles requires turbulent closure models that are capable of handling phenomena such as shock-turbulence interactions, flow separation, and strong three-dimensional effects.
- The strong anisotropies associated with such effects cannot, in general, be handled by eddy viscosity type models. The next level of closure is required, that is, Reynolds stress closure models.
- Some areas where work needs to be focused in order to effectively close the Reynolds stress transport equations are:
  - Pressure-strain models capable of handling complex flows.
  - Pressure dilatation model for compressible flows.
  - Model for the compressible dissipation rate.
  - Near-wall stiffness problems.
  - Reynolds mass and heat flux terms
- In order to compute complex compressible turbulent flows, it is also necessary to develop efficient and accurate numerical algorithms whose characteristics are well documented.



## Favre Averages and Governing Equations

- Any dependent flow variable  $f$  can be decomposed using the usual Reynolds decomposition given by

$$f = \bar{f} + f',$$

where

$$\bar{f} = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau f(\mathbf{x}, t) dt$$

or a Favre average given by

$$f = \tilde{f} + f'',$$

where

$$\tilde{f} = \frac{\overline{\rho f}}{\bar{\rho}}.$$

## Favre Averages and Governing Equations (cont.)

- The mean conservation equations for mass, momentum and total energy can then be written as

MASS

$$\partial_t \bar{\rho} + (\bar{\rho} \tilde{u}_k)_{,k} = 0,$$

MOMENTUM

$$\partial_t (\bar{\rho} \tilde{u}_i) + (\bar{\rho} \tilde{u}_i \tilde{u}_j)_{,j} = -\bar{p}_{,i} + \bar{\sigma}_{ij,j} - (\bar{\rho} \tau_{ij})_{,j},$$

where

$$\bar{\sigma}_{ij} = -\frac{2}{3} \overline{\mu u_{k,k} \delta_{ij}} + \overline{\mu (u_{i,j} + u_{j,i})} \simeq -\frac{2}{3} \bar{\mu} \tilde{u}_{k,k} \delta_{ij} + \bar{\mu} (\tilde{u}_{i,j} + \tilde{u}_{j,i})$$

$$\tau_{ij} \equiv \widetilde{u_i'' u_j''}, \quad \bar{p} = \bar{p} R \tilde{T}$$

## TOTAL ENERGY

$$\partial_t(\bar{p}\tilde{E}) + (\bar{p}\tilde{u}_k\tilde{E})_{,k} = (\bar{\sigma}_{jk}\bar{u}_j - \bar{p}\bar{u}_k - \bar{q}_k)_{,k} + (\bar{\sigma}'_{jk}\bar{u}'_j - \bar{p}'\bar{u}'_k - \bar{p}\tilde{E}''\bar{u}''_k)_{,k}$$

where  $E = C_v T + \frac{1}{2}u_i u_i$ ,

$$\bar{q}_k = -\overline{\kappa T}_{,k} \simeq -\bar{\kappa}\tilde{T}_{,k}$$

$$\begin{aligned} \widetilde{E''u''_k} &= C_v \widetilde{u''_k T''} + \tilde{u}_j \tau_{jk} + \frac{\widetilde{u''_j u''_k u''_k}}{2} \\ &\simeq -C_v C_\mu \frac{k^2}{\epsilon \sigma_T} \tilde{T}_{,k} + \tilde{u}_j \tau_{jk} + \frac{\widetilde{u''_j u''_k u''_k}}{2} \end{aligned}$$

and  $\bar{\kappa}$  is the thermal conductivity.

- The energy equation is written in terms of the total energy because this structure is necessary in order to effectively employ shock capturing techniques in the numerical solution algorithms.

## Favre Averages and Governing Equations (cont.)

- The Favre-averaged Reynolds stress transport equation is given by

### REYNOLDS STRESS TRANSPORT

$$\begin{aligned} \partial_t(\bar{\rho}\tau_{ij}) + (\tilde{u}_k \bar{\rho}\tau_{ij})_{,k} = & P_{ij} + \Pi_{ij} - T_{ij,k,k} - \epsilon_{ij} + \frac{2}{3}\overline{p'u'_{k,k}}\delta_{ij} - \overline{u''_i p_{,j}} - \overline{u''_j p_{,i}} \\ & + (\overline{\sigma'_{ik}u'_j} + \overline{\sigma'_{jk}u'_i})_{,k} + \overline{u''_i \sigma'_{j,k,k}} + \overline{u''_j \sigma'_{i,k,k}} \end{aligned}$$

where

$$\begin{aligned} P_{ij} &= -\bar{\rho}\tau_{ik}\tilde{u}_{j,k} - \bar{\rho}\tau_{jk}\tilde{u}_{i,k} \\ \Pi_{ij} &= \overline{p'u'_{i,j}} + \overline{p'u'_{j,i}} - \frac{2}{3}\overline{p'u'_{k,k}}\delta_{ij} \\ \epsilon_{ij} &= \overline{\sigma'_{ik}u'_{j,k}} + \overline{\sigma'_{jk}u'_{i,k}} \\ T_{ijk} &= \overline{\rho u''_i \tilde{u}''_j u''_k} + (\overline{p'u'_i \delta_{jk}} + \overline{p'u'_j \delta_{ik}}) \\ &\simeq -C_s \bar{\rho} \frac{k^2}{\epsilon} [\tau_{ij,k} + \tau_{jk,i} + \tau_{ik,j}] \end{aligned}$$

and

$$\overline{\sigma'_{ik}u'_j} + \overline{\sigma'_{jk}u'_i} \simeq \bar{\mu}[\tau_{ij,k} + \tau_{jk,i} + \tau_{ik,j}]$$

## Model for the Deviatoric Part of the Pressure-Strain Rate Correlation

- An analysis of pressure-strain correlation models using a dynamical systems approach has led to the development of a new model for this correlation. This improved model is only weakly nonlinear in the anisotropy tensor.
- Based on an analysis of the bifurcation diagram for rotating shear flow, it has been shown that the deficiencies of many pressure-strain correlation models are intrinsic to this general hierarchy of models and cannot be eliminated by the addition of more complex terms.
- The model performs better, overall, in incompressible, homogeneous shear, with and without rotation than previously proposed models. In the case of homogeneous shear with curvature, the model performs as well as other models.
- This overall improvement is shown to carry over to the compressible regime where a simple variable density extension of the model is used.

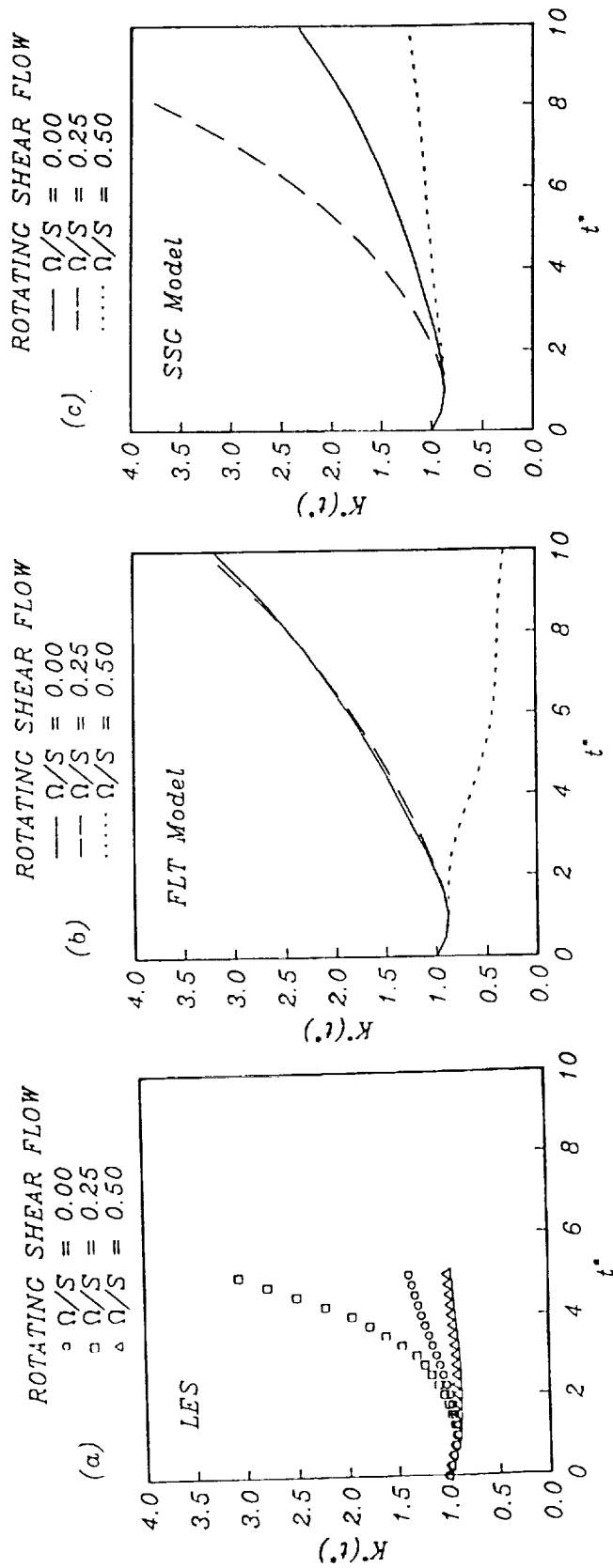
## The SSG Pressure-Strain Rate Correlation Model

$$\begin{aligned} \Pi_{ij} = & -\bar{\rho}(C_1\epsilon + C_1^*\mathcal{P})b_{ij} + C_2\epsilon(b_{ik}b_{kj} - \frac{1}{3}b_{mn}b_{mn}\delta_{ij}) + (C_3 - C_3^*II^{\frac{1}{2}})k(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\delta_{ij}) \\ & + C_4k(b_{ik}\tilde{S}_{jk} + b_{jk}\tilde{S}_{ik} - \frac{2}{3}b_{mn}\tilde{S}_{mn}\delta_{ij}) + C_5k(b_{ik}\tilde{W}_{jk} + b_{jk}\tilde{W}_{ik}) \end{aligned}$$

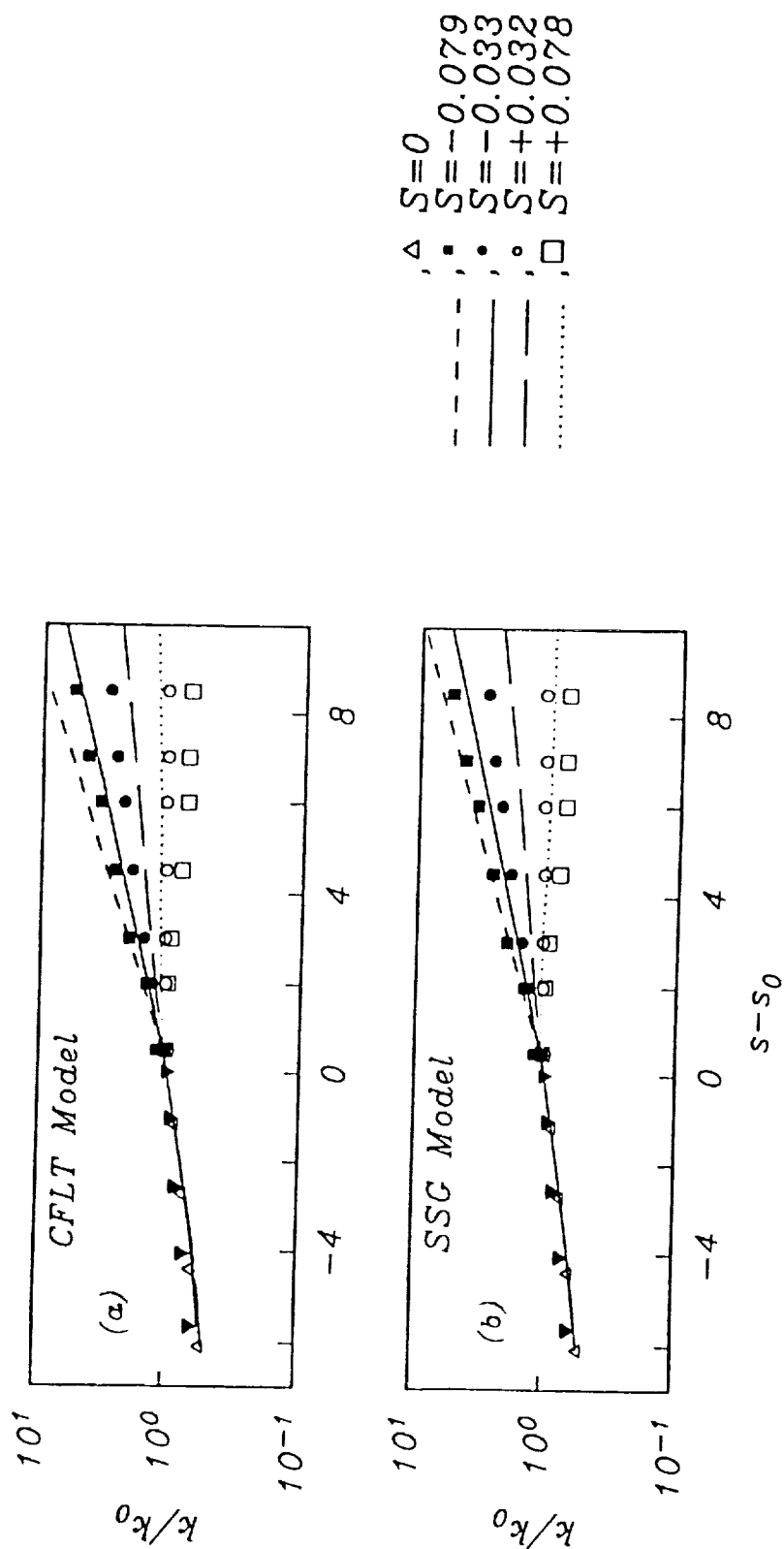
where  $\mathcal{P} = -\tau_{ij}\tilde{u}_{i,j}$  is the turbulence production,  $\tilde{S}_{ij}$  and  $\tilde{W}_{ij}$  are the mean strain and vorticity tensors, respectively, and

$$C_1 = 3.4, C_1^* = 1.80, C_2 = 4.2, C_3 = \frac{4}{5},$$

$$C_3^* = 1.30, C_4 = 1.25, C_5 = 0.40$$

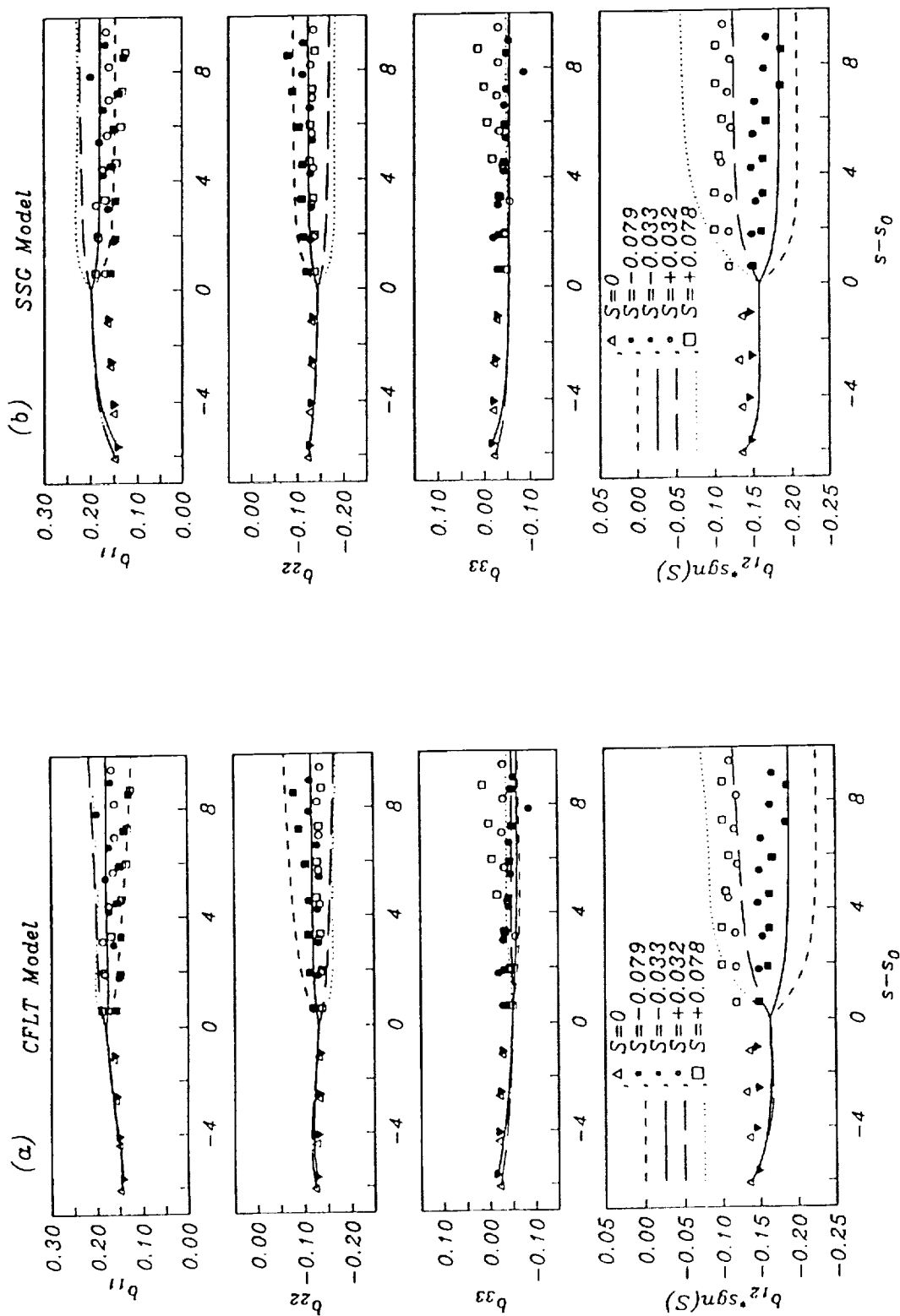


Comparison of the Speziale, Sarkar and Gatski (SSG) model and Fu, Launder and Tselepidakis (FLT) model with large-eddy simulations (LES) of rotating shear flow for different ratios of the rotation rate to shear ( $\Omega/S$ ): (a) LES of Bardina, Ferziger and Reynolds, (b) FLT model, and (c) SSG model.



Variation of Kinetic Energy with Strain for Homogeneous Shear with Longitudinal Curvature (symbols are the experimental results of Holloway and Tavoularis, 1992): a) Craft, Fu, Launder and Tselepidakis (CFLT) Model, b) SSG Model.





Variation of Anisotropies with Strain for Homogeneous Shear with Longitudinal Curvature (symbols are the experimental results of Holloway and Tavoularis, 1992): a) Craft, Fu, Launder and Tselepidakis (CFLT) Model, b) SSG Model.

## A Model for the Pressure-Dilatation $\overline{p'd'}$

- Decompose the pressure-dilatation into contributions from the incompressible pressure and the compressible pressure.

$$\overline{p'd'} = \overline{p^{I'}d'} + \overline{p^{C'}d'}$$

- The incompressible pressure satisfies the usual Poisson equation

$$\nabla^2 p^{I'} = -2\bar{\rho}\bar{u}_{m,n}^I u_{n,m}^{I'} - \bar{\rho}u_{m,n}^{I'} u_{n,m}^{I'}$$

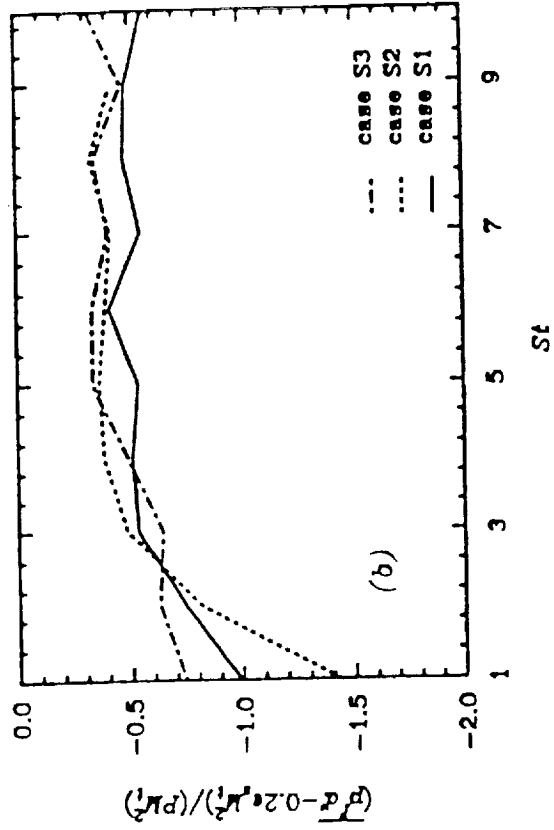
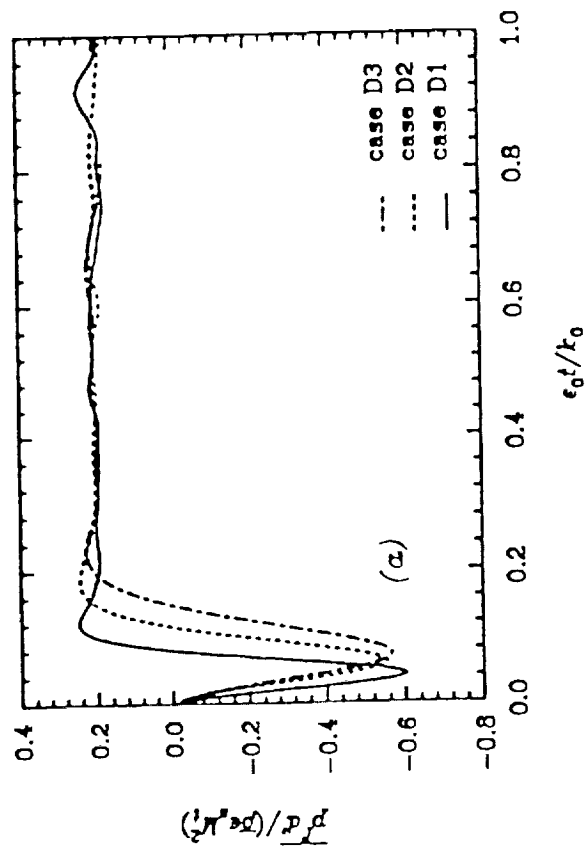
and the remainder is the compressible pressure  $p^{C'}$

- The time-integrated  $\overline{p^{C'}d'} \simeq 0$ . Therefore only  $\overline{p^{I'}d'}$  needs to be modeled.
- The Poisson equation for  $p^{I'}$  is solved and the resulting exact expression for  $\overline{p^{I'}d'}$  is used to obtain the following model

$$\overline{p'd'} = -\alpha_2 \bar{\rho} \mathcal{P} M_t^2 + \alpha_3 \bar{\rho} \epsilon_s M_t^2$$

where  $\mathcal{P} = -\tilde{u}_{i,j} \widetilde{u_i'' u_j''}$  is the production, and  $\epsilon_s$  the solenoidal dissipation.

- DNS is used to verify the model for  $\overline{p'd'}$  and to calibrate it.



Pressure-Dilatation as a Function of Nondimensional Time: a) Decaying Isotropic Turbulence, b) Homogeneous Shear Flow.

## Compressible Turbulent Dissipation Rate Model

- For the present purposes, the turbulent dissipation rate is assumed to be isotropic, so that

$$\epsilon_{ij} = \frac{2}{3}\bar{\rho}\epsilon\delta_{ij},$$

and

$$\bar{\rho}\epsilon = \overline{\sigma'_{kl}u'_{k,l}}.$$

- The compressible Navier-Stokes equations have been asymptotically analyzed at high Reynolds numbers. This leads to a decomposition of the dissipation rate  $\epsilon$  into a solenoidal part,  $\epsilon_s$ , and a compressible part,  $\epsilon_c$ ,

$$\bar{\rho}\epsilon = \bar{\rho}(\epsilon_s + \epsilon_c),$$

where

$$\epsilon_s = \overline{\nu\omega'_i\omega'_i}, \quad \epsilon_c = \frac{4}{3}\overline{\nu u'^2_{k,k}},$$

and  $\omega'_i$  is the fluctuating vorticity.

## Compressible Turbulent Dissipation Rate Model (cont.)

- There is an equi-partition between the kinetic and potential energies in high turbulent Reynolds number and low turbulent Mach number flows. The analysis leads to a model for the compressible dissipation rate,

$$\epsilon_c \simeq \alpha_1 \epsilon_s M_t^2,$$

where  $M_t$  is the turbulent Mach number based on the turbulent kinetic energy and local mean speed of sound, and the modeling coefficient  $\alpha_1$  is taken as 0.5 based on comparison with direct simulations of the compressible, homogeneous turbulence.

- An extension of the incompressible form is adopted for  $\epsilon_s$ ,

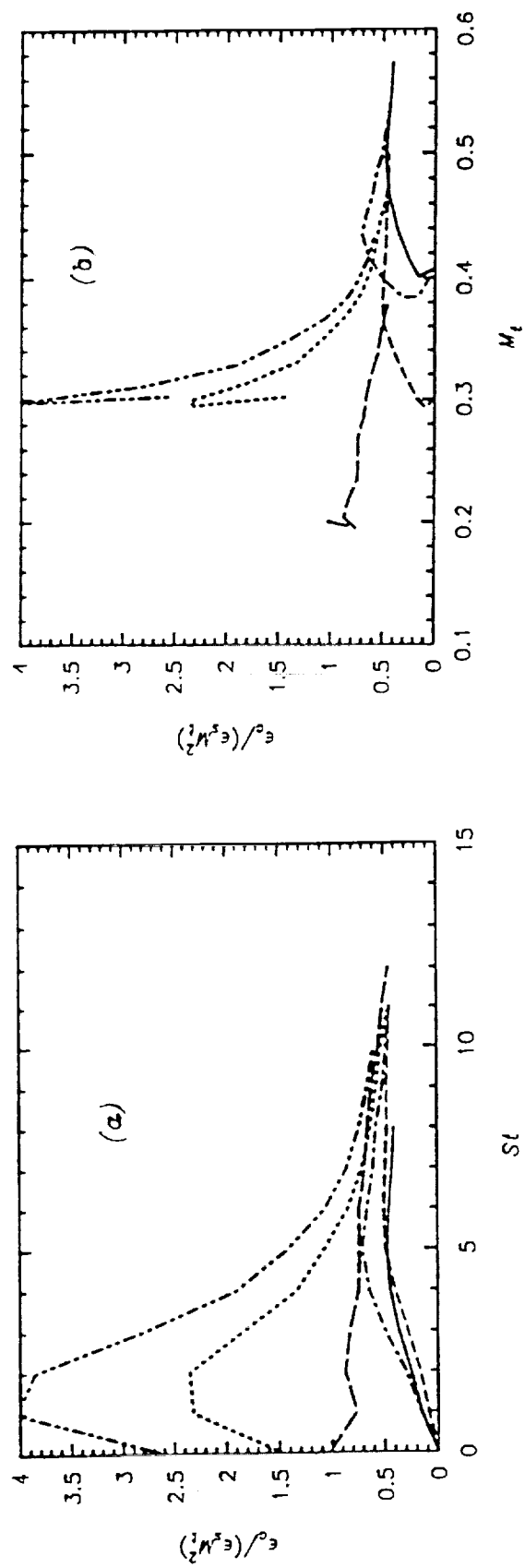
$$\begin{aligned} \partial_t(\bar{p}\epsilon_s) + (\bar{p}\tilde{u}_k\epsilon_s)_{,k} = & - C_{\epsilon 1} \frac{\epsilon_s}{k} \bar{p} \tau_{ij} (\tilde{u}_{i,j} - \frac{1}{3} \tilde{u}_{k,k} \delta_{ij}) - \frac{4}{3} \bar{p} \epsilon_s \tilde{u}_{i,i} \\ & - C_{\epsilon 2} \bar{p} \frac{\epsilon_s^2}{k} + C_{\epsilon} (\frac{\bar{p}k}{\epsilon_s} \widetilde{u_k'' u_l''} \epsilon_{s,l})_{,k} \end{aligned}$$

where the model coefficients are given by

$$C_{\epsilon 1} = 1.44, \quad C_{\epsilon 2} = 1.90, \quad C_{\epsilon} = 0.15$$

## Compressible Turbulent Dissipation Rate Model (cont.)

- This equation is applicable to moderate Mach number turbulence and may have to be modified for strongly supersonic or hypersonic flows.



Variation of Scaled Turbulent Dissipation Rate Ratio as a Function of Nondimensional Time (a) and Turbulent Mach Number (b).

## Variable Viscosity Effects

- The transport equation for the fluctuating vorticity can be written in the form
- The solenoidal dissipation is defined as  $\epsilon_s = \overline{\nu \omega'_i \omega'_i}$ . It can be obtained from the fluctuating vorticity transport equation by first using

$$\overline{2\bar{\rho}\bar{\nu}\omega'_i \left( \frac{\partial \omega'_i}{\partial t} + \tilde{u}_j \frac{\partial \omega'_i}{\partial x_j} \right) + \bar{\rho} \frac{D\bar{\nu}}{Dt} \overline{\omega'_i \omega'_i}} \equiv \frac{\partial(\bar{\rho}\epsilon_s)}{\partial t} + \frac{\partial(\bar{\rho}\tilde{u}_j \epsilon_s)}{\partial x_j}$$

- The transport equation for the solenoidal dissipation can then be written as

$$\begin{aligned} \frac{\partial(\bar{\rho}\epsilon_s)}{\partial t} + \frac{\partial(\bar{\rho}\tilde{u}_j \epsilon_s)}{\partial x_j} &= \bar{\rho} \frac{D\bar{\nu}}{Dt} \overline{\omega'_i \omega'_i} - \frac{4}{3} \bar{\rho} \epsilon_s \frac{\partial \tilde{u}_i}{\partial x_i} - C_{\epsilon 1} \frac{\epsilon_s}{k} \bar{\rho} \tau_{ij} (\tilde{u}_{i,j} - \frac{1}{3} \tilde{u}_{k,k} \delta_{ij}) \\ &\quad - C_{\epsilon 2} \bar{\rho} \frac{\epsilon_s^2}{k} + C_{\epsilon} \frac{\bar{\rho} k}{\epsilon_s} \overline{u''_k u''_l \epsilon_{s,l}}, k \end{aligned}$$

- For simplicity, a power law dependence on the temperature can be used for the molecular viscosity

$$\mu(T) = \mu(T_0) \left( \frac{T}{T_0} \right)^n$$

where  $n \approx 0.7$ .



## Variable Viscosity Effects (cont.)

- This leads to the relation (using the isentropic compression assumption; see Coleman & Mansour, 1991)

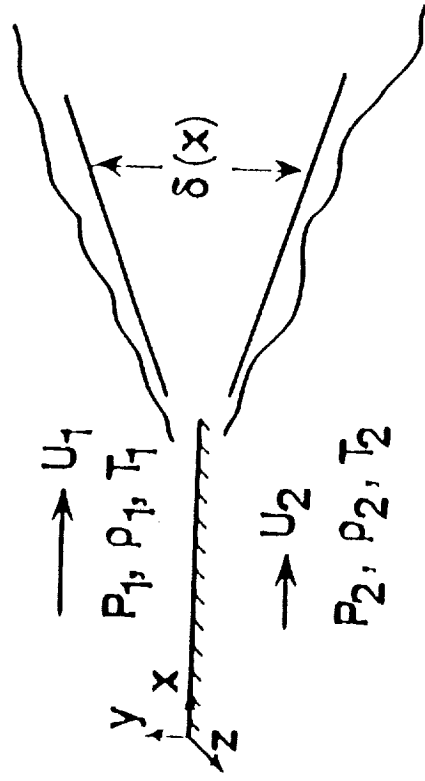
$$\bar{p} \frac{D\bar{v}}{Dt} \overline{\omega'_i \omega'_i} = \bar{p} \epsilon_s \frac{1}{\bar{v}} \frac{D\bar{v}}{Dt} = [1 - n(\gamma - 1)] \frac{\partial \tilde{u}_i}{\partial x_i} \bar{p} \epsilon_s$$

where  $\gamma = c_p/c_v$ .

- The solenoidal dissipation equation can then be written as

$$\begin{aligned} \frac{\partial(\bar{p}\epsilon_s)}{\partial t} + \frac{\partial(\bar{p}\tilde{u}_j\epsilon_s)}{\partial x_j} = & - 0.6\bar{p}\epsilon_s \frac{\partial \tilde{u}_i}{\partial x_i} - C_{\epsilon 1} \frac{\epsilon_s}{k} \bar{p} \tau_{ij} (\tilde{u}_{i,j} - \frac{1}{3} \tilde{u}_{k,k} \delta_{ij}) \\ & - C_{\epsilon 2} \bar{p} \frac{\epsilon_s^2}{k} + C_{\epsilon} (\frac{\bar{p}k}{\epsilon_s} \widetilde{u''_l u''_l} \epsilon_{s,l})_{,k} \end{aligned}$$

## Supersonic Shear Layer

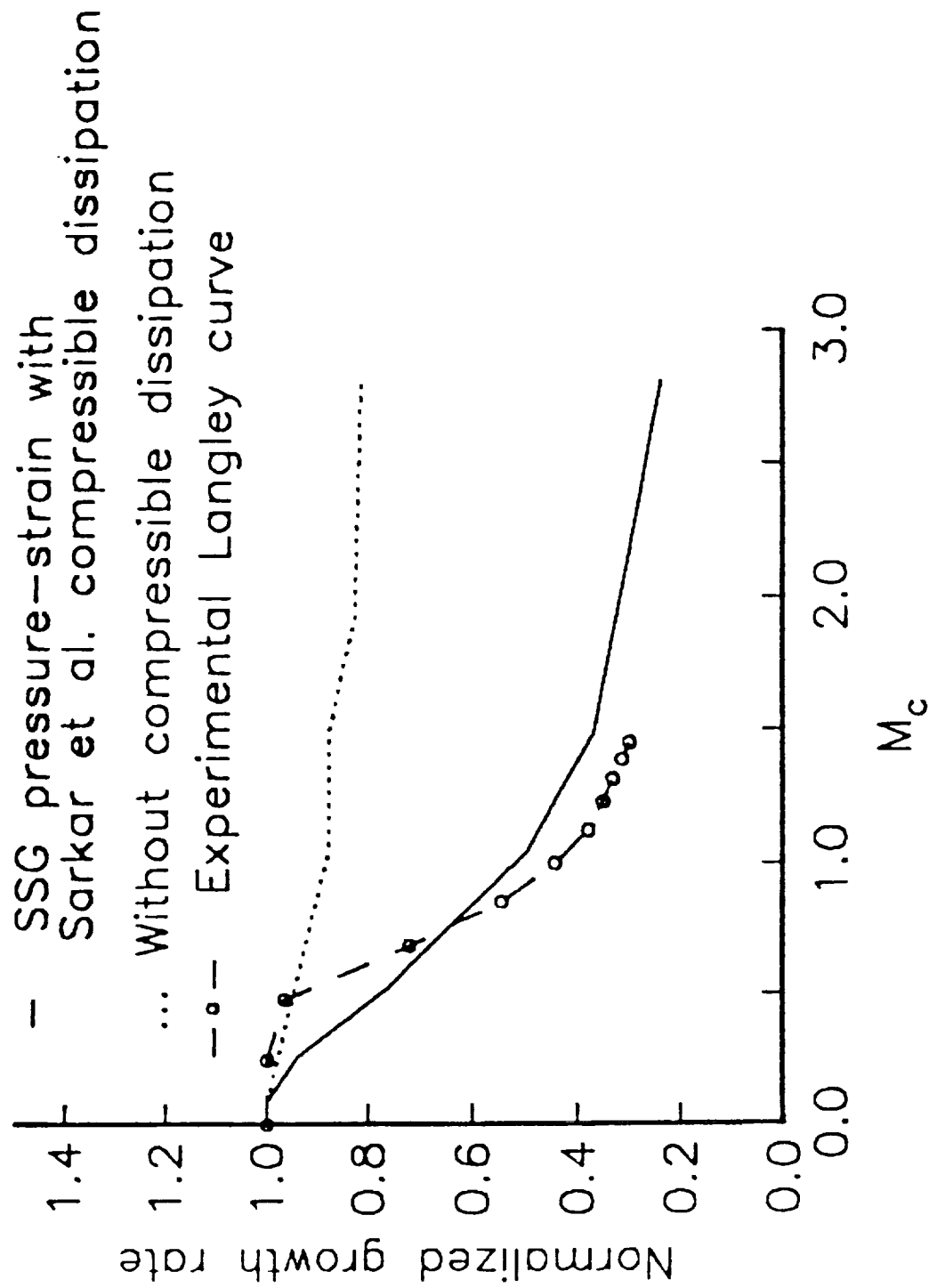


- It is observed experimentally that the fully developed turbulent shear layer spreads as

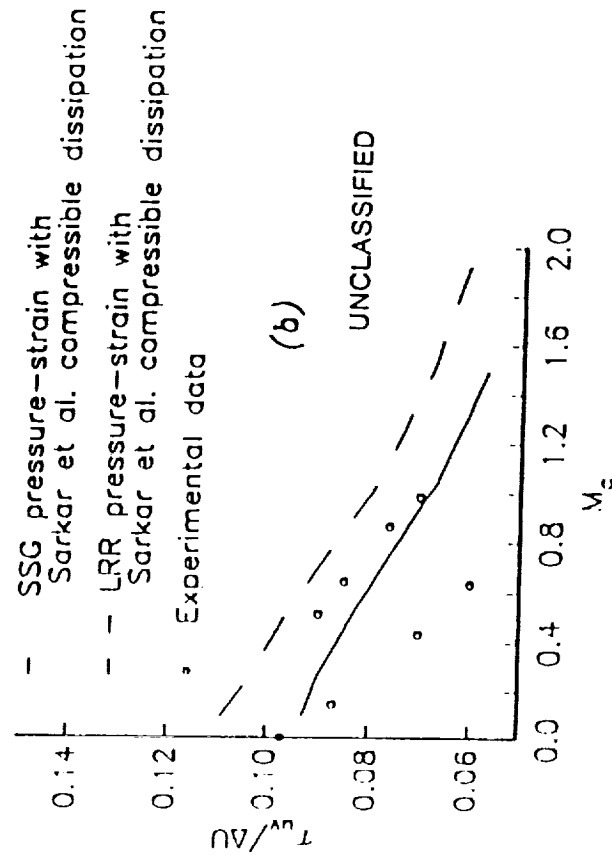
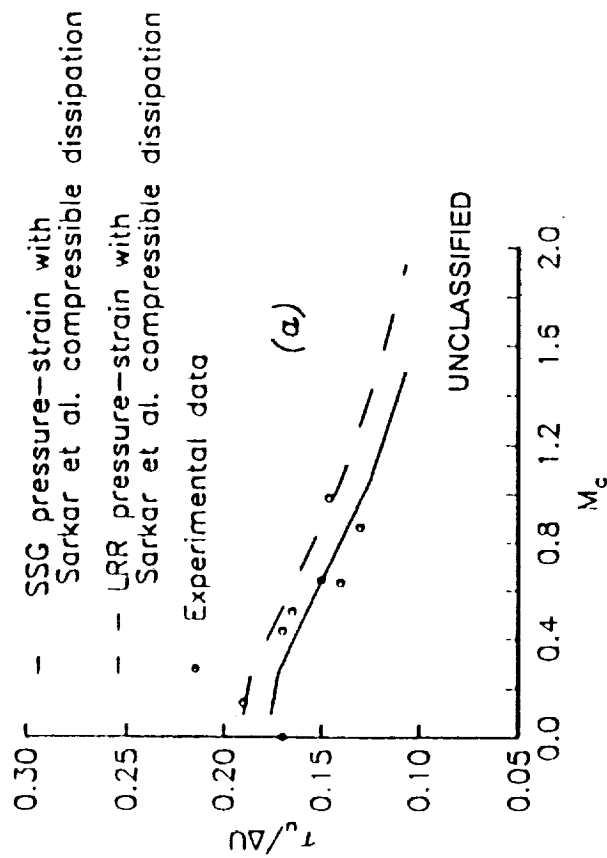
$$\frac{d\delta}{dx} = C_\delta \left( \frac{U_1 - U_2}{U_1 + U_2} \right)$$

where  $C_\delta$  is approximately constant for the incompressible case.

- Experiments show that the spreading rate parameter  $C_\delta$  decreases dramatically when the convective Mach number  $M_c = \Delta/(a_1 + a_2)$  increases.



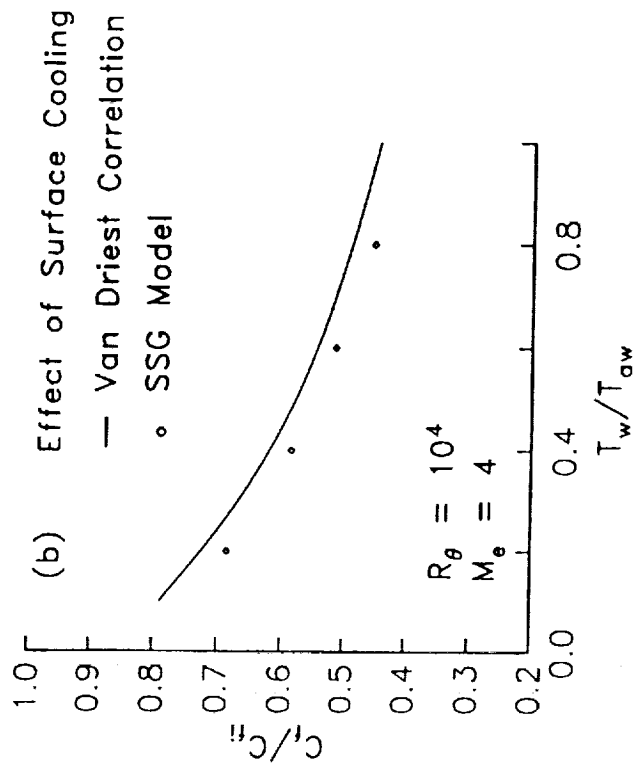
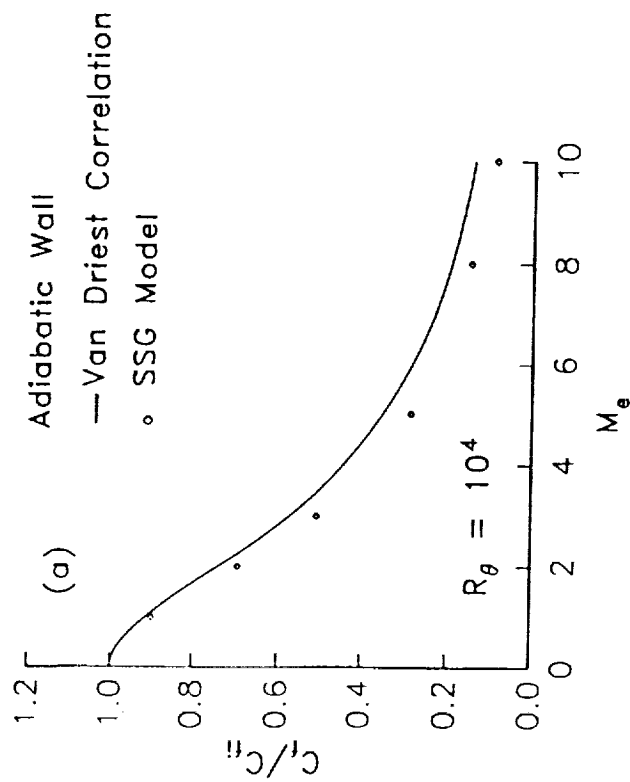
Variation of Normalized Growth Rate with Convective Mach Number for Supersonic Shear Layer.



Variation of Normalized Stress Distributions with Convective Mach Number for Supersonic Shear Layer: (a) Normalized Streamwise Stress, (b) Normalized Shear Stress.

## Near-Wall Stiffness Problems

- It is becoming more desirable to integrate directly to the wall in solving wall-bounded turbulent flows because of the need to compute more complex flowfields.
- It is necessary to have the proper balance of terms in the near-wall region in order to insure the correct asymptotic behavior of the computed turbulent variables.
- An analysis of two-equation models has shown that there are two major deficiencies associated with the solution of the isotropic dissipation rate equation: (1) the lack of natural boundary conditions for the dissipation rate,  $\epsilon$ , and (2) the appearance of higher-order correlations in the balance of terms for the dissipation rate at the wall.
- This deficiency naturally carries over to the solution of the Reynolds stress transport equations.
- In order to achieve asymptotic consistency in the near-wall region, a modification of the solenoidal dissipation equation is required.
- Two damping functions need to be introduced into the equation: (1) a damping function associated with the eddy viscosity in order to insure the correct near-wall behavior for the modeled triple correlation term, (2) a damping function associated with the destruction of dissipation term.



Normalized Skin-Friction Distribution: a) as a function of Mach number for adiabatic wall, b) as a function of wall temperature.

## Models for the Reynolds Mass and Heat Flux

- An examination of the total energy equation and the Reynolds stress transport equations how that models for both the heat flux  $\overline{u_i''T_k''}$  and the mass flux  $\overline{u_i''}$
- These quantities are also needed in the modeling of the pressure-velocity diffusion term  $\overline{(p'u_i'')_j}$

$$\overline{p'u_i''} = -\bar{\rho}R\tilde{T}\overline{u_i''} + \bar{\rho}Ru_i''\tilde{T}''$$

- When gradient transport models are used in the model for the pressure-velocity diffusion term, stiffness problems arise.

$$\overline{u_i''} = \frac{C_\mu k^2}{\bar{\rho}\sigma_\rho \epsilon} \bar{\rho}_{,i}$$

$$\overline{u_i''T''} = -\frac{C_\mu k^2}{\sigma_T \epsilon} \tilde{T}_{,i}$$

- Improvements in the gradient transport hypothesis or an improved set of models for both the Reynolds heat and mass flux are probably needed.

## Numerical Solution of Compressible Turbulent Transport Equations

- Discretization Requirements
  - Accuracy
  - Shock Capturing Ability
  - Artificial Dissipation Scheme Development
- Time Integration Requirements
  - Turbulent transport equations are stiff in near-wall region
- Geometric Complexity Requirements
  - Practical aerodynamic configurations are very complex
- The discretization requirement can be satisfied by developing an algorithm using a (upwind) Roe flux difference splitting scheme for the spatial differences.
- The time integration requirement can be satisfied by using an implicit time integration scheme.
- The extension to complex geometries can be accomplished in many configurations by using a multiblock, finite volume method.



## Concluding Remarks

- As has been shown, compressibility corrections and modifications need to be introduced into a variety of terms in the energy and Reynolds stress transport equations in order to treat high speed flows.
- The more important the role of turbulent transport processes in the flow dynamics, the more desirable/necessary it is to use turbulent stress transport equations.
- At present there is a reluctance to generally utilize Reynolds stress transport equations relative to the simpler two-equation turbulence models. This is true in the incompressible regime and even more so in the compressible regime.
- Reynolds stress turbulent modeling will be utilized on a broader scale only after it is shown that it can be efficiently used with improved predictive capability.
- A generalized time accurate Navier-Stokes code is being adapted (LaRC) to run both two-equation and Reynolds stress models. This duality eliminates the numerical differences and the efficiency of the algorithm with the turbulent closure models can be effectively compared.



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Comment on : The Present State and Future Direction of  
Second-order Closure Models for Compressible Flows P-23

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Workshop on Engineering Turbulence Modeling  
August 21-22, 1991

## Outline

- *Viegas*
  - Opening comments
  - An alternative RSE model
    - Results from VHVRK (FRAME) and LS models
  - On shear layers — spread rate comparisons
    - Results of compressibility corrections
- *Huang*
  - General comments on RSE models
    - compressibility corrections
  - On the law of the wall
  - Concluding remarks

An alternative RSE model: FRAME model (VHVRK, 1983)

- \* Developed in collaboration with colleagues from France  
(A second-order closure model for compressible turbulent boundary layers which is capable of predicting shock boundary layer flows.)
- \* Builds upon pioneering incompressible second order model of LRR.
- \* Compressibility effects included by:
  - \* Introducing Favre Averaging
  - \* Reintroducing non-zero divergence terms that were eliminated in original models
  - \* Accounting for non-zero mass weighted fluctuating velocity - compressibility terms.
- \* Near wall effects included by using:
  - \* LRR near wall pressure-rate of strain terms
  - \* wall damping of quadratic return to isotropy (slow) terms and in the dissipation terms
  - \* Using the Favre averaged form of the Hanjalic-Launder dissipation rate equation with some coefficient modifications to the near wall terms and by using wall damping in the destruction part of this equation.
- \* Uses total energy equation (including  $k$ )
- \* Uses "total" turbulent dissipation rate equation

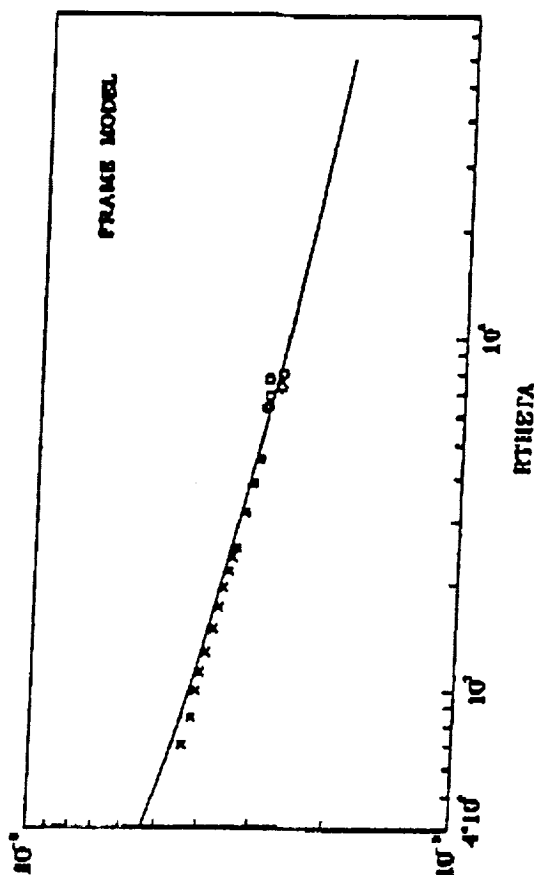
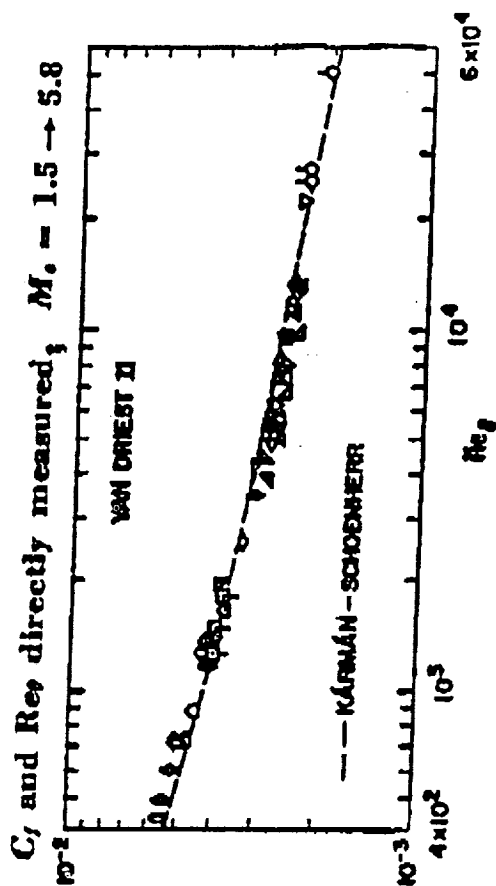
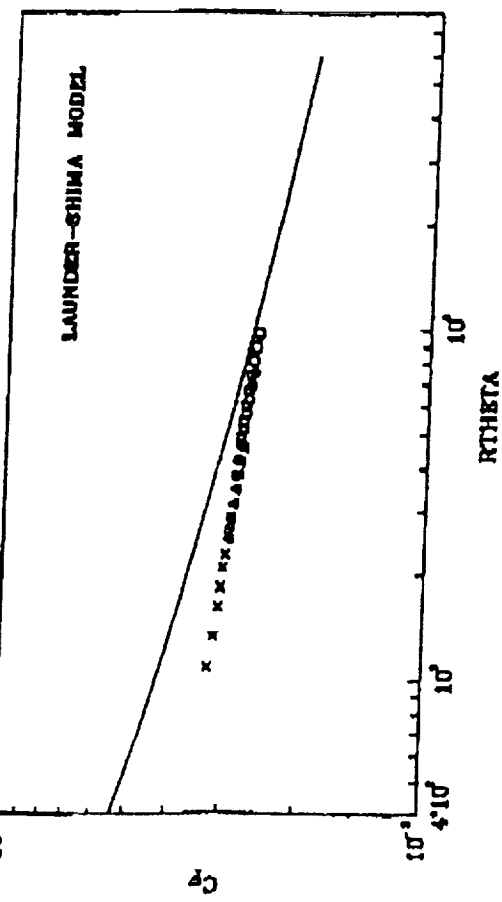
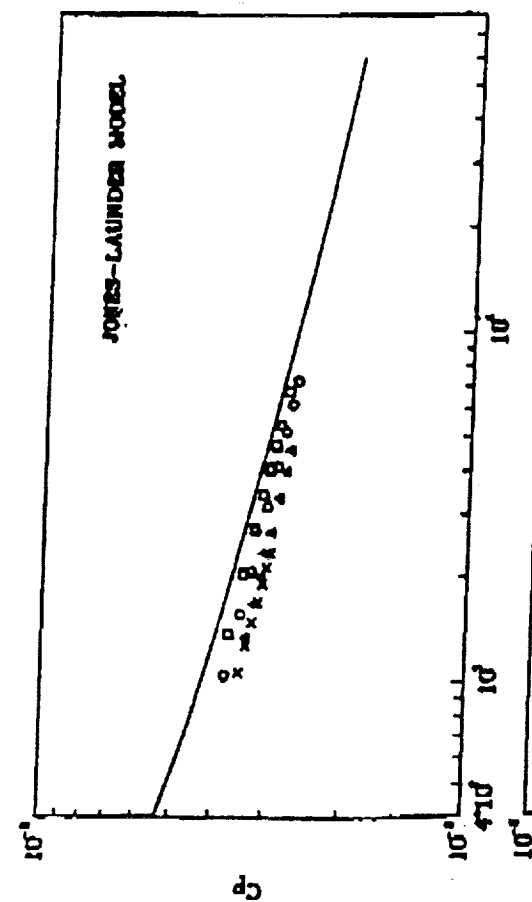
Successfully applied to:

- \* Adiabatic flat plate,  $M=3$ , to develop model
- \* Supersonic expansion at  $M=1.76$  - Dussauge
- \* Transonic shock-boundary layer interaction at  $M=1.36$  - Delery
- \* Corner flow at  $M=3$  - Settles
- \* Oscillating boundary layer on flat plate at  $M=0$  - Spalart -DNS
- \* Adiabatic and nonadiabatic flat plate to  $M=8$  - Karman-Shoenherr

# SKIN FRICTION ON AN ADIABATIC FLAT PLATE

KARMANN-SCHOENHERR LAW

- MACH NO. - 2.0
- MACH NO. - 3.0
- △ MACH NO. - 3.5
- MACH NO. - 4.0
- × MACH NO. - 4.5

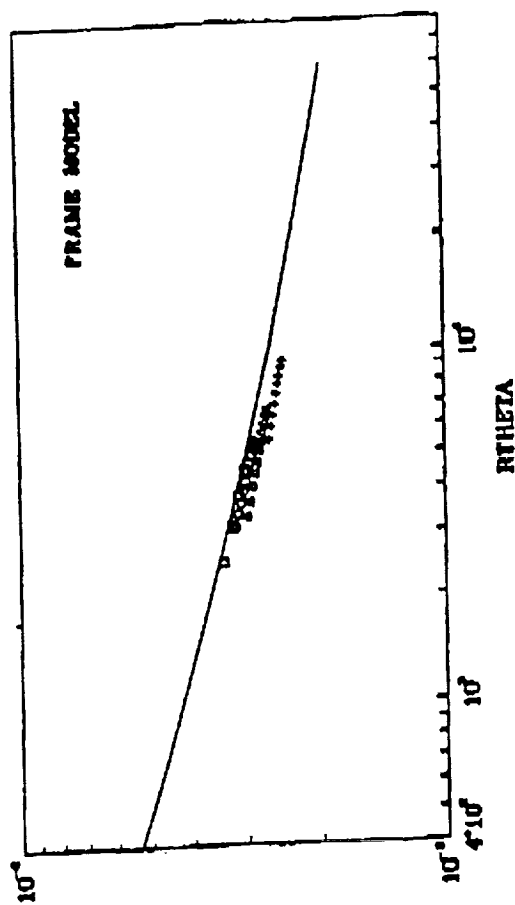
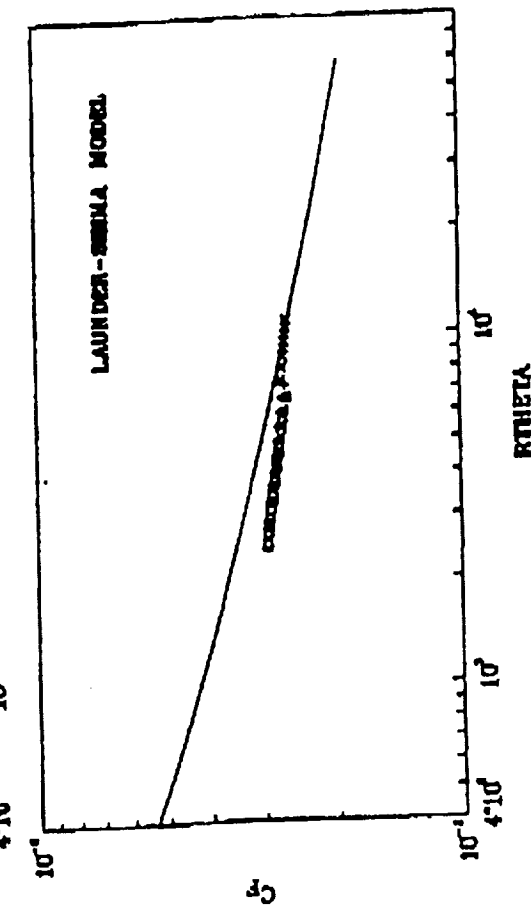
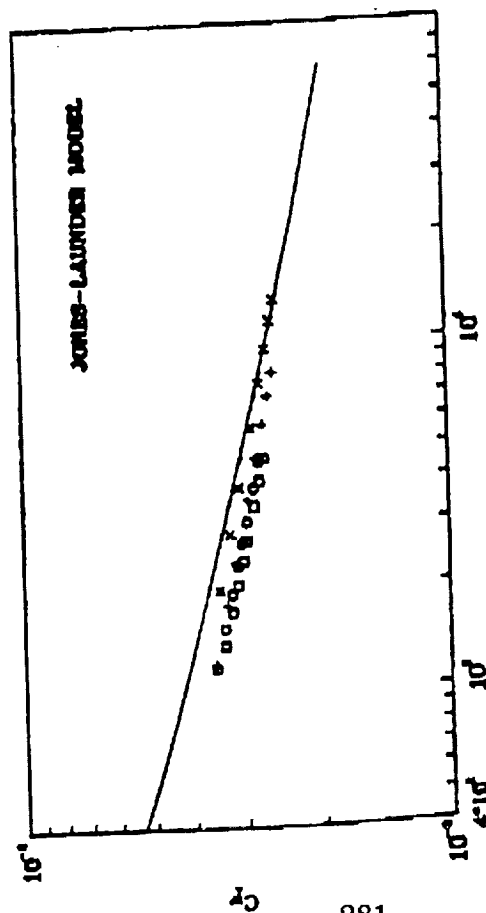
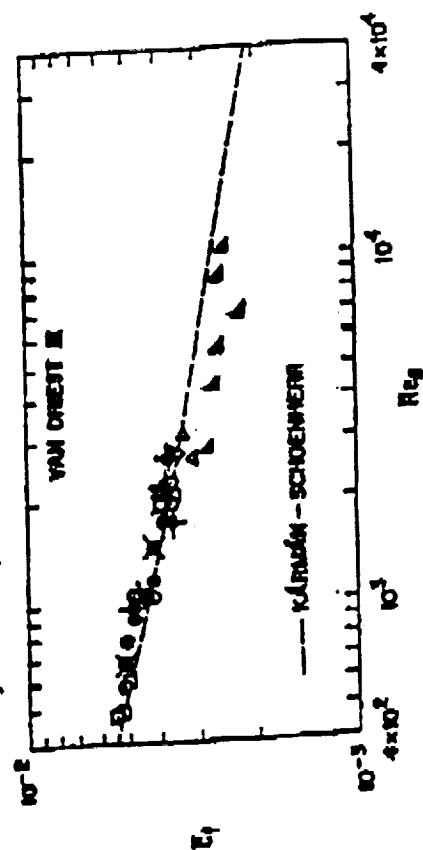


# SKIN FRICTION ON A NON-ADIABATIC FLAT PLATE AT MACH NO. = 5

## KARMANN-SCHOENHEBERG LAW

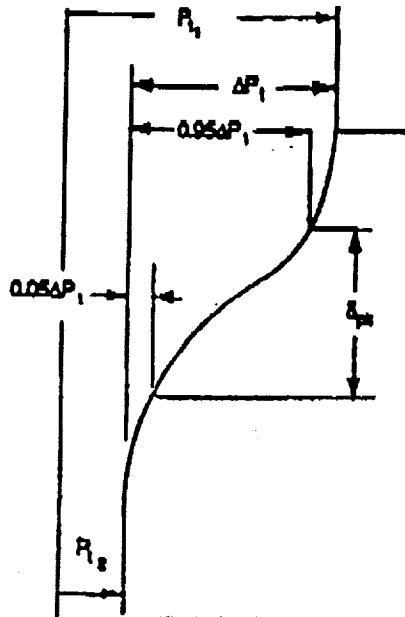
- $T_w/T_{aw} = 1.0$
- $T_w/T_{aw} = 0.9$
- △  $T_w/T_{aw} = 0.8$
- +  $T_w/T_{aw} = 0.4$
- x  $T_w/T_{aw} = 0.2$

$C_f$  directly measured;  $M_e = 2.8 \rightarrow 7.4$ .



# ALTERNATIVE CORRELATIONS OF SPREAD RATE

## \* IMPACT PRESSURE THICKNESS



PITOT PRESSURE PROFILE

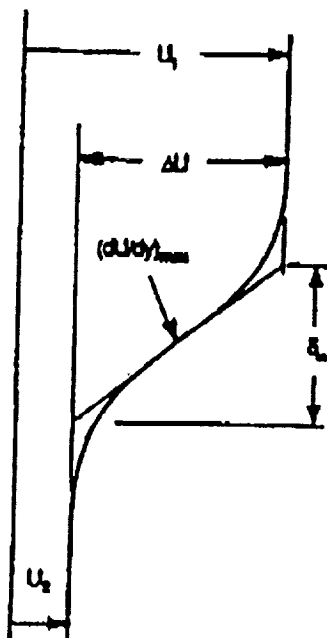
$$\delta'_{pit} = G(M_c) \frac{(1 - U_2/U_1) (1 + \sqrt{(\rho_2/\rho_1)})}{1 + \sqrt{(\rho_2/\rho_1)} (U_2/U_1)}$$

$$M_c = \frac{M_1 \sqrt{\frac{\rho_2}{\rho_1}} - M_2}{\sqrt{\frac{\rho_2}{\rho_1}} + 1.0}$$

$$G(M_c)/G(0)$$

Papamoschou and Roshko

## \* VORTICITY THICKNESS



VELOCITY PROFILE

$$\delta_\omega = (U_1 - U_2)/(\partial U/\partial y)_{max}$$

$$\delta'_\omega = C_\omega(M_c) \frac{1 - U_2/U_1}{1 + U_2/U_1}$$

$$C_\omega(M_c)/C_\omega(0)$$

Bogdanoff



# TURBULENCE MODELS APPLIED

EDDY VISCOSITY (mass weighted variables)

$$\mu_t = C_\mu \bar{\rho} \frac{\bar{k}^2}{\bar{\epsilon}} \quad C_\mu = 0.09$$

TURBULENCE FIELD EQUATIONS

$$(\bar{\rho}\bar{k})_{,t} + (\bar{\rho}\bar{u}_j\bar{k})_{,j} = -(\overline{\rho u_i'' u_j''})\bar{u}_{i,j} - \bar{\rho}\bar{\epsilon} + D_k + E_k$$

$$(\bar{\rho}\bar{\epsilon})_{,t} + (\bar{\rho}\bar{u}_j\bar{\epsilon})_{,j} = -C_{\epsilon 1} \frac{\bar{\epsilon}}{\bar{k}} (\overline{\rho u_i'' u_j''})\bar{u}_{i,j} - C_{\epsilon 2} \bar{\rho} \frac{\bar{\epsilon}^2}{\bar{k}} + D_\epsilon + E_\epsilon$$

$E_k, E_\epsilon \equiv$  Extra Compressibility Terms

STANDARD  $k - \epsilon$  MODEL

$$E_k = 0 \text{ and } E_\epsilon = 0$$

SARKAR ET AL. (SEHK)  $k - \epsilon$  MODEL

$$E_\epsilon = 0$$

$$E_k = -\alpha_1 M_t^2 \bar{\rho}\bar{\epsilon} \quad M_t = \frac{\sqrt{2\bar{k}}}{\bar{a}} \quad \bar{a} = \sqrt{\gamma R\bar{T}}$$

$$\alpha_1 = 1.0$$

ZEMAN  $k - \epsilon$  MODEL

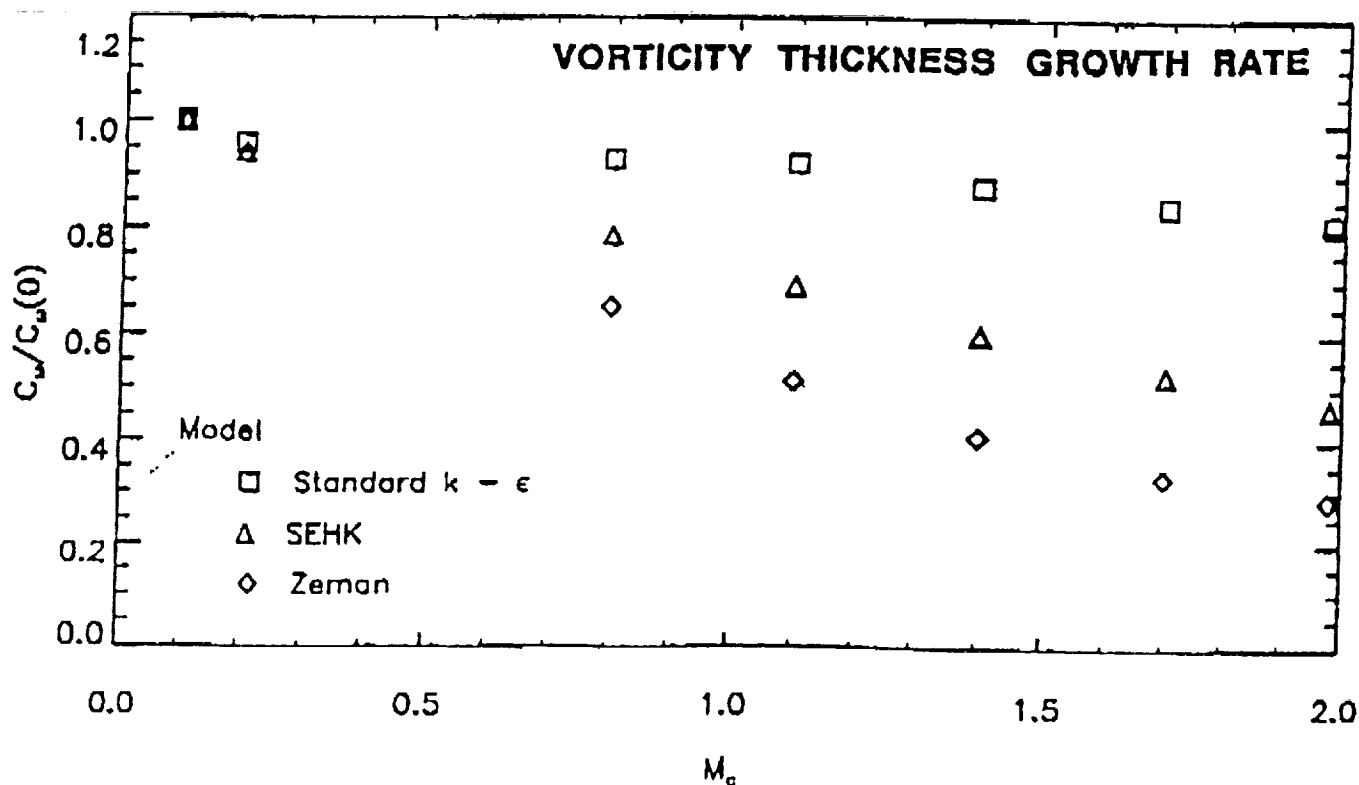
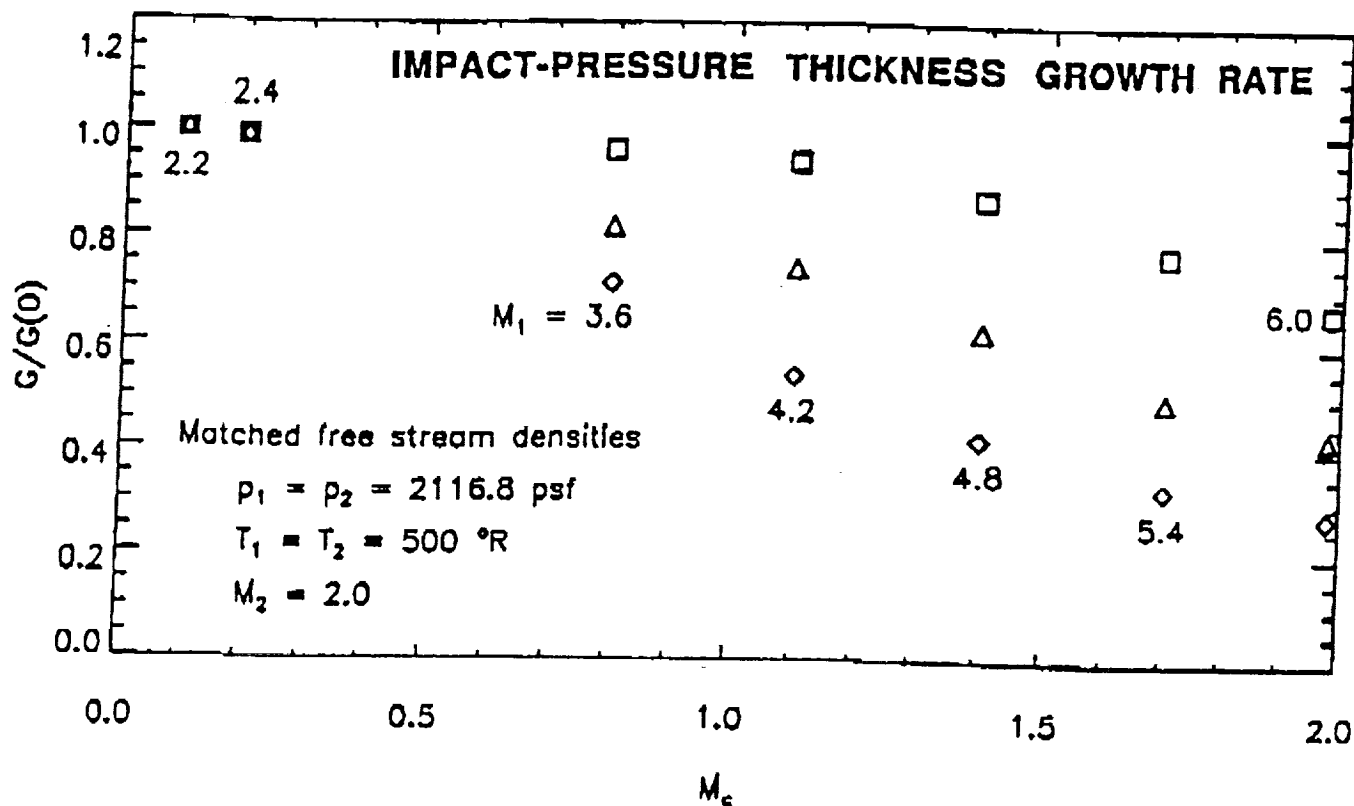
$$E_\epsilon = 0$$

$$E_k = -C_d F(M_t) \bar{\rho}\bar{\epsilon} \quad F(M_t) = 1.0 - \exp\left[-\left((M_t - 0.1)/.6\right)^2\right]$$

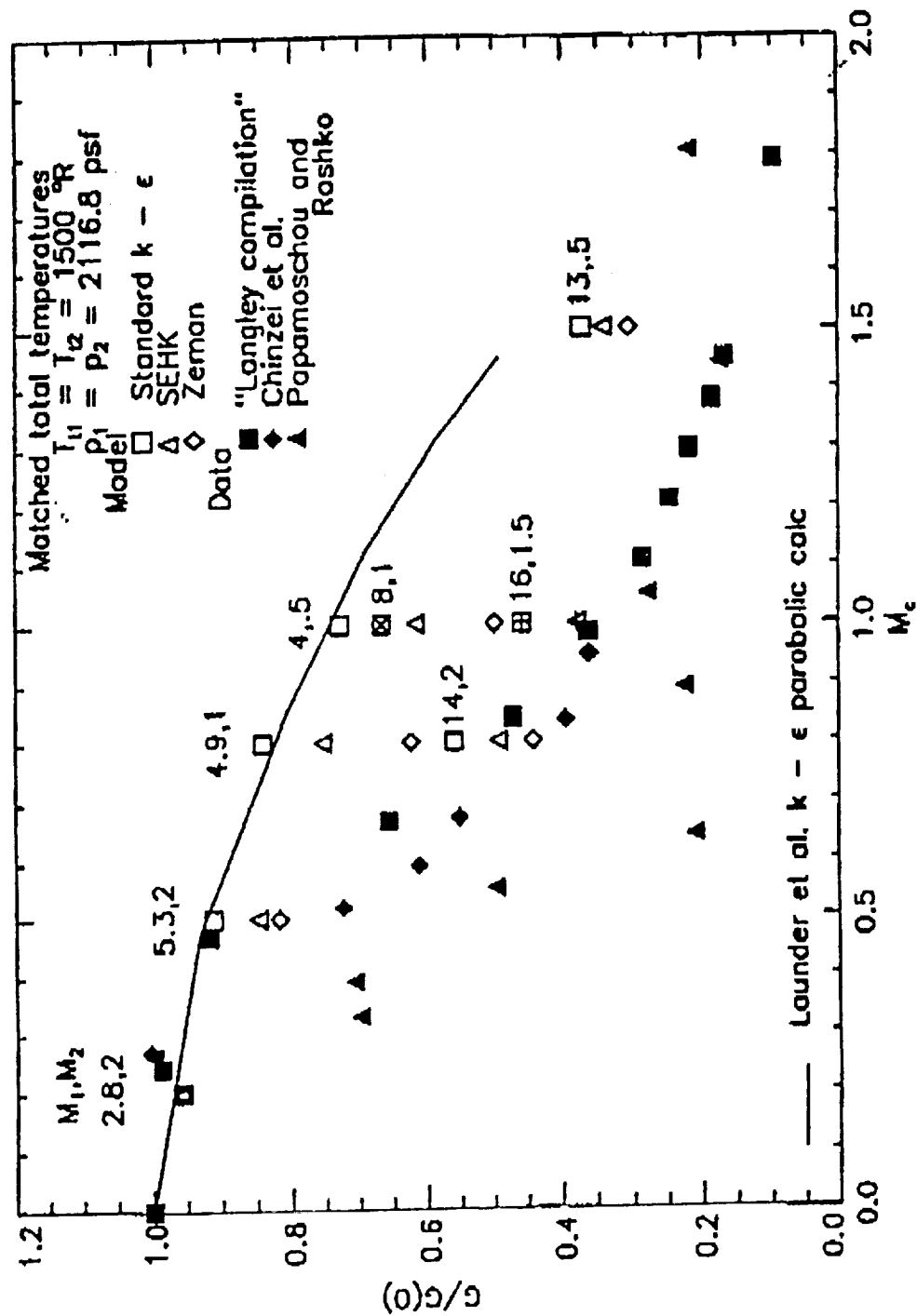
$$\text{and } F(M_t) = 0, \text{ if } M_t < 0.1,$$

$$C_d = 0.75 \quad M_t = \frac{\sqrt{2\bar{k}}}{\alpha^*} \quad \alpha^* = \sqrt{2\gamma R\bar{T}/(1+\gamma)}$$

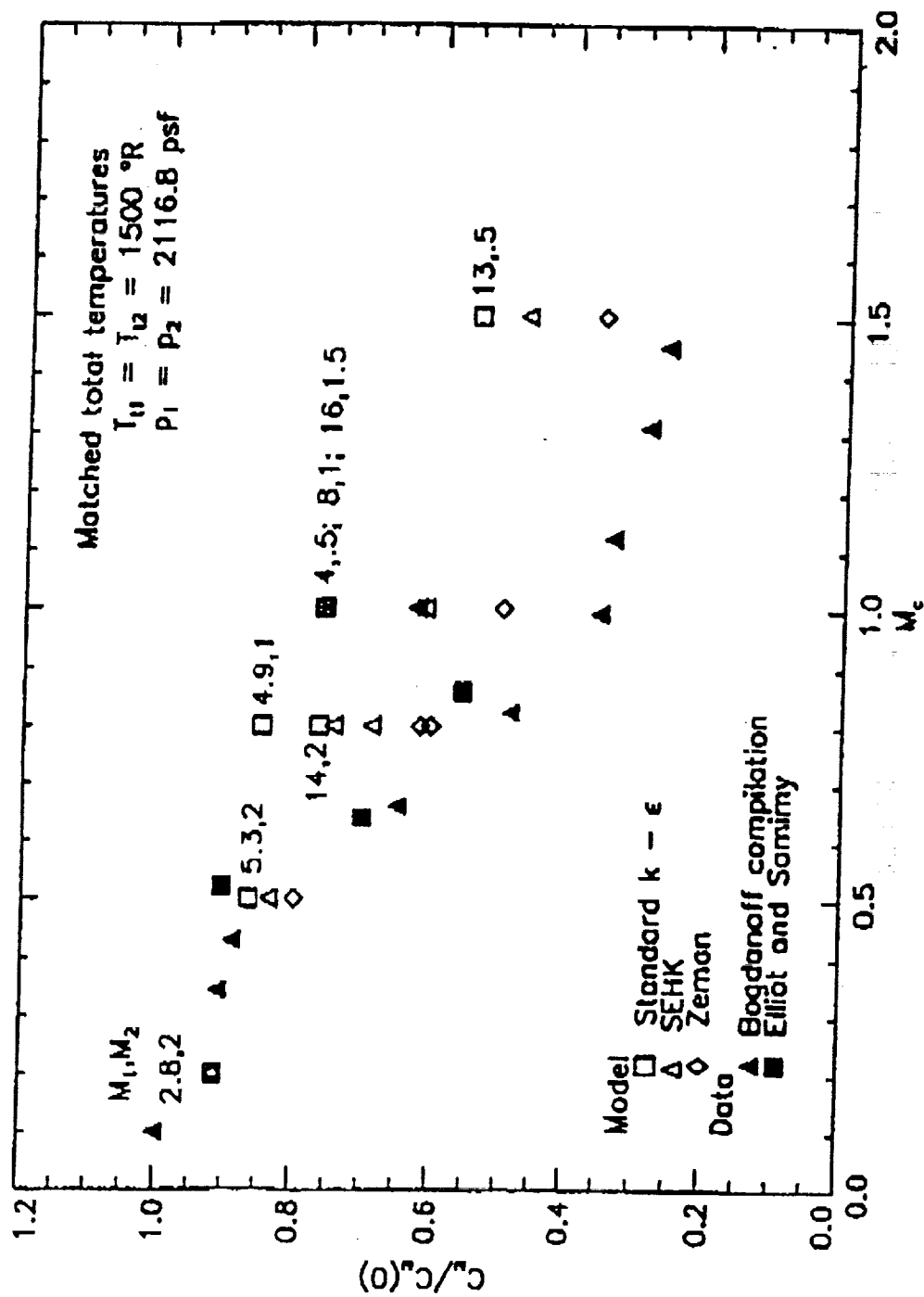
# EFFECT OF COMPRESSIBILITY CORRECTIONS ON PREDICTED FREE-SHEAR SPREAD RATES



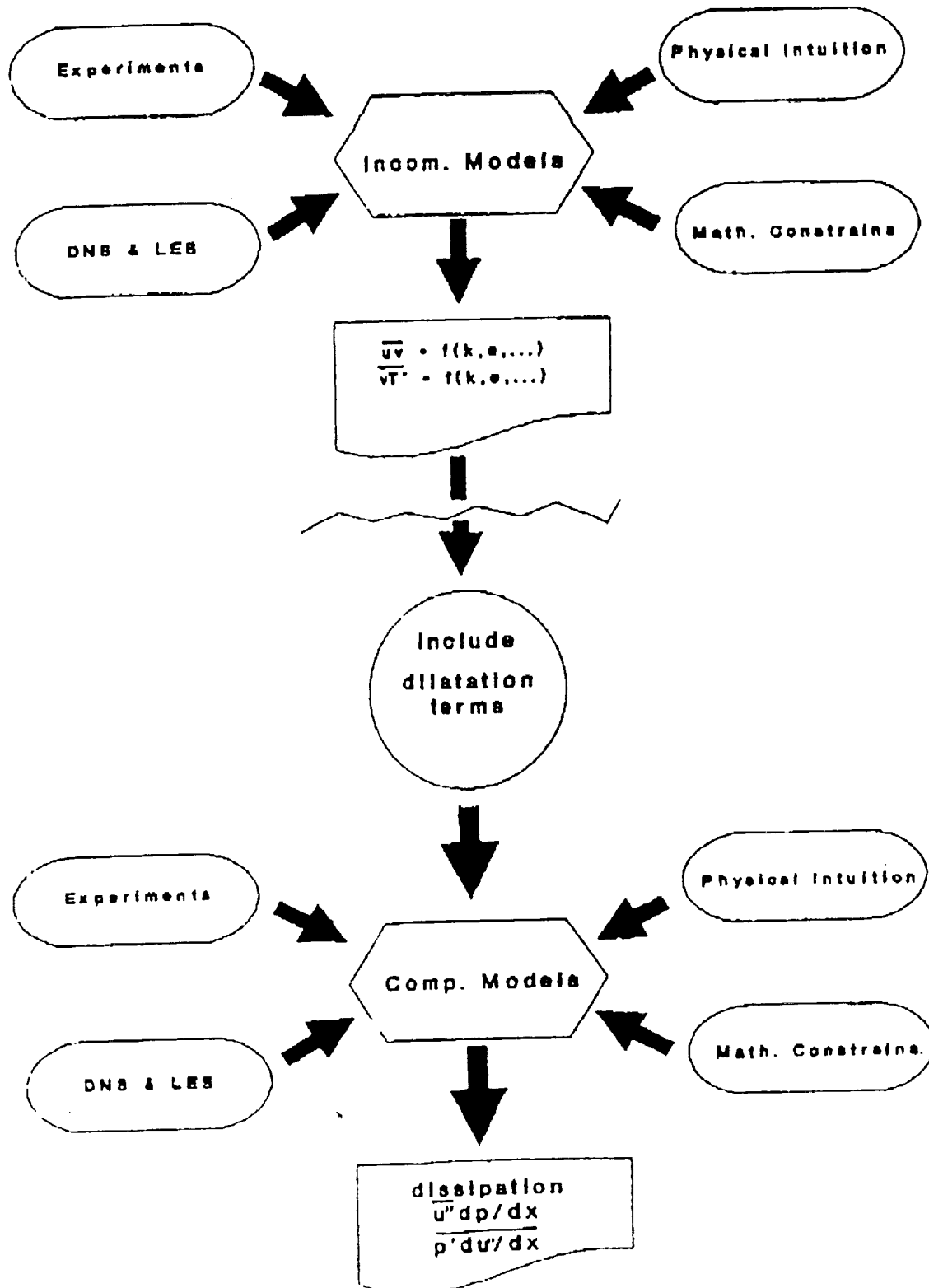
# EFFECT OF COMPRESSIBILITY CORRECTIONS ON IMPACT- PRESSURE THICKNESS GROWTH RATE



# EFFECT OF COMPRESSIBILITY CORRECTIONS ON VORTICITY THICKNESS GROWTH RATE



# Evolution of Turbulence Models



## Modeling

$$\frac{D \overline{\rho u_i'' u_j''}}{Dt} = d_{ij} + P_{ij} + \Phi_{ij} - \rho \epsilon_{ij} + \text{Compressibility terms}$$

### Incompressible Modeling

- Turbulence diffusion,  $d_{ij}^T$ 
  - $\frac{\partial}{\partial x_k} \left[ c_s \rho \frac{k}{\epsilon} \left( \overline{u_i'' u_m''} \frac{\partial \overline{u_j'' u_k''}}{\partial x_m} + \overline{u_j'' u_m''} \frac{\partial \overline{u_k'' u_i''}}{\partial x_m} + \overline{u_k'' u_m''} \frac{\partial \overline{u_i'' u_j''}}{\partial x_m} \right) \right]$
  - $\frac{\partial}{\partial x_k} \left[ \frac{2}{3} c_s \rho \frac{k^2}{\epsilon} \left( \frac{\partial \overline{u_j'' u_k''}}{\partial x_m} + \frac{\partial \overline{u_k'' u_i''}}{\partial x_m} + \frac{\partial \overline{u_i'' u_j''}}{\partial x_m} \right) \right]$
- Pressure Interaction,  $\Phi_{ij} = \Phi_{ij,1} + \Phi_{ij,2} + \Phi_{ij,w}$ 
  - Slow Term,  $\Phi_{ij,1}$ 
    - $-c_1 \rho \epsilon b_{ij}$
    - $-c_1 \rho \epsilon b_{ij} + c_1' \rho \epsilon (b_{ik} b_{kj} - \frac{1}{3} II \delta_{ij})$
  - Fast Term,  $\Phi_{ij,2}$ 
    - Launder, Reece and Rodi, 1975
    - Fu, Launder and Tselepidakis, 1987
    - Craft, Fu, Launder and Tselepidakis, 1991
    - Shih and Lumley, 1985
    - Speziale, Sarkar and Gatski, 1990

## Incompressible Modeling (continue)

- Wall-Reflection Term,  $\Phi_{ij,w}$ 
  - Hanjalic and Launder, 1976
  - Gibson and Launder, 1978
- Dissipation rate,  $\epsilon_{ij}$ 
  - isotropic,  $= \frac{2}{3}\delta_{ij}\epsilon$ , with constants in  $\Phi_{ij}$  functions of II and III (Launder and Shima, 1989 and Launder, 1990)
  - anisotropic
    - $\epsilon_{ij} = [(1 - f)\frac{2}{3}\delta_{ij} + f\overline{u_i''u_j''}/k]\epsilon$  [Hanjalic and Launder, 1976]
    - models satisfying asymptotic near-wall behavior [Launder and Reynolds, 1983; So, 1991 and Shih, 1991]
    - transport models model for  $\epsilon_{ij}$  [Kollman, 1991]
- Heat Fluxes,  $\overline{u_i''\theta}$ 
  - $\overline{u_i\theta} = -\frac{2}{3}c_T\frac{k^2}{\epsilon}\frac{\partial T}{\partial x_k}$
  - $\overline{u_i\theta} = -c_T\frac{k}{\epsilon}\overline{u_i''u_j''}\frac{\partial T}{\partial x_k}$
  - ASM type heat-flux equation
  - Transport heat-flux equations

## Compressible Modeling

- Pressure dilatation,  $\overline{p'u''_{k,k}}$ 
  - Gatski et al., 1991 ( $f_1(M_t)P_k$  and  $f_2(M_t)\epsilon$ )
  - Zeman, 1991 ( $\overline{p'^2}$ -equation)
  - Rubesin, 1990 ( $\overline{\rho'^2}$ -equation + polytropic process)
  - Taulbee and VanOsdol, 1991 ( $\overline{\rho'^2}$ -equation + modeled  $\overline{p'u''_{k,k}}$  Poisson solution)
- Fluctuation velocity average,  $\overline{u''_i}$ 
  - Gatski et. al., 1991 (density-gradient model)
  - Zeman, 1991 (transport equation for  $\overline{u''_i}$ )
  - Taulbee and VanOsdol, 1991 (transport equation for  $\overline{u''_i}$ )
  - Rubesin, 1991 (Constant total enthalpy + polytropic process)
- Dilatation dissipation
  - $\epsilon = \epsilon_s + \epsilon_d$  and  $\epsilon_d = f(M_t)\epsilon_s$ 
    - Zeman, 1990
    - Sarkar et. al., 1989
  - Rubesin, "total"  $\epsilon$ -transport equation.
- Rapid Compression Model,  $\rho L^n = \text{Constant}$  (Reynolds, 1980; Morel and Mansour, 1982; Young and Coakley, 1987; Coakley and Huang, 1991; Rubesin, 1990 and Zeman, 1991)



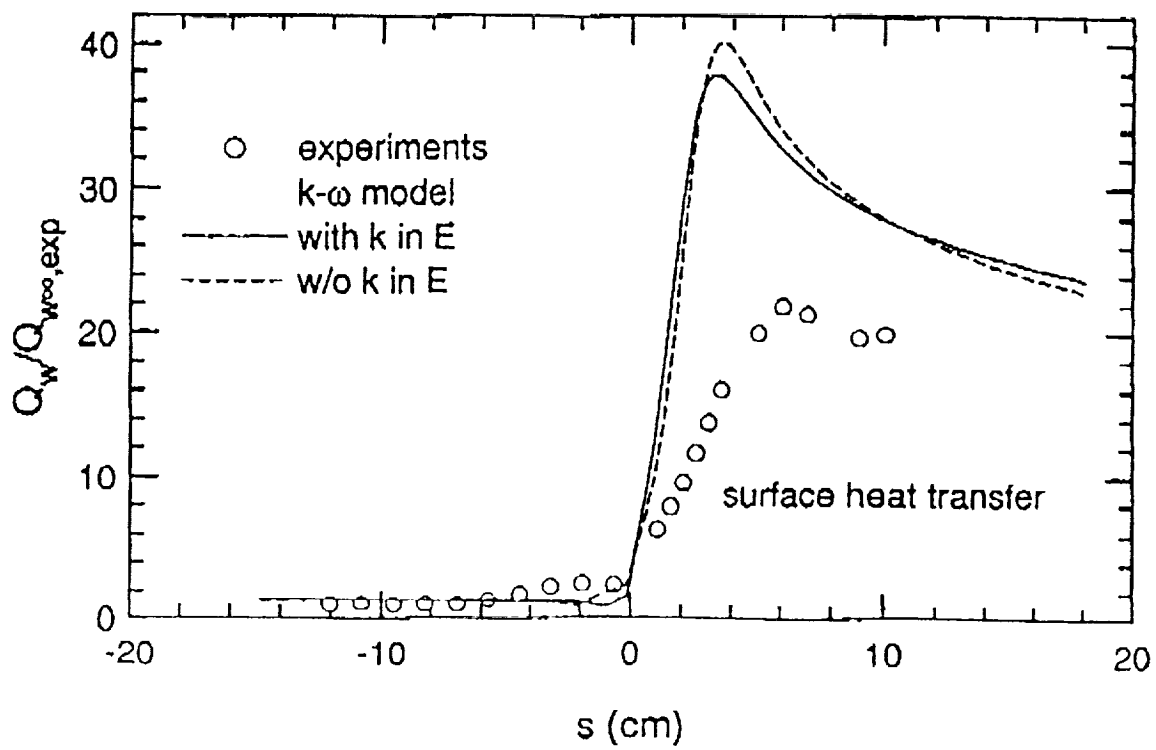
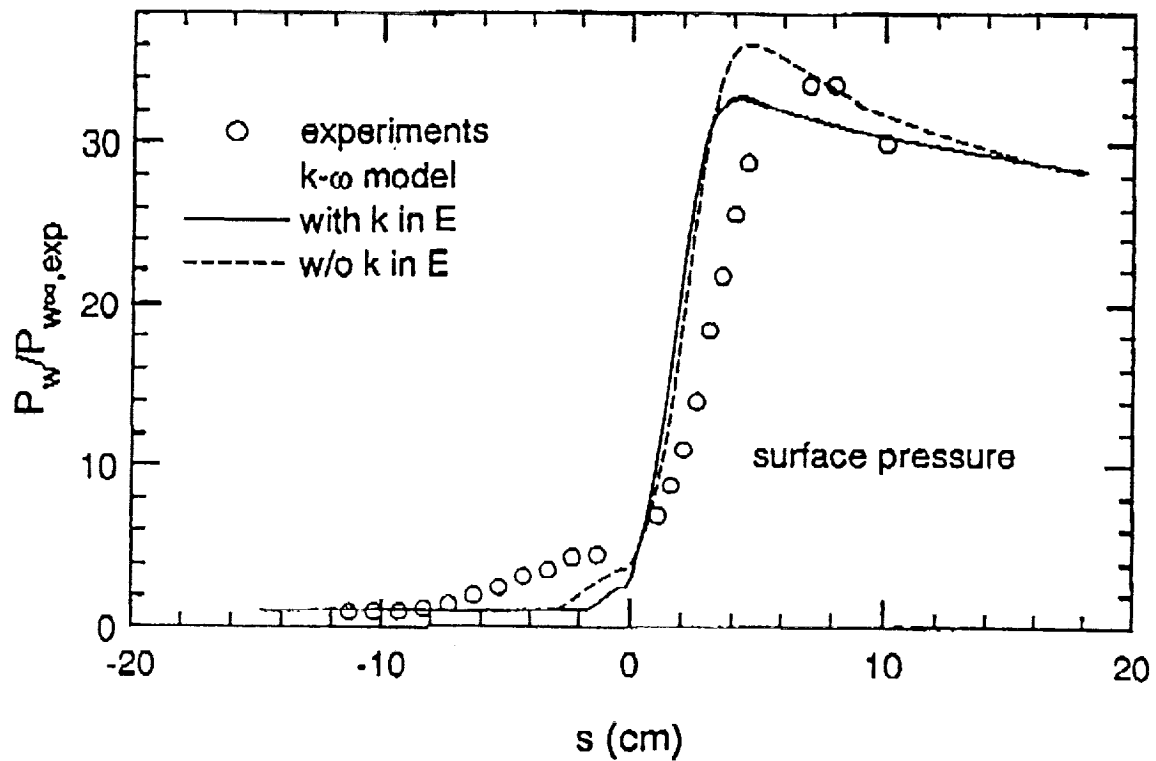
## Some Remarks

- There are more models than what has been presented in the position paper. Some have been tested in many "real" flows with success.
- Comparison of the models based only on simple homogenous-type flows may be misleading.
- Near-wall modeling is still a challenging problem for 2nd moment closure.
- Due to strong coupling among governing equations and the absence of numerical stabilizing turbulence viscosity in the mean-flow equations, the solution of Reynolds stress equations requires special attention.
- Currently, LRR, FRAME and Launder-Shima models have been implemented in a N-S code and comparison of models against real-flow experimental data is underway.
- The use the total energy is necessary for hypersonic flow calculations:  $E_T = E + k$ , where  $E = c_v T + \frac{1}{2} \bar{u}^2$

$$\frac{D\rho E}{Dt} = \dots - P_k + \rho\epsilon$$

# Hypersonic Cylinder-Flare

35° flare angle  $M = 7.05$

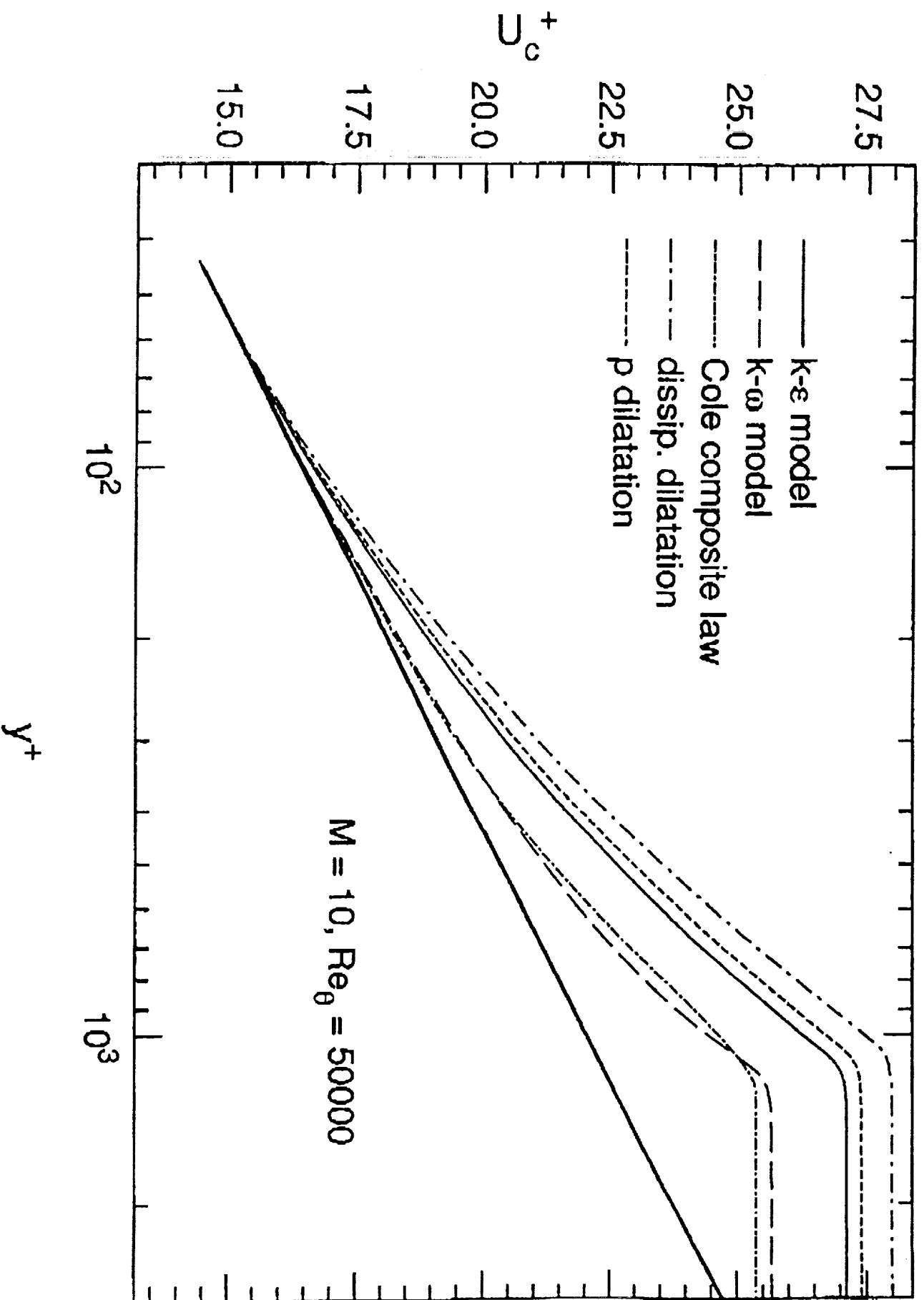


### Some Remarks (continue)

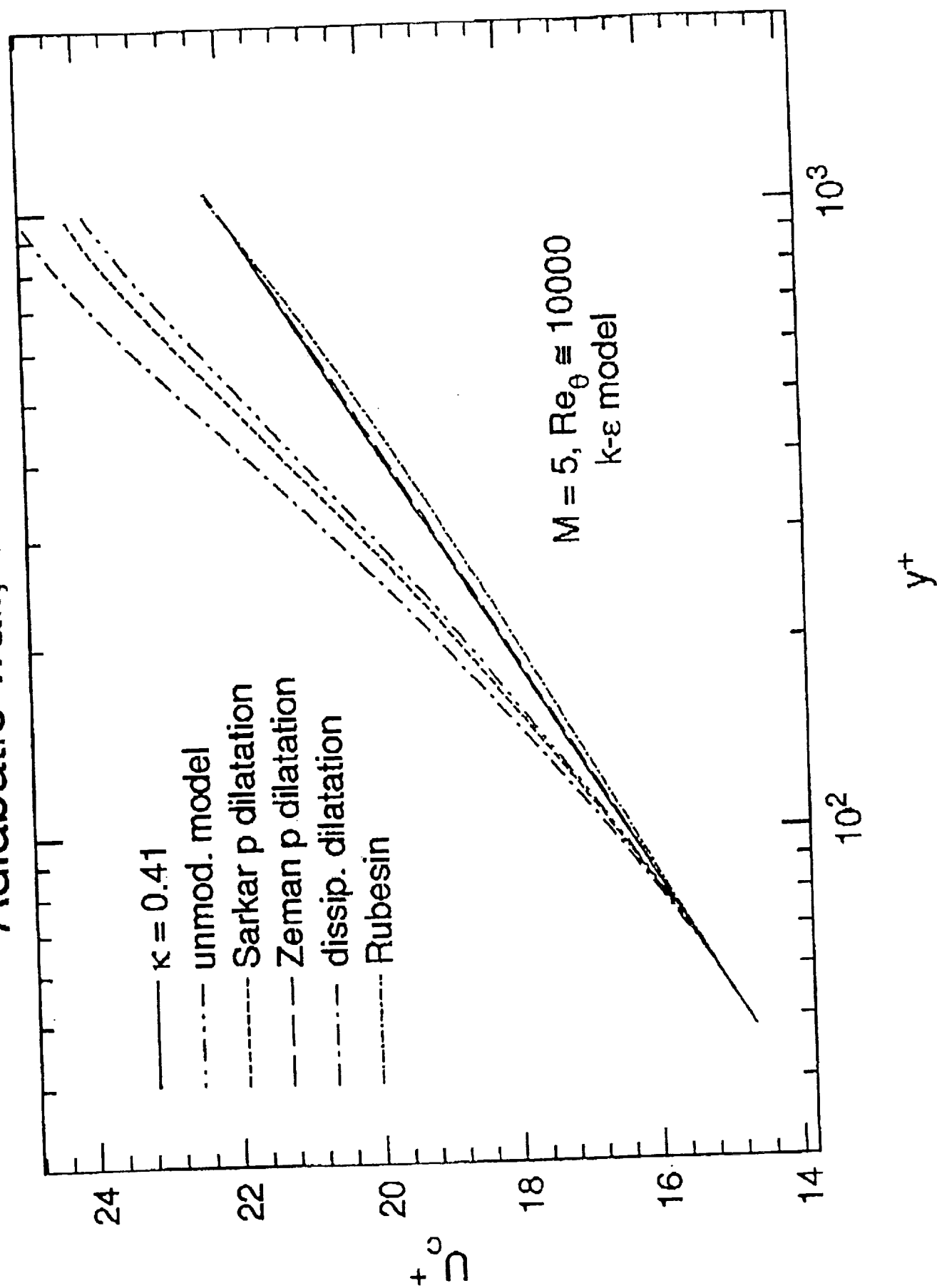
- *Wall flows* — The law of the wall is independent of the Mach number if the comparison is made based on the Van Driest transformed variables.
  - Models using  $\epsilon$ -equation produce lower Von Karman constant,  $\kappa$ .
  - Model constants can be derived as functions of density gradients.
  - $k - \omega$  model is less sensitive to Mach number effects (coincidence ?) — only for wall flows.
  - Need more turbulent energy !!
  - Dilatation dissipation concepts make the flow more "laminar" — wrong direction.
  - The new pressure dilatation model shown in the position paper also lowers  $\kappa$ .
  - Rubesin's total  $\epsilon$ -model approach goes into the right direction.
  - Zeman's new pressure dilation model does an excellent job.

# Compressible law of the wall

## Adiabatic wall, boundary layer flow



# Compressible law of the wall Adiabatic wall, couette flow



## Some Remarks (continue)

- Modeling of the  $\epsilon$  equation is still a challenging problem — both for incompressible and compressible flows — Experience has shown that a 2-equation-level model can be used to improve the weakness of the  $\epsilon$ -equation.
- Two baseline test problems are recommended - one is the compressible mixing layer and the other is the compressible law of the wall. Experience has shown that these two flows display completely different behavior.
  - *Mixing layer* — As Mach number increases, the spreading rate decreases.
    - All unmodified turbulence models fail to predict this behavior.
    - This leads to models designed to increase the total dissipation rate as the turbulence Mach number increases.

# **Comment on : The Present State and Future Direction of Second-Order Closure Models for Compressible Flows**

**W. W. Liou**

**Center for Modeling of Turbulence and Transition  
ICOMP/NASA Lewis Research Center**

Comment 1 : Turbulent Dissipation Rate -  $\bar{\rho}\epsilon$

$$\bar{\rho} \epsilon = \bar{\rho} ( \epsilon_s + \epsilon_c ) \quad , \quad \epsilon_s = \overline{\nu \omega_i'' \omega_i''}$$

$$\frac{D\epsilon_s}{Dt} = \text{Incompressible Models} - 0.6 \bar{\rho} \epsilon_s \tilde{u}_{k,k} \text{ (in the position paper)}$$

$$-\overline{r_{ij}'' T_{,j}'' S_{,i}''} + \dots \text{ (Liou and Shih (1991))}$$

- Preliminary analysis (Liou and Shih (1991)) has shown that the third order moments,  $\overline{r_{ij}'' T_{,j}'' S_{,i}''}$ , may be as important as the terms that being retained and may need to be modeled as well.

Comment 2 : Turbulent Mass Flux -  $\overline{u_i'}$

Compressibility Effects: .., .., reduced spreading rate, ...

entrainment

- Turbulent mass flux terms in the Favre-averaged equations may have a fair amount of effect on the mean flow development if they are modeled more rigorously, especially for wall-bounded flows.





## DISCUSSION

B. E. Launder (T.B. Gatski)

I must say that I feel preferably unpersuaded by the practice of adding, just multiplying the dissipation rate which comes out of the dissipation rate transport equation by a factor that is proportions to the Mach number. A concept of the dissipation equation is that it is really representing the spectral transfer rate of energy. It is looking at the large scales; the small scales are totally irrelevant. Admittedly, you won't get the right behavior with the so-called standard dissipation equation. But surely one must look at how to improve the transport equation rather than having a quick fix.

A comment on what John Viegas said earlier. He looks at two flows and believes you need a correction of different sign - wrong! I believe we need to look at twenty-two or on hundred and twenty-two. All we are looking at is the desperate sparsity of the compressible flow data base. That is why one can get away with these simplistic ideas. Just by having  $\epsilon_t$  equal to the quantity one plus a function of Mach number, quantity times epsilon won't work.

S. Sarkar (reply)

First, this was not meant to be the only compressible fix. And, as far as you're saying that the compressible dissipation should not add to the solenoidal dissipation, because after all our conventional wisdom is that this is a fine scale thing, therefore it shouldn't affect epsilon. This does not violate that. What you are saying is that this has an extra irrotational component which has large scales and small scales. It is not as if the compressibility is just changing the small scale end of the spectrum; what it is doing is creating an irrotational component that is both small and large scales. We are just choosing to look at the small scales because it's simpler to do it.

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G. Huang (S. Sarkar)

You can compose that anyway you like, but only for homogeneous and constant property flow.

S. Sarkar (reply)

Yes, absolutely.

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p. 26

THE PRESENT STATE AND  
FUTURE DIRECTIONS OF  
PDF METHODS

S. B. Pope  
Cornell University

## WORKSHOP OBJECTIVE

- "To discuss the present status and the future direction of various levels of engineering turbulence modeling related to CFD computations for propulsion"
- Combustion is an essential part of propulsion
- Discuss PDF methods for turbulent combustion

## TURBULENT COMBUSTION MODELS

- Essential to integrate the development of:
  - turbulence model
  - chemical kinetics
  - numerical method
- Turbulent/combustion interactions
- Tractable thermochemistry

## TURBULENT COMBUSTION MODELS IN USE IN INDUSTRY

Typically:

- $k-\epsilon$
- equilibrium/mixing-limited combustion
- finite-volume codes

## IMPROVEMENTS SOUGHT

- Finite-rate kinetics —  $\text{NO}_x$ , CO, soot  
— extinction, ignition
- Generality — beyond idealized premixed and diffusion flames
- PDF Methods can provide these improvements

## PDF METHODS

- Solve modelled evolution equation for a one-point joint pdf

- $\underline{\phi}(\mathbf{x}, t)$  — compositions,  $\underline{\phi} = \{\phi_1, \phi_2, \dots, \phi_\sigma\}$   
mass fractions, enthalpy

$\mathbf{U}(\mathbf{x}, t)$  — velocity

$\omega(\mathbf{x}, t)$  — turbulence frequency  $= \varepsilon/k$

- Hierarchy of PDF methods

- $\underline{\phi}$
- $\mathbf{U}, \underline{\phi}$
- $\mathbf{U}, \omega, \underline{\phi}$

- Non-linear reaction rates in closed form



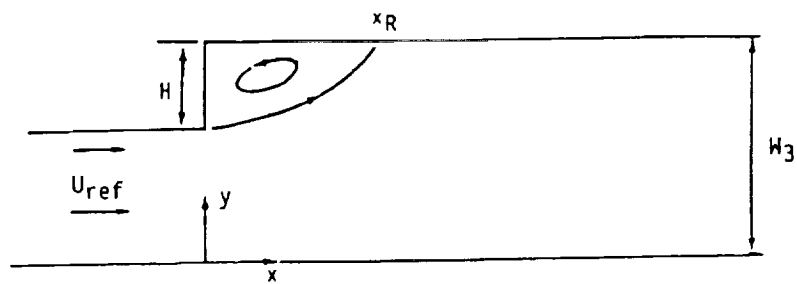
## COMPOSITION JPDF

- Need turbulence model  
( $k$ - $\epsilon$  or  $\langle u_i u_j \rangle - \epsilon$ )
- Gradient-diffusion model of turbulent transport  
of  $\phi$
- e.g. J.-Y. Chen et al.
  - $\langle u_i u_j \rangle - \epsilon$
  - 4-step reduced scheme for methane
  - solve for jpdf of 5 compositions
  - Masri/Bilger/Dibble piloted diffusion flame

## VELOCITY-COMPOSITION JPDP

- $\langle U \rangle$ ,  $\langle u_i u_j \rangle$  etc. obtained from jpdf
- Need  $\varepsilon$  equation (or equivalent)
- All convective transport in closed form (no gradient-diffusion modelling)
- Connection to Reynolds-stress models
- e.g. Haworth & El Tahry (GM)  
Anand et al. (Allison GT)

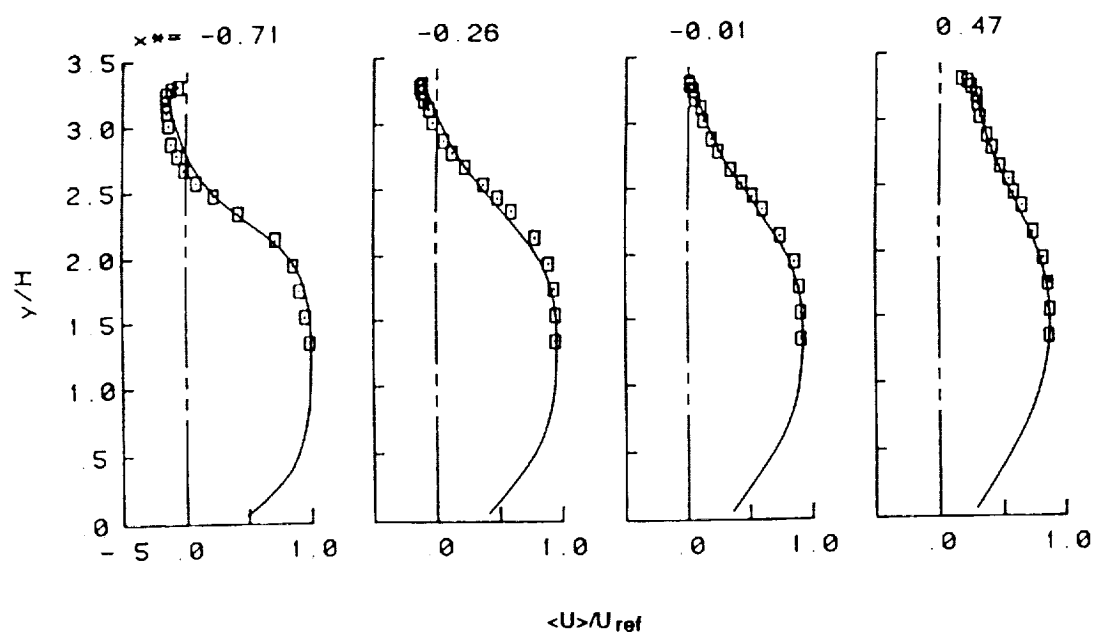
## FLOW OVER A BACKWARD-FACING STEP



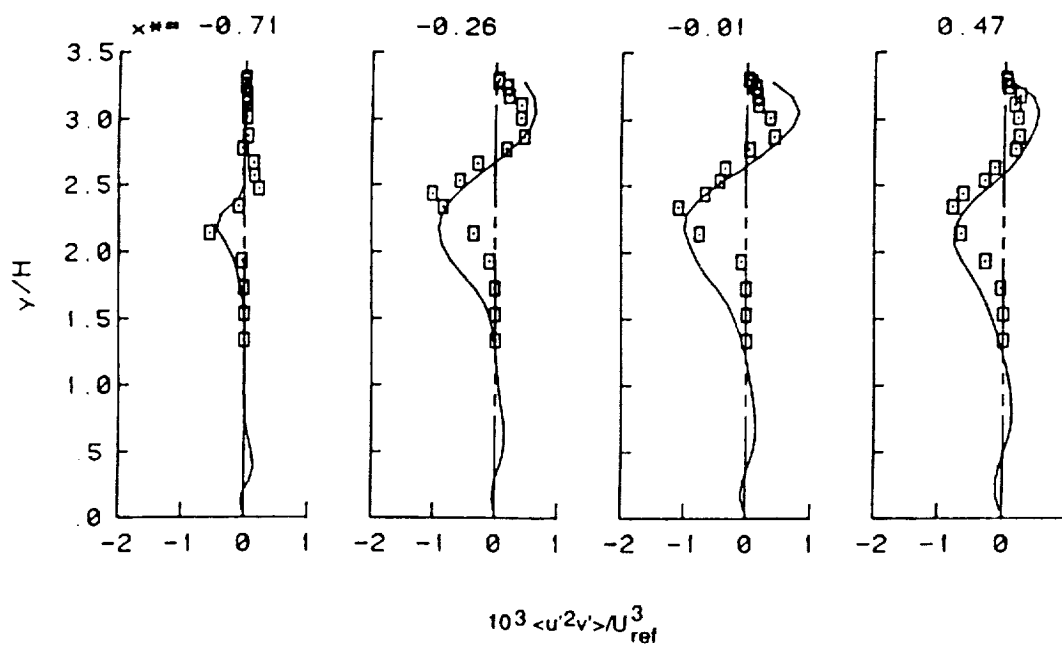
$$\begin{aligned} H &= 0.0762 \text{ m} & W_3/(W_3-H) &= 1.43 \\ W_3 &= 0.254 \text{ m} & U_{\text{ref}} &= 0.196 \text{ m/s} \end{aligned}$$

MEASUREMENTS: PRONCHICK & KLINE (1983)  
 PDF CALCULATIONS: ANAND, POPE & MONGIA (1990)

# FLOW OVER A BACKWARD-FACING STEP MEAN AXIAL VELOCITY



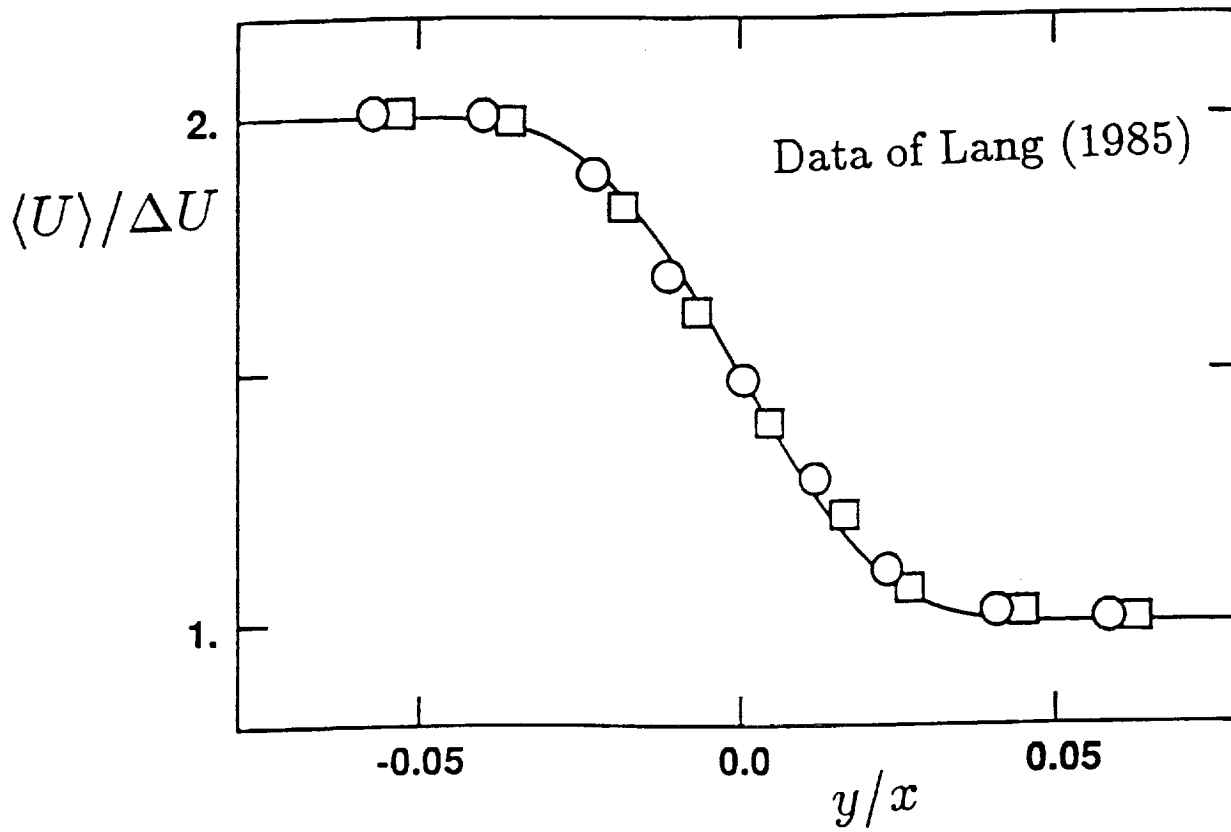
# FLOW OVER A BACKWARD-FACING STEP TRIPLE CORRELATIONS



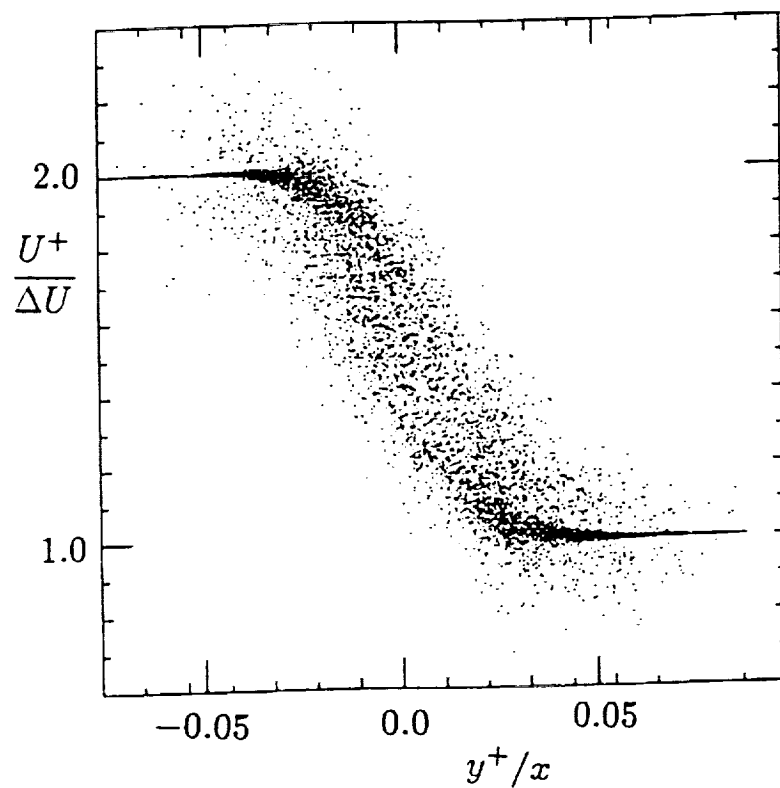
## VELOCITY-FREQUENCY- COMPOSITION JPDF

- Single, self-contained model equation
- Describes distribution of  $\epsilon$ .
- e.g. plane mixing layer

# PLANE MIXING LAYER MEAN VELOCITY PROFILE

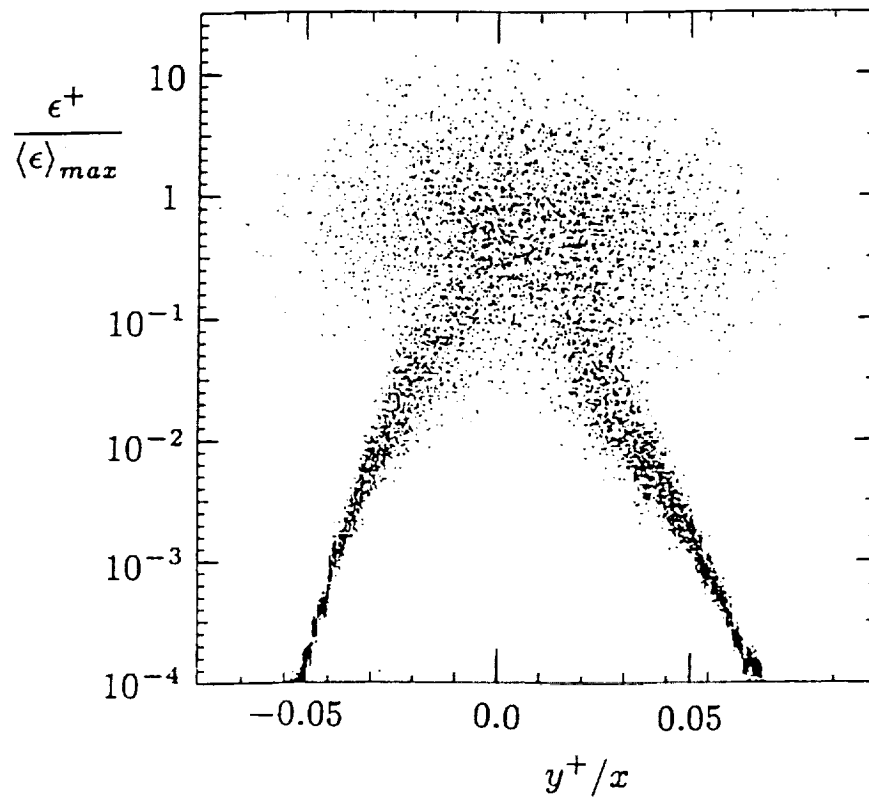


JPDF CALCULATION OF THE PLANE MIXING LAYER  
SCATTER PLOT: AXIAL VELOCITY vs. LATERAL POSITION





JPDF CALCULATION OF THE PLANE MIXING LAYER  
SCATTER PLOT: DISSIPATION vs. LATERAL POSITION



## PRESENT STATE OF PDF METHODS

- Much research and development work remains to be done, but:
  - Realistic finite-rate kinetics have been incorporated
  - Applications have been made to complex 2D and 3D flows
  - Accuracy—should be at least as good as a second-order closure

## THE FUTURE OF PDF METHODS

- El Tahry & Haworth (General Motors):

*"...in our opinion, the PDF method is the most appealing of the one-point statistical approaches for in-cylinder reacting flows. Applications to in-cylinder combustion can be expected within a few years."*

- Correa (General Electric):

*"The prevalent  $k$ - $\epsilon$ /assumed shape pdf closure model...must be improved upon or replaced before other quantities can be usefully predicted. An alternative is the Monte-Carlo/pdf approach; although well proven for fully-developed shear flows, this method needs to be adapted to pressure-dominated flow in complex geometries."*

- From Proposal for PDF research and development by Rolls Royce, SNECMA, MTU.....to European Community:

*...a joint velocity-composition pdf...method allows relatively complex chemistry to be simulated and also fully couples the turbulence with the chemistry. It seems the only way forward from the present position.*

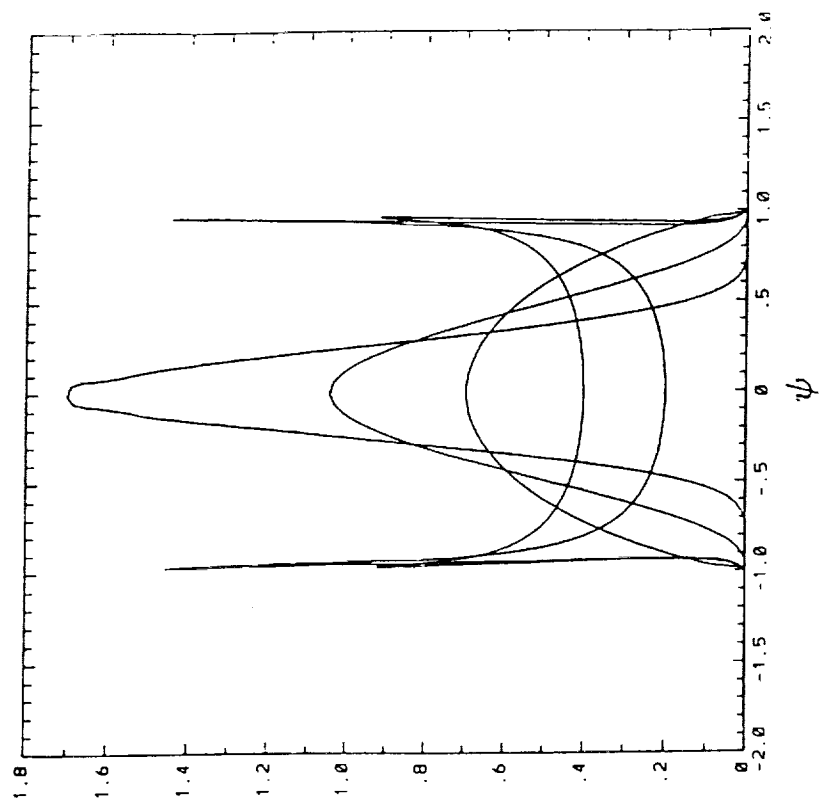
## FUTURE DIRECTIONS OF PDF METHODS

1. Improvements and extensions
2. Applications to practical combustion devices

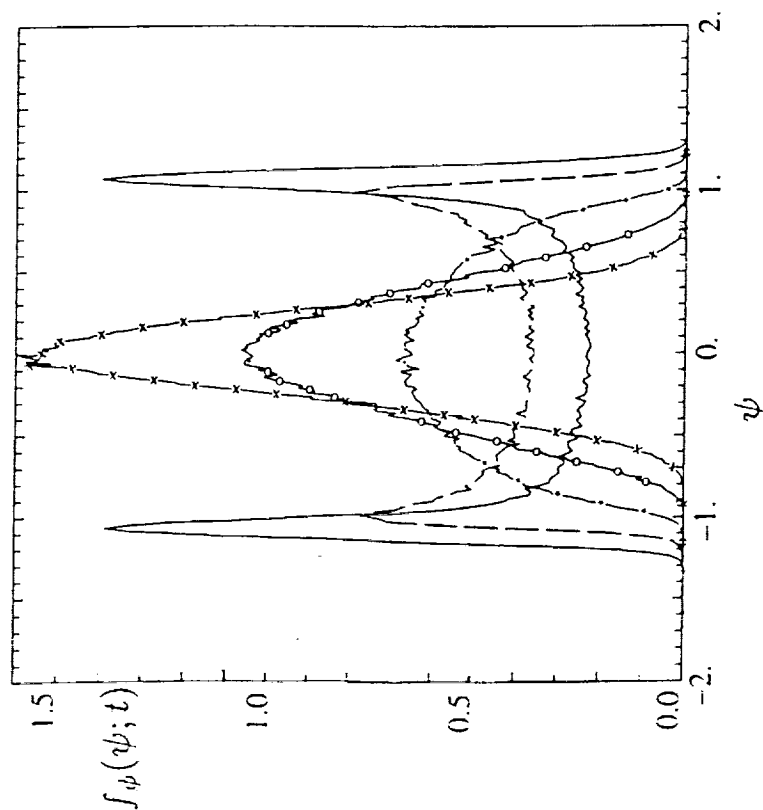
## IMPROVEMENTS

1. Reduced kinetics — Maas & Pope (1991)
2. Mapping closures — Chen, Chen & Kraichnan (1989), Pope (1991), Gao (1991)

# DECAY OF THE PDF OF A CONSERVED SCALAR



MAPPING CLOSURE



DIRECT NUMERICAL SIMULATIONS

## EXTENSIONS

- Incorporate  $\nabla\phi$  in jpdf  
  
Meyers & O'Brien 1981  
Pope 1991, Gao & O'Brien 1991, Dopazo 1991
- Represent coupling between reaction and mixing
- Contains information on:
  - jpdf of  $\xi$  and  $\chi$  (diffusion flames)
  - $\langle c \rangle$  and  $\Sigma$  (premixed flames)
- Reconciles flamelet and non-flamelet approaches

## NUMERICAL DEVELOPMENT FOR APPLICATIONS (WORK AT CORNELL)

1. Reduced kinetics—automatic generation and tabulation procedures
2. Improved Monte Carlo/particle method
  - second-order accurate in space and time
  - low statistical error
3. General, robust pressure algorithm



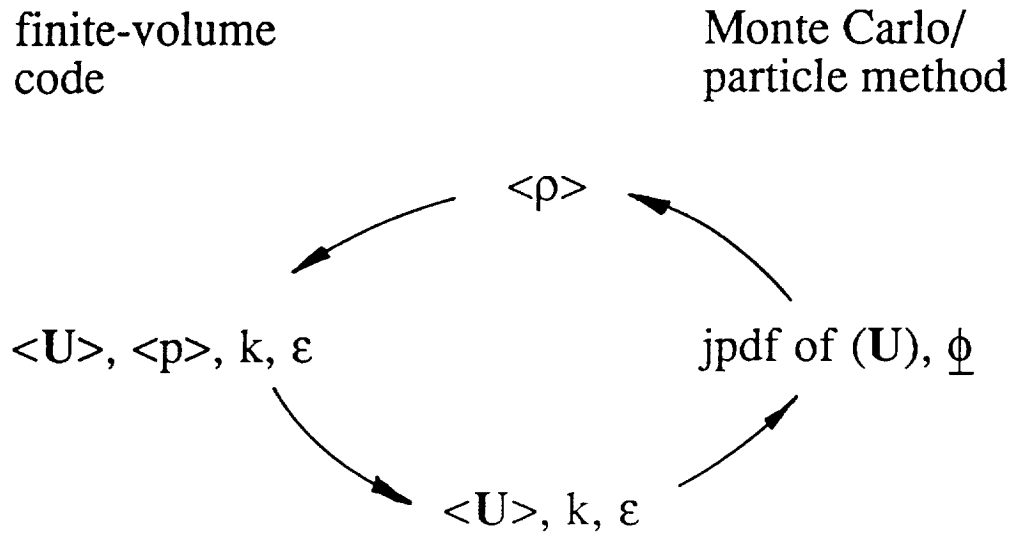
## TRANSITION FROM $k$ - $\epsilon$ to PDF

- Industrial combustor codes:
  - complex geometry—grid generation
  - models for other processes—sprays, soot, radiation
  - post-processing/integration in design procedures
- Incorporate PDF methods within existing codes
- Numerical method fundamentally different (particle method vs. finite-volume method)

## 4 STAGE TRANSITION

0. Starting point: finite volume code for  $\langle \mathbf{U} \rangle$ ,  $\langle p \rangle$ ,  $k$ ,  $\epsilon$ ,  $\langle \underline{\phi} \rangle$
1. jpdf of  $\mathbf{U}$ ,  $\underline{\phi}$ ; discard  $\mathbf{U}$  information  
(PDF method determines  $\langle \underline{\phi} \rangle$ ,  $\langle \rho \rangle$ : incorporate reduced kinetics)
2. jpdf of  $\mathbf{U}$ ,  $\underline{\phi}$   
(finite-volume code determines  $\langle p \rangle$  and  $\epsilon$ )
3. jpdf of  $\mathbf{U}$ ,  $\omega$ ,  $\underline{\phi}$   
(finite-volume code determines  $\langle p \rangle$ )
4. jpdf of  $\mathbf{U}$ ,  $\omega$ ,  $\underline{\phi}$ —self-contained particle method

## FIRST STAGE



- essentially jpdf of  $\underline{\phi}$
- but simple transition (2nd stage) to  
jpdf of  $\langle \mathbf{U} \rangle, \underline{\phi}$  (avoids gradient-diffusion modelling)
- reduced kinetics can be incorporated

## CONCLUSIONS

- Turbulent combustion modelling:  
need to integrate
  - turbulence model
  - chemical kinetics
  - numerical method
- PDF methods
  - $\underline{\phi}$ ;  $\underline{U}$ ,  $\underline{\phi}$ ;  $\underline{U}$ ,  $\underline{\omega}$ ,  $\underline{\phi}$
  - reaction and convective transport in closed form
  - finite-rate kinetic effects
- Future model development:
  - mapping closures
  - reduced kinetics
  - add  $\nabla \underline{\phi}$
- Future numerical developments:
  - more accurate particle methods
  - general pressure algorithm
  - incorporation in combustor codes

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COMMENTS ON  
THE PRESENT STATE AND  
FUTURE DIRECTIONS OF  
PDF METHODS

*E.E. O'Brien*

SUNY at Stony Brook

My first comment on the presentation of S.B. Pope is to note that Professor Pope is almost single-handedly responsible for the development of the one-point PDF method to the state in which it can now be reasonably expected to address actual engineering problems.

My second comment is that I am in accord with virtually all of the points he has made including his first, which was to express surprise that a conference on "modeling related to CFD computations for propulsion" should be so thin on combustion modeling. The PDF method he reviewed is relatively complicated, but it appears to be the only format available to handle the non-linear stochastic difficulties caused by typical reaction kinetics. Turbulence modeling, if it is to play a central role in combustion modeling, as it must, has to be integrated with the chemistry in a way which produces accurate numerical solutions to combustion problems. It is questionable whether the development of turbulent models in isolation from the peculiar statistics of reactant concentrations is a fruitful line of development as far as propulsion is concerned.

There are three issues, two mentioned by S.B. Pope, for which I have prepared additional outlines which are appended to this note.

- a. The one-point PDF method
- b. The amplitude mapping closure
- c. A hybrid strategy for replacing a full two-point PDF treatment of reacting flows by a single-point PDF and correlation (& cross-correlation) functions.

Finally, I would like to appeal for a concerted effort to obtain an adequate data base for compressible flow with reactions for Mach numbers of unity or higher. DNS results have played an important role in aiding the development of PDF models for incompressible flows. A similar role can be played in the efforts to elucidate the many interactions of pressure with other flow variables including species concentrations.

## **PDF Method Outline**

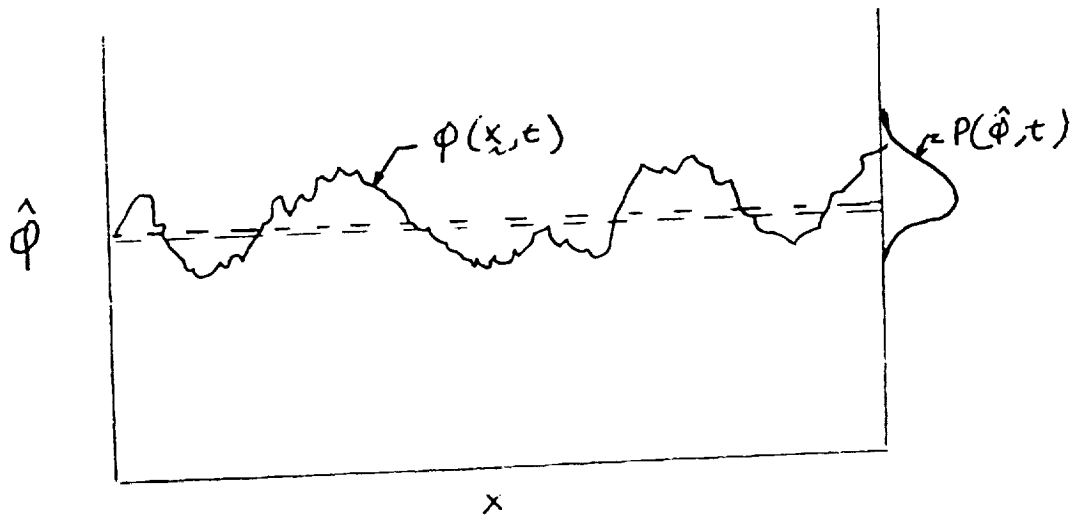
**From**      1)    N-S eqn.  
              2)    Energy eqn.  
              3)    Equation of State  
              4)    Species conservation eqn.

- **Generate an evolution equation for the 1-point PDF**  
          **(T.S. Lundgren, 1957; C. Dopazo, 1990)**
- **Close the PDF equation where necessary by 'suitable' closures**
- **Use Monte Carlo/particle methods for numerical solution**  
          **(S. Pope, 1981 +)**

**Major advantage ÷ linear increase of numerical effort with the number of dimensions**

**Example: A 1-Point PDF Equation**

single species  $\phi(x,t)$ ;  
Statistically homogeneous system



$$\frac{\partial \phi}{\partial t} = -u \cdot \nabla \phi + D \nabla^2 \phi + \dot{w}(\phi)$$

**1-point PDF equation**

$$\frac{\partial P(\hat{\phi}, t)}{\partial t} = - \frac{\partial^2}{\partial \hat{\phi}^2} [DE\{(\nabla \phi)^2 | \hat{\phi}, t\} P(\hat{\phi}, t)] + \frac{\partial}{\partial \hat{\phi}} [\dot{w}(\hat{\phi}) P(\hat{\phi}, t)]$$

- $DE\{(\nabla \phi)^2 | \hat{\phi}\}$  is expected value of scalar dissipation conditioned by the scalar value.

## General 1-Point PDF Equation

- Easily generated, and can include multispecies inhomogeneity, compressibility, etc.

- $\rho \left( \hat{U}, \hat{\phi}, \dots, \hat{\phi}_N, \hat{p}, \hat{H}; \underline{x}, t \right) \dots \dots \dots (1)$

- Closed terms : advection and reaction
- unclosed terms : pressure, molecular diffusion of all quantities in (1)
- Closure strategies : Satisfy PDF realizability and
  - a) Reproduce second-order moment closures for physical space terms (Pope, 1985)
  - b) Approximate density effect by ignoring the coupling with pressure oscillation phenomena, i.e.  $\rho = \rho(\phi_1, \dots, \phi_N, H_{ref})$
  - c) Represent molecular diffusion in velocity - composition - enthalpy space by models. Most recently mapping closures.



## The Amplitude Mapping Closure

(Chen, Chen, Kraichnan 1989; Gao, 1991)

- **Attractive for strongly non-Gaussian processes**
- **Simplest example: Non-reacting single scalar  $\phi(\underline{x}, t)$  in statistically homogeneous system.**
- **1-Pt PDF**

$$\frac{\partial P(\hat{\phi}, t)}{\partial t} = - \frac{\partial^2}{\partial \hat{\phi}^2} [DE\{(\nabla \phi)^2 | \hat{\phi}, t\} P(\hat{\phi}, t)]$$

- **Define  $\theta(Z)$  time-independent, homogeneous, isotropic, normalized Gaussian r.v. All statistics of  $\theta(z)$  are known if**

$$\overline{\theta} = 0, \overline{\theta^2} = 1 \text{ and } \overline{f_{\theta}(r)} = \overline{\theta(\underline{z}) \theta(\underline{z} + \underline{r})}$$

are given.

- **Define a scalar field  $\phi^s(\underline{x}, t)$  generated from  $\theta(z)$  from the mapping**

$$\underline{x} = \underline{z} / J(t)$$

$$\text{and } \phi^s(\underline{x}, t) = X(\theta(\underline{z}), t)$$

- **Demand  $P(\hat{\phi}, t) = P(\hat{\phi}^s, t)$**

## Consequences of the Mapping

- Since statistics of  $\theta(z)$  are completely known the statistics of  $\phi^s(x,t)$  are also completely known if  $J(t)$  and  $X$  are specified.

- N & S condition for  $P(\hat{\phi}, t) = P(\hat{\phi}^s, t)$

$$is \quad E\{(\nabla\phi)^2 | \hat{\phi}, t\} = E\{(\nabla\phi^s)^2 | \hat{\phi}^s, t\}$$

- It turns out that substitution of the mapping into the PDF equation produces a solvable equation for  $X$

$$\frac{\partial X}{\partial \tau} = -\phi \frac{\partial X}{\partial \phi} + \frac{\partial^2 X}{\partial \phi^2}$$

- $\tau$  is a normalized time scale  $d\tau = \frac{DJ^2(t)}{\lambda_\theta^2} dt$ .

Note:  $J(t)$  &  $\lambda_\theta$  (the only parameter of  $f_\theta$  that matters)

Appear only in the time scale

$\therefore$  the shape of  $P(\hat{\phi}, \tau)$  depends only on the mapping  $X(\theta, t)$ .

### Some Results of the Mapping

- **Symmetric binary mixing (initial double delta PDF)**

$$P(\hat{\phi}, 0) = \frac{1}{2} \delta(\hat{\phi}+1) + \frac{1}{2} \delta(\hat{\phi}-1)$$

$$\text{soln: } E\{(\nabla\phi)^2 | \hat{\phi}, \tau\} / F(\tau) = \exp\{-2 [\text{erf}^{-1}(\hat{\phi})]^2\}$$

- **Unsymmetric binary mixing**

$$P(\hat{\phi}, 0) = a \delta(\hat{\phi}+1) + (1-a) \delta(\hat{\phi}-1), \quad 0 < a < 1$$

Same mapping closure solution for  $E\{(\nabla\phi)^2 | \hat{\phi}, \tau\} / F(\tau)$

- **General soln. has been obtained (Gao, 1991)**
- **Current status: Formal solutions have been obtained for multispecies cases (Gao & O'Brien, 1991)**  
**But, no reported success in incorporating it in numerical codes for more than one species**  
**Also seems to misrepresent asymptotic behavior in time**

## 2-Point PDF

### Advantages

- Spatial structures explicitly included
- Self-contained time and length scales as in spectral description of turbulence and unlike 1-Point PDF & K- $\epsilon$  or other moment closures

### Disadvantages

- Dimensions doubled
- Closures harder to construct
- Numerical work so far limited to isothermal reactions of type  $A + B \rightarrow P$ .

#### Closure approximations

Advection  
Diffusion  
Reaction

EDQNM  
Linear Mean Square Estimate  
None needed

### New Wrinkles

- Use 1-point joint PDF of quantities and their gradients

$$P(\phi, \nabla\phi; x, t)$$

- Hybrid closures

### Hybrid Strategy

- Aimed at more than 2 species, statistically homogeneous.

- Numerical method is fractional steps (Yanenko, 1971)

*i.e.* 
$$P(t+\Delta t) \approx (I+O_A\Delta t)(I+O_D\Delta t)(I+O_R\Delta t)P(t)$$
  
*I is the identity operator*

- Replaces the full 2-point PDF method by a correlation function - 1 point PDF approach..

In a cycle of computations in both composition & physical space

- a) Advection has no effect on 1-point PDF but it modifies the correlation and cross-correlation functions  $f(r,t)$  [EDQNM]
  - b) Molecular diffusion modifies the 1-point PDF (LMSE or mapping closure, if workable and the correlation functions (known)
  - c) Chemical reaction effects the 1-point PDF (known) and, inadvertently, may alter the correlation functions (assume similarity)
- Reproduces full 2-point PDF results for





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*Workshop on Engineering Turbulence Modeling*

*Comments on Pdf Methods*

*J.-Y. Chen  
Combustion Research Facility  
Sandia National Labs  
Livermore , CA*

# *Grand Challenge of Combustion Engineering*

---

## *Challenge: Significant Reduction of Combustion Generated Pollutants*

### *Facts:*

- *Pollutant Formation Rate  $\ll$  Fuel Oxidation Rate*
- *Small Quantity*
- *Highly Sensitive to Interactions between Turbulence and Chemistry*

### *Difficulty:*

- *Not Capable of Solving Navier-Stokes Equations with Detailed Chemistry*

### *Approach:*

- *"Rational" Modeling*



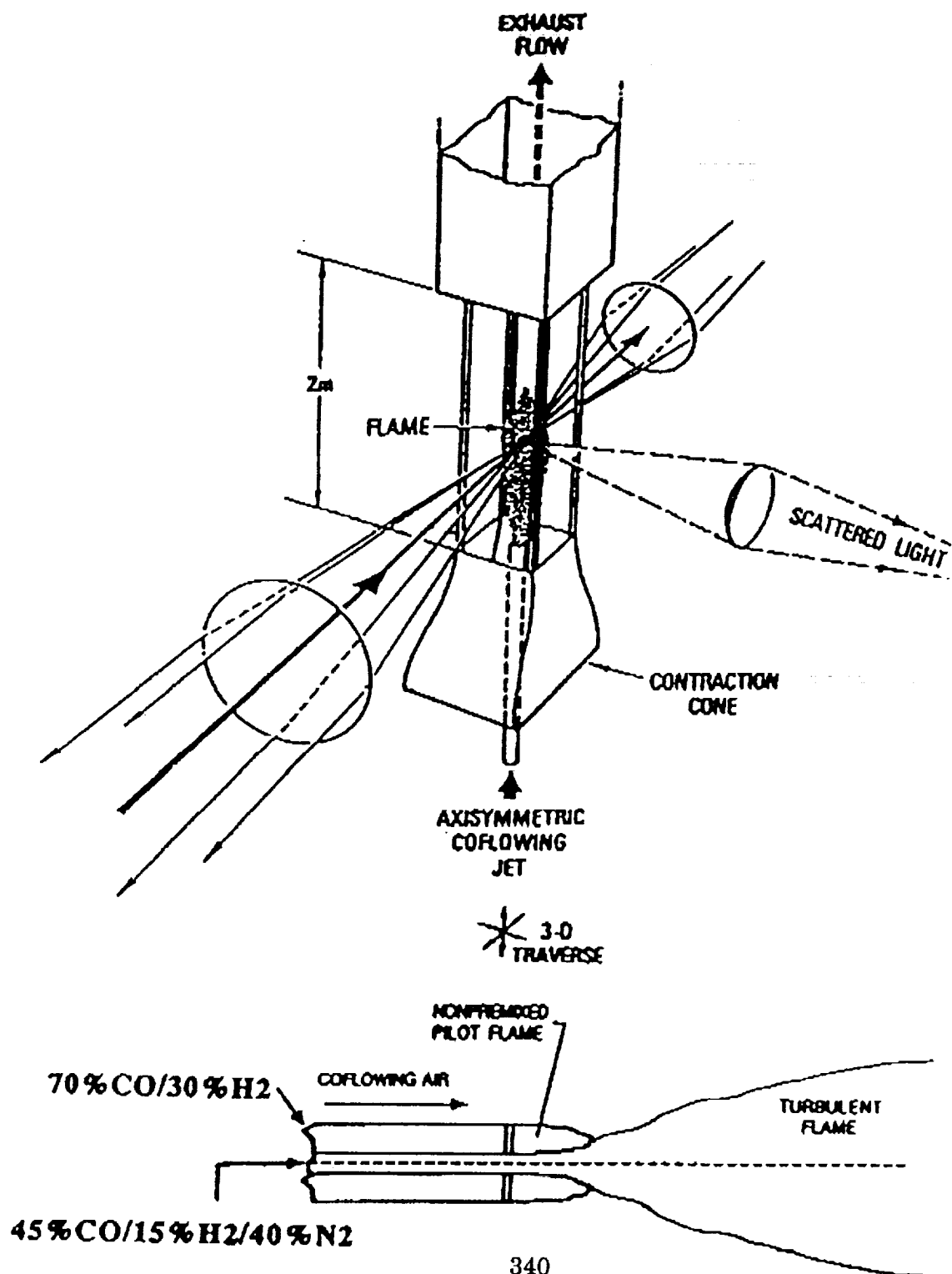
## *Research of PDF Methods at Sandia*

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- *Simple Geometry (Parabolic Flow)*
- *Reduced Reaction Mechanisms:*
  - *Two-step H<sub>2</sub> Flames*
  - *Three-step CO/H<sub>2</sub> Flames*
  - *Four-step CH<sub>4</sub> Flames*
  - *Five-step CH<sub>3</sub>OH Flames*
  - *up to six reactive scalars*
- *Thermal NO Formation in Turbulent Hydrogen Jet Flames*
- *Soot Formation in C<sub>2</sub>H<sub>4</sub> flames*

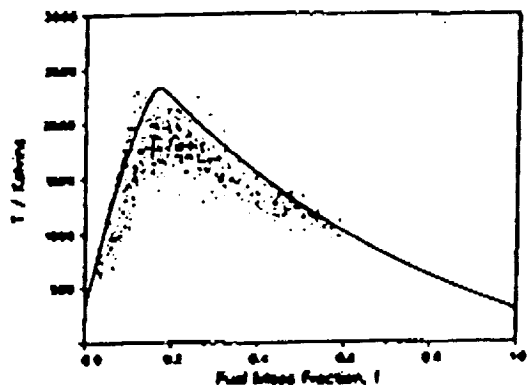
# Experiments of Turbulent Jet Flames (Masri & Dibble, 1988)

(Fuel Jet: 45% CO/15% H<sub>2</sub>/40% N<sub>2</sub>)  
(Pilot Jet: 70% CO/30% H<sub>2</sub>)

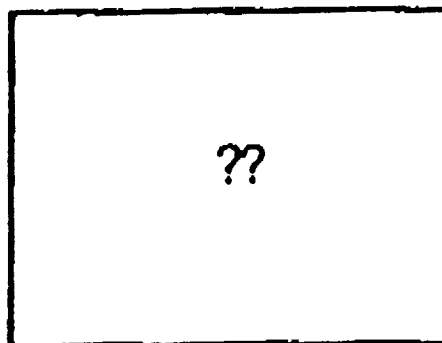


# Departures From Chemical Equilibrium

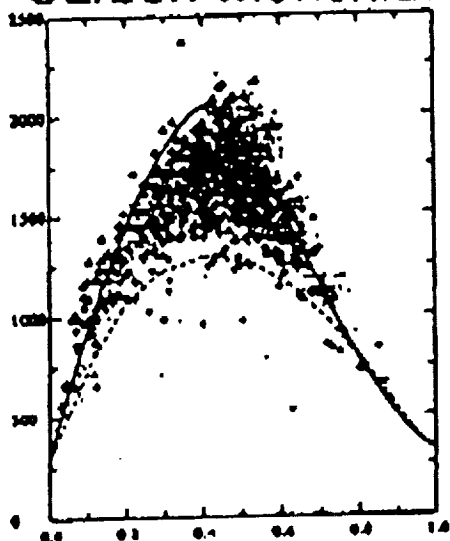
Hydrogen



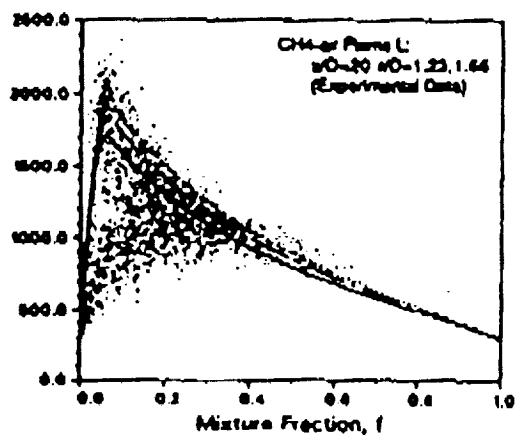
Methanol



Carbon Monoxide

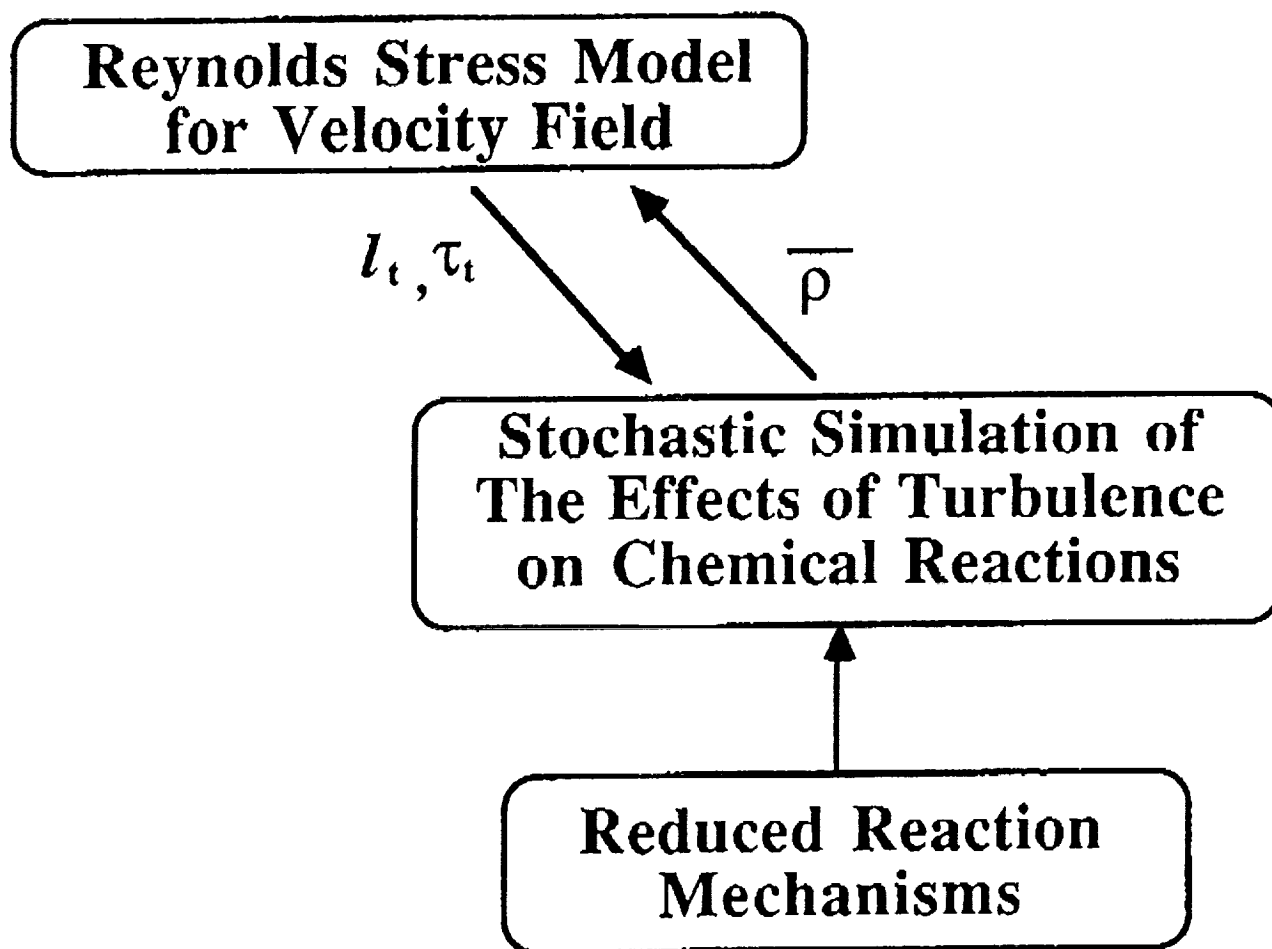


Methane



# ***Modeling Turbulent Reacting Flows***

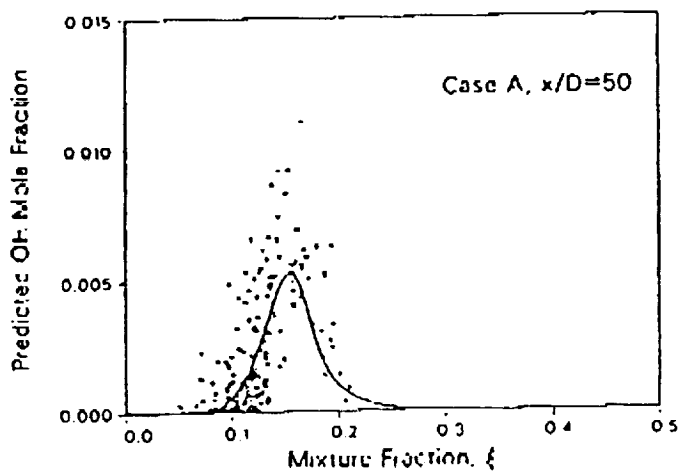
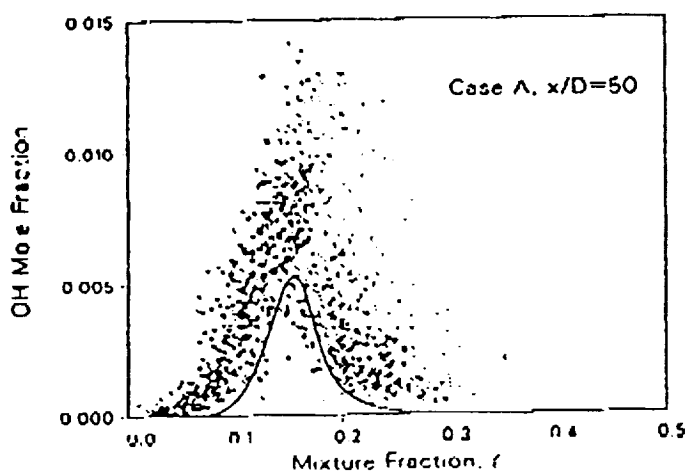
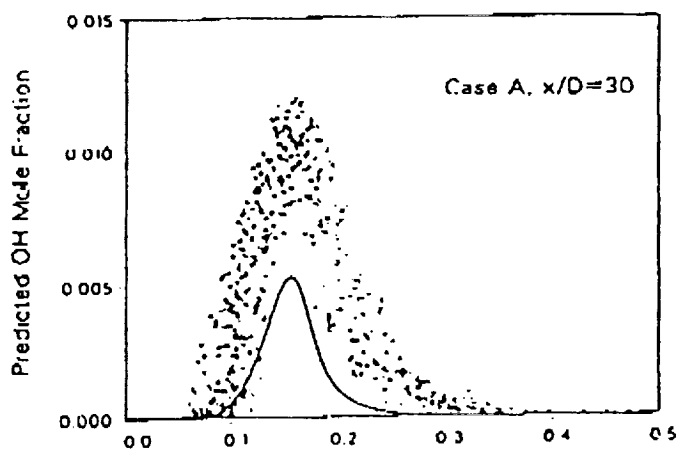
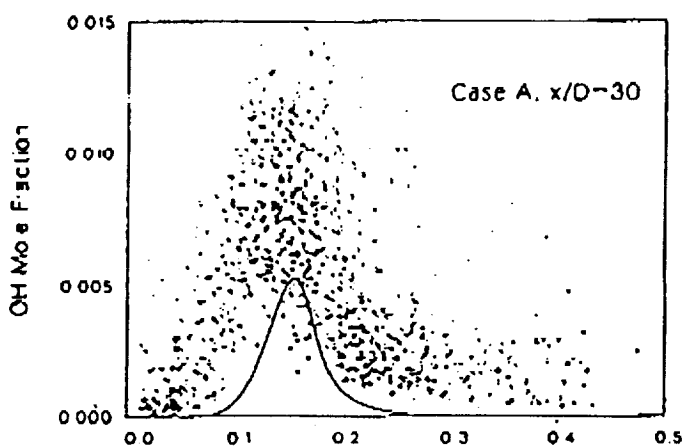
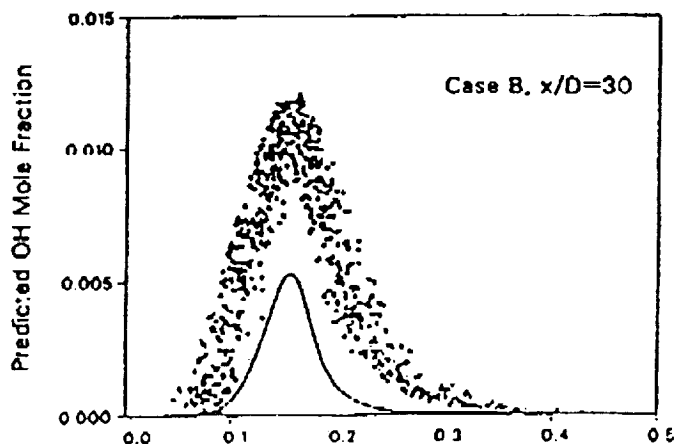
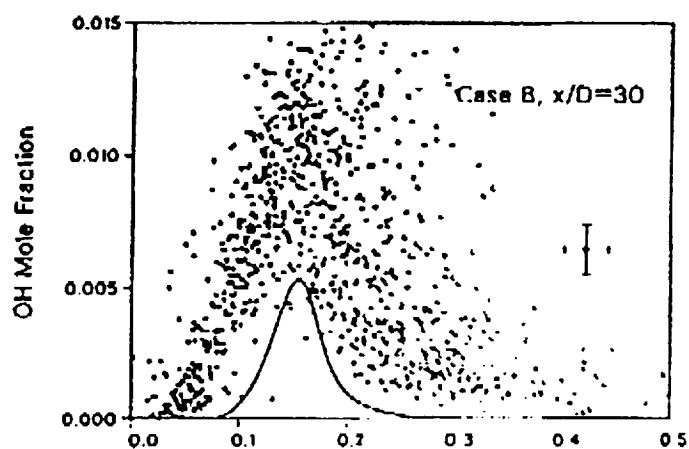
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# Superequilibrium OH Radical

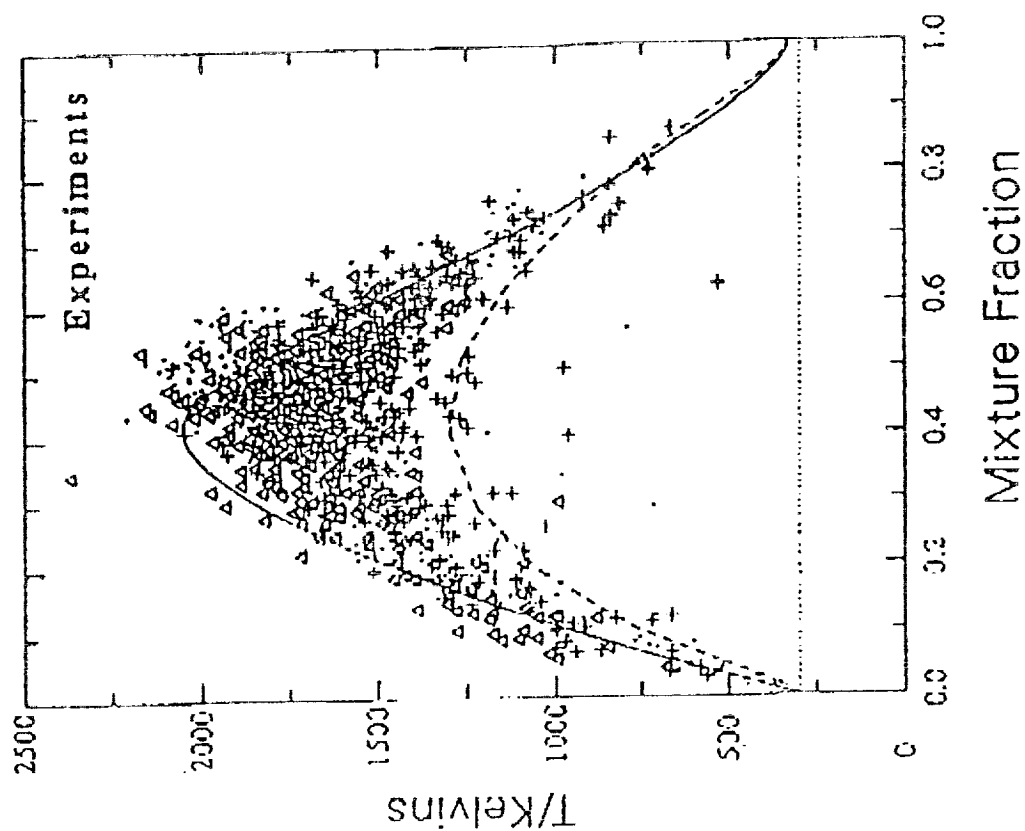
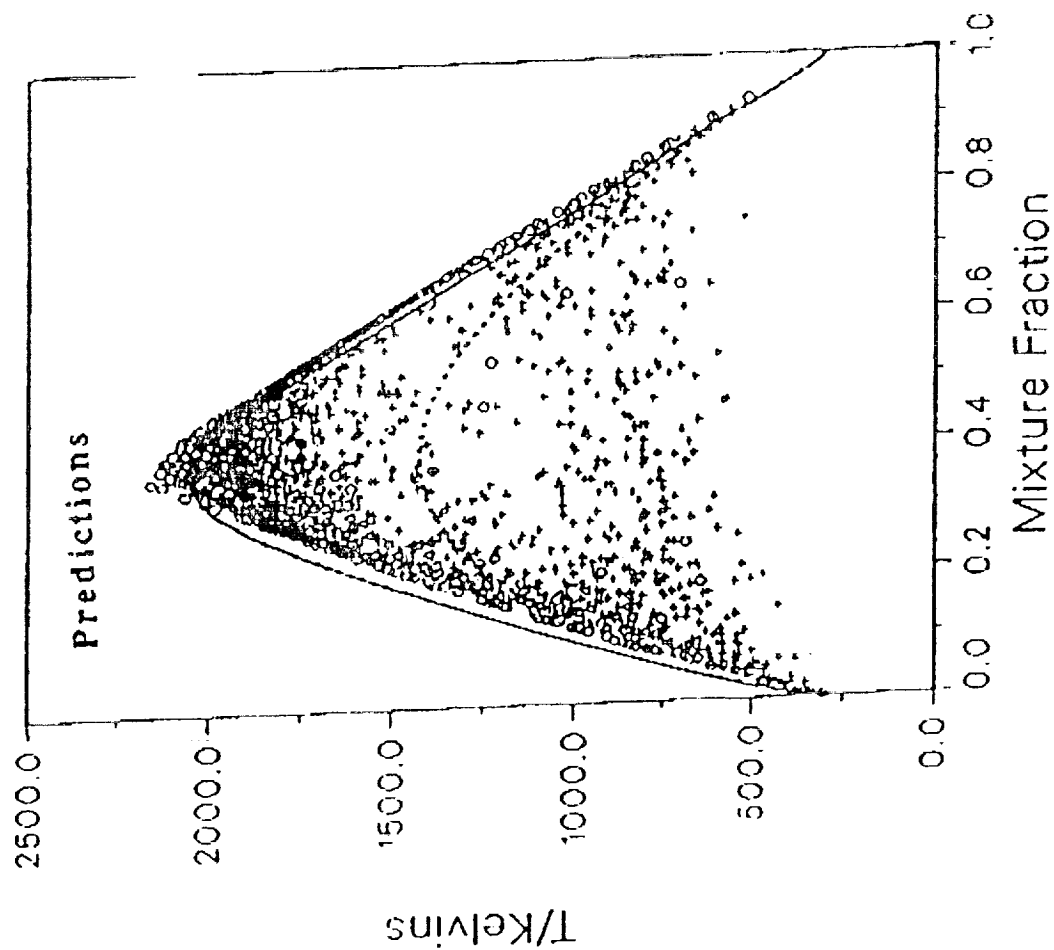
## Measurements

## Predictions



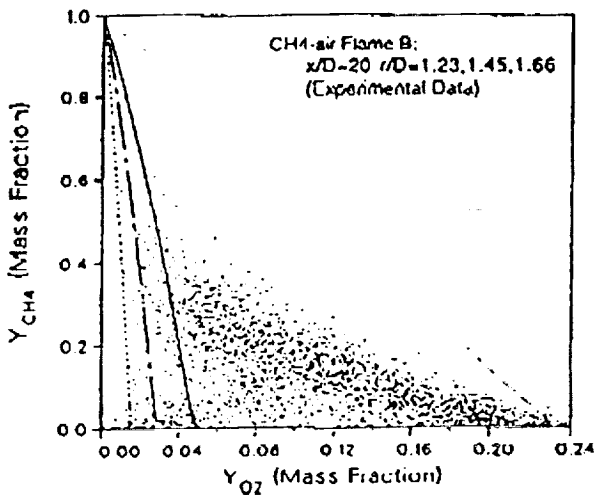
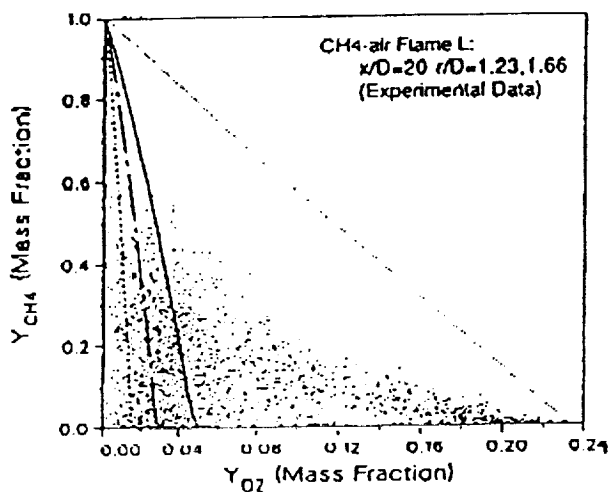
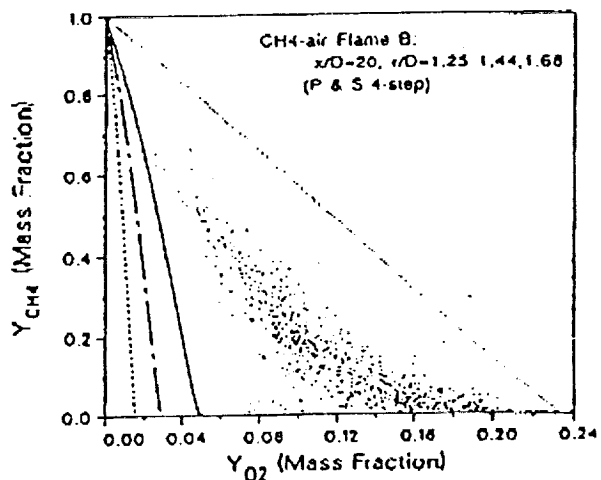
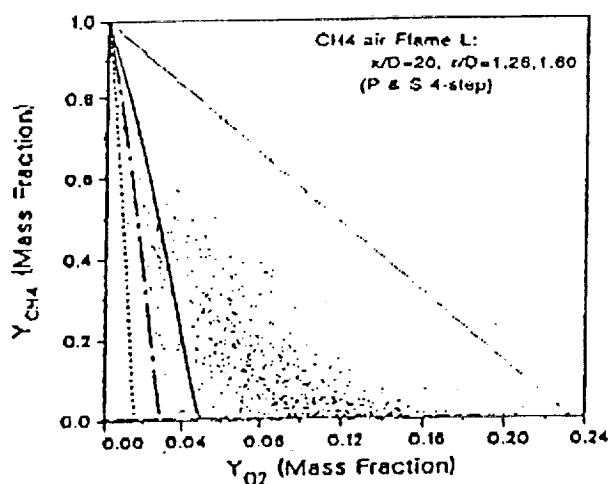
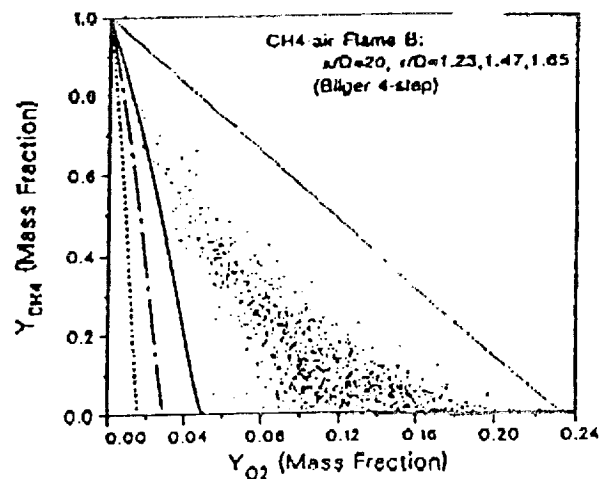
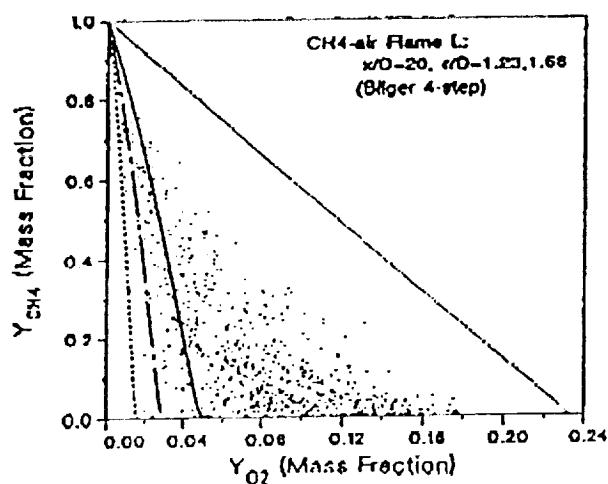
# PDF Modeling of Turbulent Jet Flames

(Fuel Jet: 45%CO/15%H<sub>2</sub>/40%N<sub>2</sub>)  
(Pilot Jet: 70%CO/30%H<sub>2</sub>)



# Scatter Plot for CH<sub>4</sub> and O<sub>2</sub>

## (CH<sub>4</sub> Turbulent Jet Flames)

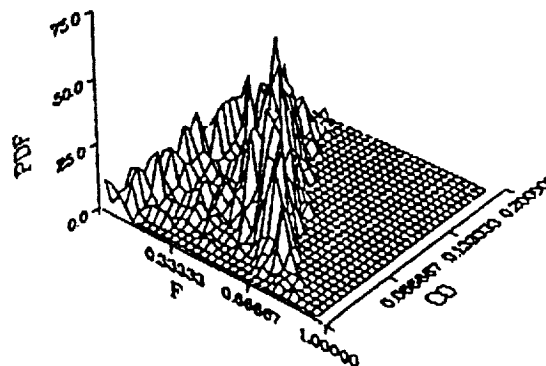
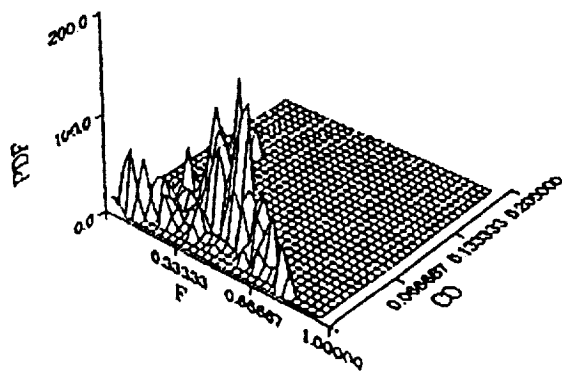


# Methanol Turbulent Jet Flames

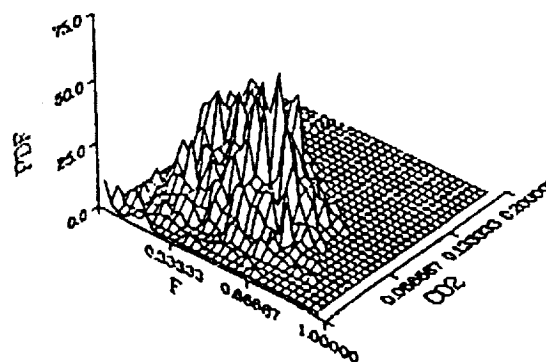
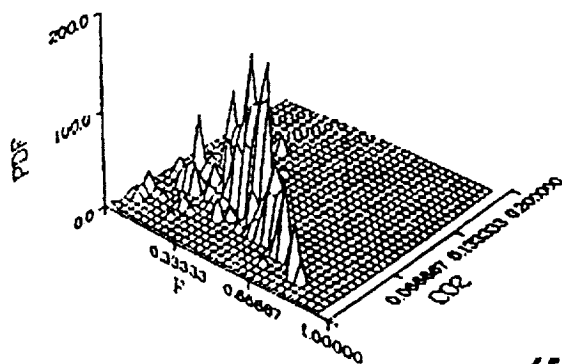
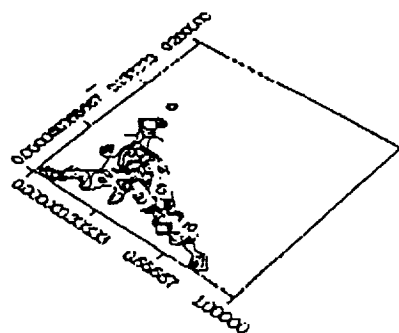
## Joint PDF of $(f, \text{CO}_2)$ , $(f, \text{CO})$

### Predictions

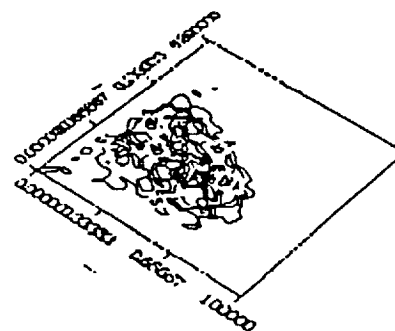
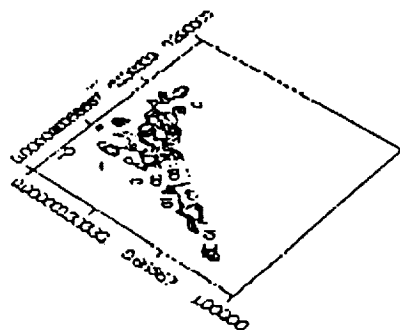
### Measurements



$(f, \text{CO})$



$(f, \text{CO}_2)$

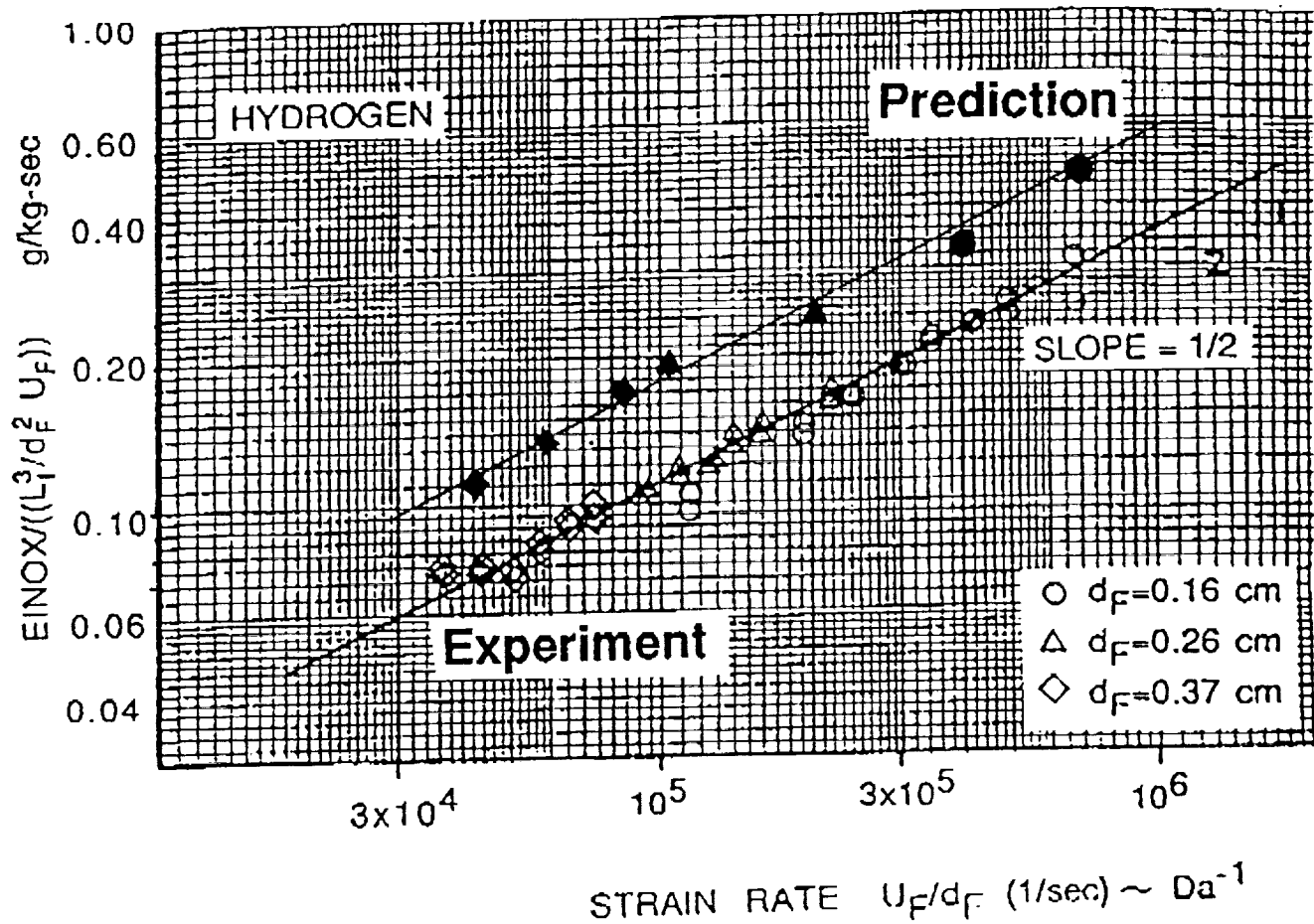




# Comparison Between Predictions and Experimental Data

Experimental Data for H<sub>2</sub> Turbulent Jet Flames  
by Chen & Driscoll (1990)

$$(U_{\text{coflow}} / U_{\text{fuel}} \approx 0.001)$$



## Turbulent $C_2H_4$ Jet Flames

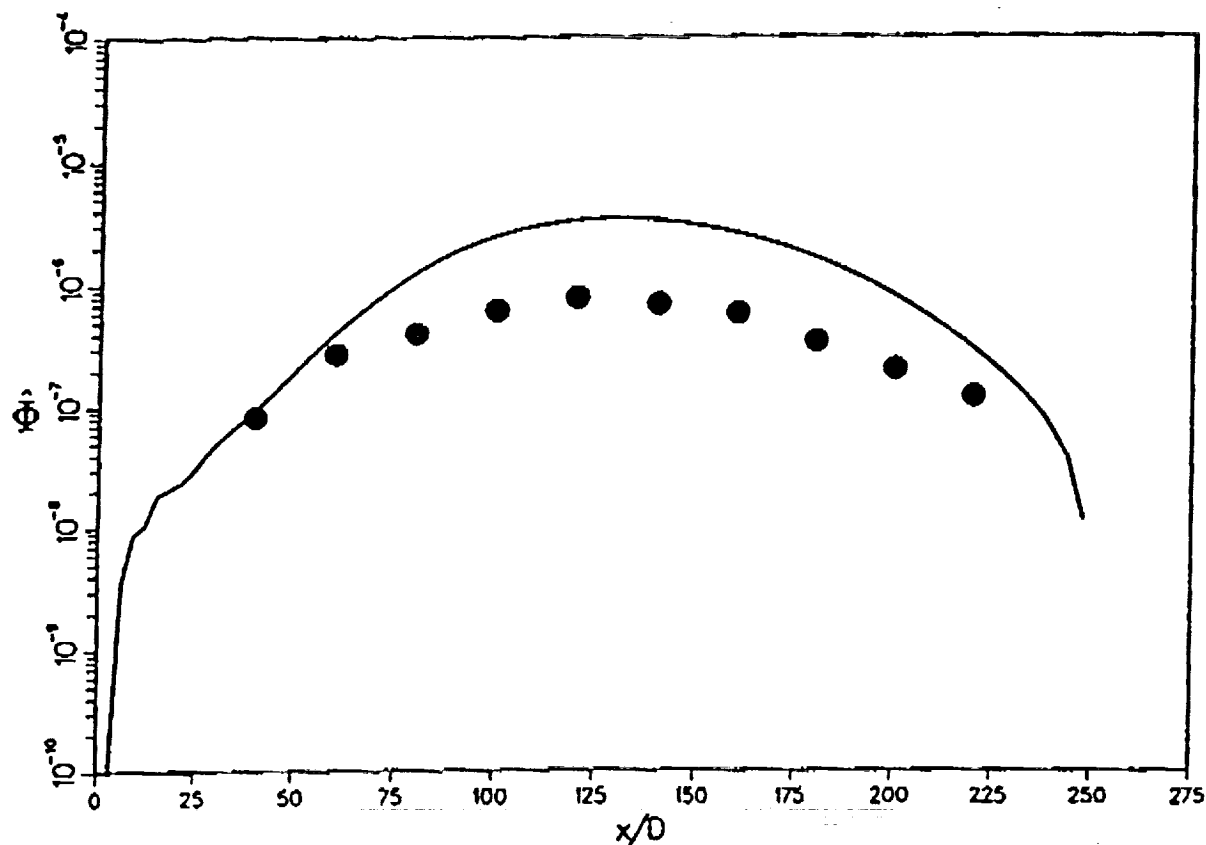


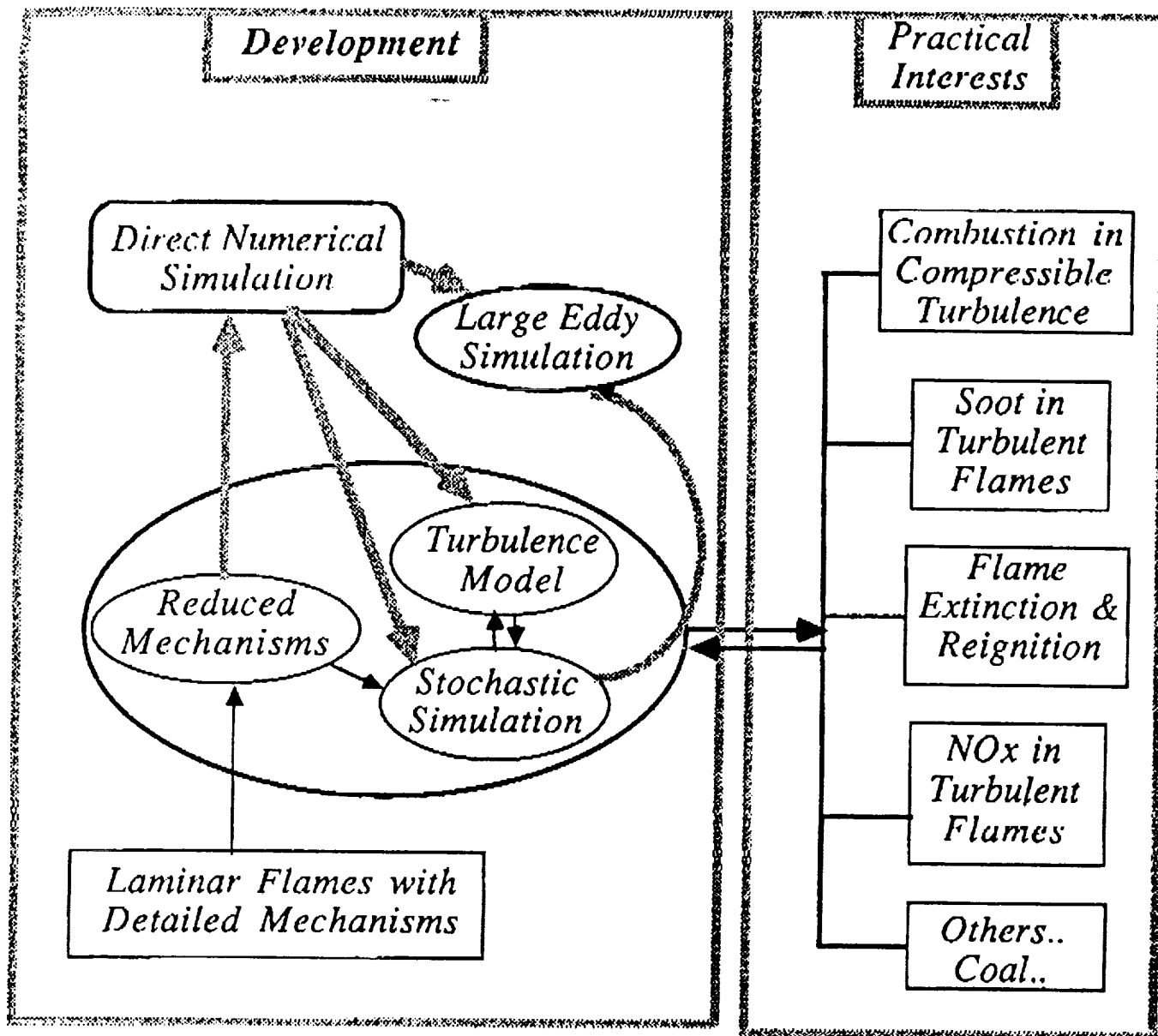
Fig. 2. Mean soot volume fraction  $\bar{\Phi}$  along the axis compared to measurements by Neill and Kennedy (1990) for a turbulent  $C_2H_4$  flame with zero pressure gradient.

## *Needed Improvements of PDF Methods for Combustion Applications*

---

- *Computationally Expensive*
- *Direct Calculation of Detailed  
Chemical Kinetics - Not Feasible*
  - *Capabilities of Reduced  
Reaction Mechanisms ??*
- *Primitative Status of Mixing Model*
- *Interactions between Turbulence  
and Chemical Reactions*

## Development of "Rational" Models

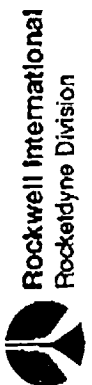


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# **SOME COMMENTS ON TURBULENCE MODELING FROM AN INDUSTRIAL PERSPECTIVE**

**MUNIR M. SINDIR**

**21 AUGUST 1991**



# TURBULENCE MODELING IS ONE OF THE KEY PROBLEMS IN CFD USE

- IN PROPULSION INDUSTRY COMPUTATIONAL TECHNIQUES (NAMESLY CFD) GRADUALLY BECOMING ENGINEERING DESIGN AND ANALYSIS TOOLS
  - HIGH PERFORMANCE AIRCRAFT ENGINES
  - HIGH SPEED AIR-BREATHING PROPULSION (e.g. NASP)
  - ROCKET PROPULSION (e.g. SSME, STME, NLS)
  - ADVANCED PROPULSION (e.g. NUCLEAR, ELECTRIC)
- THE KEY ISSUES IN ACCEPTANCE OF CFD ARE TURN-AROUND TIME, COST AND ACCURACY
  - TURN-AROUND TIME OF ANALYSIS HAS TO FIT PROGRAM SCHEDULE
  - COST IS THE BOTTOM LINE, THE VALUE OF CFD TO THE PROGRAM SHOULD AT LEAST BALANCE ITS COST (ASSUME \$800 PER CRAY HOUR FOR COMPUTER COST IN ADDITION TO LABOR COST)
  - UNCERTAINTY IN PREDICTIONS HAVE TO BE QUANTIFIED. IS IT 20% OR 200%? THESE FIX THE SAFETY FACTORS AND MARGINS REQUIRED FOR DESIGN
- IF CFD CAN BE DIVIDED INTO 2 MAJOR PARTS - NUMERICS AND PHYSICAL MODELS
  - NUMERICS - RELATIVELY MATURE, INACCURACIES TRACTABLE, STEADY PROGRESS
  - PHYSICAL MODELS - CRITICAL PROBLEM, LIMITED DATABASE, FEW PRACTICAL NEW IDEAS

# TURBULENCE MODELING FROM AN APPLICATIONS PERSPECTIVE

- CODE NUMERICS AND MODELS SHOULD BE VIEWED TOGETHER IN ASSESSING MODEL ACCURACY AND PERFORMANCE

- NUMERICAL ISSUES

- SPATIAL ACCURACY (MOST CODES 1ST -3RD ORDER)

- HOW NUMERICALLY DIFFUSIVE IS THE METHODOLOGY?
      - 1ST ORDER RESULTS CAN BE VASTLY DIFFERENT FROM 3RD ORDER FOR A GIVEN GRID

- TEMPORAL ACCURACY (MOST CODES 1ST - 2ND ORDER)

- CAN NUMERICAL SCHEME RESOLVE TRANSIENTS?

- GRID RESOLUTION

- WHAT IS THE MINIMUM RESOLUTION REQUIRED?
    - WHAT I CAN AFFORD IS NOT THE ANSWER

- CONVERGENCE

- HOW DO YOU ASSESS CONVERGENCE?
    - WHAT DO YOU MONITOR IN COMPLEX FLOWS
    - RESIDUALS OFTEN MISLEADING

- HOW SENSITIVE ARE RESULTS TO BOUNDARY CONDITION IMPLEMENTATION

- e.g. PRESSURE b. c. ON A ROTATING SURFACE

# TURBULENCE MODELING FROM AN APPLICATIONS PERSPECTIVE (CONT.)

- MODELING ISSUES

- TURBULENCE MODELS ARE VERY SENSITIVE TO GRID DISTRIBUTION AND SPACING ESPECIALLY IN REGIONS OF HIGH SHEAR (e.g. WALLS)

- REQUIRED GRID HOWEVER IS RARELY DISCUSSED (AT BEST ONLY  $y^+$  IS GIVEN FOR THE FIRST POINT)

- EQUALLY IMPORTANT QUESTION FOR COMPLEX GEOMETRIES IS THE REQUIRED GRID FOR THE ENTIRE DOMAIN

- THERE ARE VERY FEW WELL ESTABLISHED ANSWERS TO THESE QUESTIONS. RESULTS ARE GENERALLY NOT UNIVERSAL BUT ARE

- PROBLEM DEPENDENT

- CODE DEPENDENT



## FOR TURBULENCE MODEL EVALUATION IT IS DIFFICULT TO SEPARATE NUMERICAL INACCURACIES FROM MODEL LIMITATIONS

- UNLESS
  - SAME CODE EMPLOYED
  - RESOLUTION REQUIREMENTS FOR EACH MODEL DETERMINED AND USED (SAME GRID IS NOT APPROPRIATE FOR ALL MODELS)
  - EVALUATION DONE BY THIRD PARTY (LIMITED BIAS)
- EVALUATION SHOULD INCLUDE
  - MORE COMPLICATED TEST CASES TO CLEARLY DIFFERENTIATE BETWEEN MODELS (FLAT PLATES ARE LIMITED IN VALUE)
  - PERFORMANCE DATA
    - COMPUTATION TIME (ACTUAL CPU)
    - STABILITY CHARACTERISTICS
    - CONVERGENCE RATE
    - ACCURACY
    - EASE OF USE
- THERE IS A CRITICAL NEED FOR "WELL DEFINED" EXPERIMENTAL DATA TO SUPPORT MODEL EVALUATION TASKS

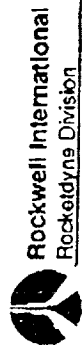


Rockwell International  
Rockadyne Division

# WHAT DOES INDUSTRY DO?

## ROCKETDYNE PERSPECTIVE

- TURBULENCE MODELING DONE ON COMPONENT LEVEL
  - e.g. WHAT MODEL DO WE USE FOR HIGH SPEED INLETS?  
WHAT IS APPROPRIATE FOR COOLING CAVITIES IN TURBOPUMPS?
- CONCENTRATE ON FIVE CRITICAL COMPONENTS
  - INLETS
  - COMBUSTORS/INJECTORS
  - NOZZLES
  - TURBINES
  - PUMPS
- ACCEPT THAT FOR MOST APPLICATIONS TURBULENCE MODELS ARE "NON-UNIVERSAL"
  - SELECT 1 CFD CODE FOR EACH COMPONENT
  - IMPLEMENT ALL APPROPRIATE MODELS IN THAT CODE
  - IDENTIFY RELATIVE ACCURACY VS COST OF EACH MODEL THROUGH A SYSTEMATIC EVALUATION PROCESS AND ESTABLISH MODEL PERFORMANCE DATABASE
  - MATCH MODEL TO LEVEL OF ANALYSIS/ACCURACY REQUIRED IN APPLICATIONS



## DISCUSSION

D. Wilcox (to Sindir)

I understand that there is some concerns with NASP contractors that RSM is too expensive. I find only a 20% increase in CPU time to compute a full RSM with the newer algorithms, relative to the two equation models.

M. Sindir (reply)

The model that you came up with is not a tool unless it gets into the methodology of established codes that contractors use for validation. And that is a major activity.

T. Gatski (to Sindir)

You describe an extremely complex situation and then use a Baldwin Lomax model; I have a confidence level of zero in that calculation!

M. Sindir (reply)

I know and agree, but that is what is being done in industry. But time constraints keep this problem from being handled properly.

A. Hsu

I feel that two items ought to be added to the list of tasks to pursue in PDF modeling. First, we have to examine PDF for high speed flows, like flows with shocks. Second, for the particle Monte Carlo method, problems involved in solution over a realistic geometry should be addressed.

A. Singhal (to S. Pope)

How do we transition this new and evolving technology into industry? In the first stage of transition, you introduced the joint PDF of velocity and composition, not just the first level of PDF which was shown in the hierarchy to be just composition. I'm curious why.

S. Pope (reply)

The reason for that is that the numerical algorithm for joint PDF of velocity and composition is really simple and more economical than just composition alone. It sounds strange, but the reason is that in the PDF for composition, the diffusion terms, turbulent transport,

### **Session III**

## **Unconventional Turbulence Modeling**



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p-29

The Present State of DIA Models  
and  
Their Impact on One Point Closures

Akira Yoshizawa  
Institute of Industrial Science  
University of Tokyo

I. Objectives

- A. Outline of DIA
- B. Outline of TSDIA (two-scale DIA) and some suggestions to turbulence modeling
- C. Proposals:
  - Helicity for the study of the effects of swirling and cross flow
  - Density variance for the study of highly compressed flows

## II. Basic Laws

### Compressible fluid

$$(\partial/\partial t) \rho + \nabla \cdot (\rho u) = 0$$

$$(\partial/\partial t) \rho u_i + (\partial/\partial x_j) (\rho u_j u_i) = - (\partial/\partial x_i) p \\ + (\partial/\partial x_j) \mu s_{ji}$$

$$(\partial/\partial t) \rho e + \nabla \cdot (\rho u e) = - p \nabla \cdot u + \phi + \nabla \cdot (\lambda \nabla \theta)$$

where

$\rho$ : Density;  $u$ : Velocity;  $p$ : Pressure;

$e$ : Internal energy;  $\theta$ : Temperature;

$\mu$ : Viscosity;  $\lambda$ : Heat conductivity;

$$s_{ij} = \partial u_j / \partial x_i + \partial u_i / \partial x_j - (2/3) \nabla \cdot u \delta_{ij}$$

$$\phi = \mu [ (\partial u_j / \partial x_i)^2 + (1/3) (\nabla \cdot u)^2 ]$$



[Note] Thermodynamic relations for a perfect gas:

$$p = \rho R \theta \quad (R: \text{Gas constant})$$

$$= (\gamma - 1) e \quad (\gamma: \text{Ratio of specific heats})$$

$$e = C_v \theta \quad (C_v: \text{Specific heat at constant volume})$$

Incompressible approximation

$$\nabla \cdot \mathbf{u} = 0$$

$$(\partial/\partial t) u_i + (\partial/\partial x_j) (u_j u_i) = - (\partial/\partial x_i) p + \nu \Delta u_i$$

where

$$p/\rho \rightarrow p$$

$$\nu = \mu/\rho: \text{Kinematic viscosity}$$

### III. Outline of DIA<sup>1</sup>

Premise: Vanishing mean velocity

#### Homogeneity

$$\langle u_i(x;t) u_j(x';t') \rangle = \langle u_i(x - x';t) u_j(0;t') \rangle$$

Independence of the coordinate origin

→ Infinite or periodic region

→ Fourier-integral or -series representation

#### Fourier integral

$$f(x;t) = \int f(k;t) \exp(-ik \cdot x) dk$$

## Homogeneous turbulence

$$k \cdot u(k; t) = 0$$

$$\begin{aligned} (\partial/\partial t) u_i(k; t) - ik_j \iint \delta(k - p - q) dp dq \\ \times u_j(p; t) u_i(q; t) = ik_i p(k; t) - \nu k^2 u_i(k; t) \end{aligned}$$

[Note] Elimination of pressure:

$$\begin{aligned} p(k; t) = - (k_i k_j / k^2) \iint \delta(k - p - q) dp dq \\ \times u_i(p; t) u_j(q; t) \end{aligned}$$

[Note] Green's function:

$$\begin{aligned} (\partial/\partial t) G_{ij}'(k; t, t') - iM_{imn}(k) \iint \delta(k - p - q) dp dq \\ \times u_m(p; t) G_{nj}'(q; t, t') - \nu k^2 G_{ij}'(k; t, t') \\ = D_{ij}(k) \delta(t - t') \end{aligned}$$

where

$$D_{ij}(k) = \delta_{ij} - k_i k_j / k^2$$

$$M_{ijk}(k) = [k_j D_{ik}(k) + k_k D_{ij}(k)] / 2$$

## Fundamental variables

$$Q_{ij}(k; t, t') = \langle u_i(k; t) u_j(k'; t') \rangle / \delta(k + k')$$

$$G_{ij}(k; t, t') = \langle G_{ij}'(k; t, t') \rangle$$

## Difficulties in incorporating inhomogeneity<sup>2</sup>

- ① Necessity of the orthogonal function satisfying the noslip condition

[Note] Dannevik's work:<sup>3</sup>

Turbulent Rayleigh-Benard convection between two parallel plates (no mean flow)

- ① Coexistence of slow modes (mean field) and fast modes (fluctuation)

→ Simultaneous treatment of different modes

#### IV. Outline of TSDIA (Two-Scale DIA)

Departure from a complete two-point scheme

- ⊙ Passive: Difficulty in dealing with a boundary
- ⊙ Positive: Difficulty of obtaining "formulae" applicable to general flows

#### Two scales

$$\xi \quad (= x), \quad X \quad (= \delta x); \quad \tau \quad (= t), \quad T \quad (= \delta t)$$

where

$\delta$ : A small scale-expansion parameter

Then

$$f = F(X; T) + f'(\xi, X; \tau, T), \quad F = \langle f \rangle$$

## Two-scale expressions

$$\nabla_{\xi} \cdot u' = - \delta (\partial/\partial X_j) u_j', \quad \nabla_{\xi} = (\partial/\partial \xi_i)$$

$$\begin{aligned} & (\partial/\partial \tau) u_i' + (U \cdot \nabla_{\xi}) u_i' + (\partial/\partial \xi_i) p' - \nu \Delta_{\xi} u_i' \\ & = \delta [- u_j' (\partial/\partial X_j) U_i - (D/DT) u_i' - (\partial/\partial X_i) p' \\ & \quad - (\partial/\partial X_j) u_j' u_i'] + \nu\text{-related terms} \end{aligned}$$

[Note] The effects of slow modes:

Direct effects : Through U

Indirect effects: Through X and T in u'

## Fourier representation of $\xi$

$$f'(\xi, X; \tau, T) = \int f'(k, X; \tau, T) \exp[-ik \cdot (\xi - U\tau)] dk$$

## Scale expansion

$$f'(k, X; \tau, T) = \sum_{n=0}^{\infty} \delta^n f_n'(k, X; \tau, T)$$

[Note] Lowest-order or basic field  $u_0'$ :

The same system of equations as for homogeneous turbulence, except the X and T dependence

## Isotropic and helical field

$$\begin{aligned} & \langle u_{Bi}'(k, X; \tau, T) u_{Bj}'(k', X; \tau', T) \rangle / \delta(k + k') \\ &= D_{ij}(k) Q_B(k, X; \tau, \tau', T) \\ &+ \underbrace{(i/2) (k_m/k^2) \epsilon_{ijm} H_B(k, k', X; \tau, \tau', T)}_{\text{helicity effect}} \end{aligned}$$

[Note]

$$\langle u_{B'}'^2/2 \rangle = \int Q_B(k, X; \tau, \tau, T) dk$$

$$\langle u_{B'}' \cdot \omega_{B'}' \rangle = \int H_B(k, X; \tau, \tau, T) dk, \quad \omega_{B'}' = \nabla \times u_{B'}'$$

## Important correlation functions

Calculation of the Reynolds stress etc. using  
DIA based on  $Q_B$  and  $G_B$

→ Extended eddy-viscosity representation for  
the Reynolds stress

## V. Main Results from TSDIA: No Helicity

### Reynolds stress<sup>4</sup>

$$- \langle u_i' u_j' \rangle = - (2/3) K \delta_{ij} + \nu_e S_{ij}$$

$$- \sum_{n=1}^3 \tau_n [T_{nij} - (1/3) T_{nmm} \delta_{ij}]$$

$$- \tau_4 (D/Dt) S_{ij} + \dots$$

where

$$K = \langle u'^2 / 2 \rangle$$

$$S_{ij} = \partial U_j / \partial x_i + \partial U_i / \partial x_j$$

$$T_{1ij} = (\partial U_i / \partial x_m) (\partial U_j / \partial x_m)$$

$$T_{2ij} = [(\partial U_i / \partial x_m) (\partial U_m / \partial x_j)$$

$$+ (\partial U_j / \partial x_m) (\partial U_m / \partial x_i)] / 2$$

$$T_{3ij} = (\partial U_m / \partial x_i) (\partial U_m / \partial x_j)$$

[Note] See Speziale<sup>5</sup> and Rubinstein and Barton<sup>6</sup>



## Turbulent scalar flux<sup>7</sup>

$$\langle u_i' \theta' \rangle = - \kappa_{ij} (\partial / \partial x_j) \Theta$$

where

$$\begin{aligned} \kappa_{ij} = & \kappa \delta_{ij} - [\gamma_1 (\partial U_j / \partial x_i + \partial U_i / \partial x_j) \\ & - \gamma_2 (\partial U_j / \partial x_i - \partial U_i / \partial x_j)] \end{aligned}$$

[Note] See Rubinstein and Barton<sup>8</sup>

## Some other suggestions

### ● Triple correlations<sup>9</sup>

$$\langle (u'^2 / 2) u' \rangle = - \nu_K \nabla K + \underline{\nu_K' \nabla \epsilon}$$

### ● Equations for the dissipation rates of energy and scalar variance ( $\epsilon$ and $\epsilon_\theta$ )<sup>7</sup>

## V. Proposal: Helicity

### Motivation

Explanation of the generation of "white spot" in Saturn (the spiral vortical structures along the equator)<sup>10</sup>

Fig. 1: Saturn's huge white spot  
(NASA Hubble Space Telescope)

### What is the helicity?

$$(\partial/\partial t) u = u \times \omega - \nabla(p + u^2/2) + \nu \Delta u$$

$$u \parallel \omega \rightarrow u \times \omega = 0 \text{ (no energy cascade)}$$

Helicity  $u \cdot \omega$ : A measure of the break of  
reflectional symmetry in flow

Fig. 2. Helicity

## Importance of helicity

- ⊙ A measure of "smallness" of energy cascade
- ⊙ A "conserved" quantity in the absence of the injection and loss due to viscous effects

[Note]  $\langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle = \mathbf{U} \cdot \boldsymbol{\Omega} + \langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle$

- ⊙ No mean helicity:  $\mathbf{U} \cdot \boldsymbol{\Omega} = 0$  ( $\mathbf{U} \perp \boldsymbol{\Omega}$ )

Two-dimensional mean flow

channel flow, jet, wake, mixing layer, etc.

- ⊙ Finite mean helicity:  $\mathbf{U} \cdot \boldsymbol{\Omega} \neq 0$

Swirling flow, Three-dimensional mean flow

[Note] Swirling flow:

Question: Why do the eddy-viscosity models  
break the swirling motion so fast?

Answer : No consideration of helicity or the  
decreasing effect of cascade (virtual  
decrease of eddy viscosity)

[Note] Cross-flow effects:

- ① Lag of the turbulent stresses in response to the cross-flow gradient (also important in aerodynamical flows<sup>11, 12)</sup>)

Fig. 2 Spinning cylinder<sup>12)</sup>

- ② Curvature effects leading to secondary flows

Three-equation model<sup>13)</sup>

$K = \langle u'^2/2 \rangle$ : Turbulent kinetic energy

$\varepsilon$  : Energy dissipation rate

$H = \langle u' \cdot \omega' \rangle$ : Turbulent helicity

[Mean equation]

$$\nabla \cdot \mathbf{U} = 0$$

$$(D/Dt) U_i = - (\partial/\partial x_i) P + (\partial/\partial x_j) R_{ji}$$

[Reynolds stress]

$$\begin{aligned} R_{ji} &= - \langle u_i' u_j' \rangle \\ &= - (2/3) K \delta_{ij} + \nu_e (\partial U_j / \partial x_i + \partial U_i / \partial x_j) \\ &\quad - [\Omega_i \gamma_j + \Omega_j \gamma_i - (2/3) \Omega \cdot \gamma \delta_{ij}] \end{aligned}$$

where

$$\nu_e = C_1 K^2 / \epsilon, \quad \gamma = C_2 (K^4 / \epsilon^3) \nabla H$$

[Note] Symmetry-breaking factor:

Reynolds stress: Reflectionally symmetric

→ Necessity of another symmetry-breaking factor, that is, inhomogeneity such as  $\nabla H$

[K equation]

$$(D/Dt) K = P_K - \varepsilon + \nabla \cdot [C_3 (K^2/\varepsilon) \nabla K]$$

where

$$P_K = R_{ij} (\partial U_j / \partial x_i)$$

[H equation]

$$(D/Dt) H = P_H - \varepsilon_H + \nabla \cdot T_H$$

where

$$P_H = R_{ij} (\partial \Omega_j / \partial x_i) - \Omega_i (\partial / \partial x_j) R_{ji}$$

$$\varepsilon_H = C_4 (\varepsilon / K) H$$

$$T_H = K \Omega + C_5 (K^2 / \varepsilon) \nabla H$$

[\varepsilon equation]

$$(D/Dt) \varepsilon = C_6 (\varepsilon / K) P_K - C_7 (\varepsilon^2 / K) + C_8 (K^2 / \varepsilon) \Omega \cdot \nabla H \\ + \nabla \cdot [C_9 (K^2 / \varepsilon) \nabla \varepsilon]$$

# Swirling flow in a pipe $[(r, \theta, z); z: \text{axial}]$

$$- \langle u_r' u_{\theta}' \rangle = \nu_e r (\partial/\partial r) (U_{\theta}/r) - \eta \Omega_{\theta} \partial H / \partial r$$

$$- \langle u_r' u_z' \rangle = \nu_e (\partial U_z / \partial r) - \eta \Omega_z \partial H / \partial r$$

$$- \langle u_{\theta}' u_z' \rangle = \nu_e (\partial U_{\theta} / \partial z) - \eta \Omega_{\theta} \partial H / \partial z$$

where

$$\eta = C_1 K^4 / \epsilon^3 > 0$$

## Comparison with observation<sup>14</sup>

Fig. 4. Mean velocity

Fig. 5.  $-\langle u_r' u_{\theta}' \rangle$

(Broken lines: weak swirl region)

$$- \langle u_r' u_{\theta}' \rangle = \underbrace{\nu_e r (\partial/\partial r) (U_{\theta}/r)}_A - \underbrace{\eta \Omega_{\theta} \partial H / \partial r}_B$$

Around  $r = 0.7$

$$A > 0$$

$$B > 0 \quad (\Omega_{\theta} = - \partial U_z / \partial r < 0, \quad \partial H / \partial r < 0)$$

[Note]  $H = 0$  at the wall  $\rightarrow \partial H / \partial r < 0$  near it

## Application to SGS modeling<sup>13</sup>

Appearance of streamwise vortices (streaks)  
dependent on the strength of shear<sup>15</sup>

- Nonvanishing helicity on the SGS, but not on the ensemble or time mean
- Importance in the SGS modeling
- Incorporation of the helicity effect
- Virtual change of the Smagorinsky constant in channel flow, mixing layer, isotropic flow in accordance with the strength of shear



## VII. Proposal: Density Variance

Prominent difference between incompressible and  
compressible turbulence

Decelerated streamwise velocity effects

① Compressible (shock wave):

Decrease in turbulence level

Increase in temperature etc.

② Incompressible:

Increase in turbulence level

Mass-weighted mean

$$\hat{f} = \{f\} = \langle \rho f \rangle / \bar{\rho}, \quad \bar{\rho} = \langle \rho \rangle$$

$$f' = f - \hat{f}$$

[Mean equation]

$$(\partial/\partial t) \bar{\rho} + \nabla \cdot (\bar{\rho} \hat{u}) = 0$$

$$(\partial/\partial t) \bar{\rho} \hat{u}_i + (\partial/\partial x_j) \bar{\rho} \hat{u}_j \hat{u}_i = - (\partial/\partial x_i) [(\gamma - 1) \bar{\rho} \hat{e}]$$

$$+ (\partial/\partial x_j) R_{ji}$$

$$(\partial/\partial t) \bar{\rho} \hat{e} + \nabla \cdot (\bar{\rho} \hat{u} \hat{e}) = - (\gamma - 1) \bar{\rho} \hat{e} \nabla \cdot \hat{u} + \bar{\rho} (\epsilon + \chi)$$

$$+ \nabla \cdot (-H)$$

where

$$R_{ij} = - \langle \rho u_i' u_j' \rangle = - \bar{\rho} \{ u_i' u_j' \}$$

$$H = \langle \rho u' e' \rangle = \bar{\rho} \{ u' e' \}$$

$$\chi = - (\gamma - 1) \langle \rho e' \nabla \cdot u' \rangle / \bar{\rho} = - (\gamma - 1) \{ e' \nabla \cdot u' \}$$

$R_{ij}$ : Reynolds stress

$H$  : Internal energy flux

$\chi$  : Fluctuating dilatation effect

[Note] Importance of  $\chi$ :

Trace of the pressure-strain correlation  
(no contribution in the incompressible case)

[Note] Inference of  $\chi$ :

Large positive in a highly compressed region

→ Virtual increase of energy dissipation

Three-equation model<sup>16, 17</sup>

$K = \{u'^2/2\}$ ,  $\epsilon$ ,  $K_d = \langle \rho'^2 \rangle$ : Density variance

[Reynolds stress]

$$R_{ij} = - (2/3) \bar{\rho} K \delta_{ij} + \bar{\rho} \nu_e \hat{s}_{ij}, \quad \nu_e = C_1 K^2 / \epsilon$$

[Internal energy flux]

$$H = - \bar{\rho} \kappa_e \nabla \hat{e}, \quad \kappa_e = C_2 K^2 / \epsilon$$

[Fluctuating dilatation effect]

$$\chi = C_3 (\epsilon/K) (K_d/\bar{\rho}^2) \hat{e}$$

[K equation]

$$(\partial/\partial t) K + \nabla \cdot (\bar{\rho} \hat{u} K) = P_K - \bar{\rho} (\epsilon + \chi) + \nabla \cdot T_K$$

[\epsilon equation]

$$(\partial/\partial t) \epsilon + \nabla \cdot (\bar{\rho} \hat{u} \epsilon) = C_4 (\epsilon/K) P_K - C_5 \bar{\rho} (\epsilon/K) (\epsilon + \chi) + \nabla \cdot T_\epsilon$$

[K<sub>d</sub> equation]

$$(\partial/\partial t) K_d + \nabla \cdot (\hat{u} K_d) = - K_d \nabla \cdot \hat{u} - C_6 (\epsilon/K) K_d + \nabla \cdot T_d$$

exact!

Can the turbulence level decrease behind a shock  
wave?

$\nabla \cdot \hat{u} < 0$  near a shock wave

→ Production of K<sub>d</sub> (K<sub>d</sub> equation)

→ Larger  $\epsilon$ , larger  $\chi$  (than elsewhere)

→ Decrease in K and  $\epsilon$  (K and  $\epsilon$  equations)

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- 16) A.Yoshizawa: Phys.Fluids A2 (1990) 838.
- 17) A.Yoshizawa: submitted to J.Phys.Soc.Jpn.

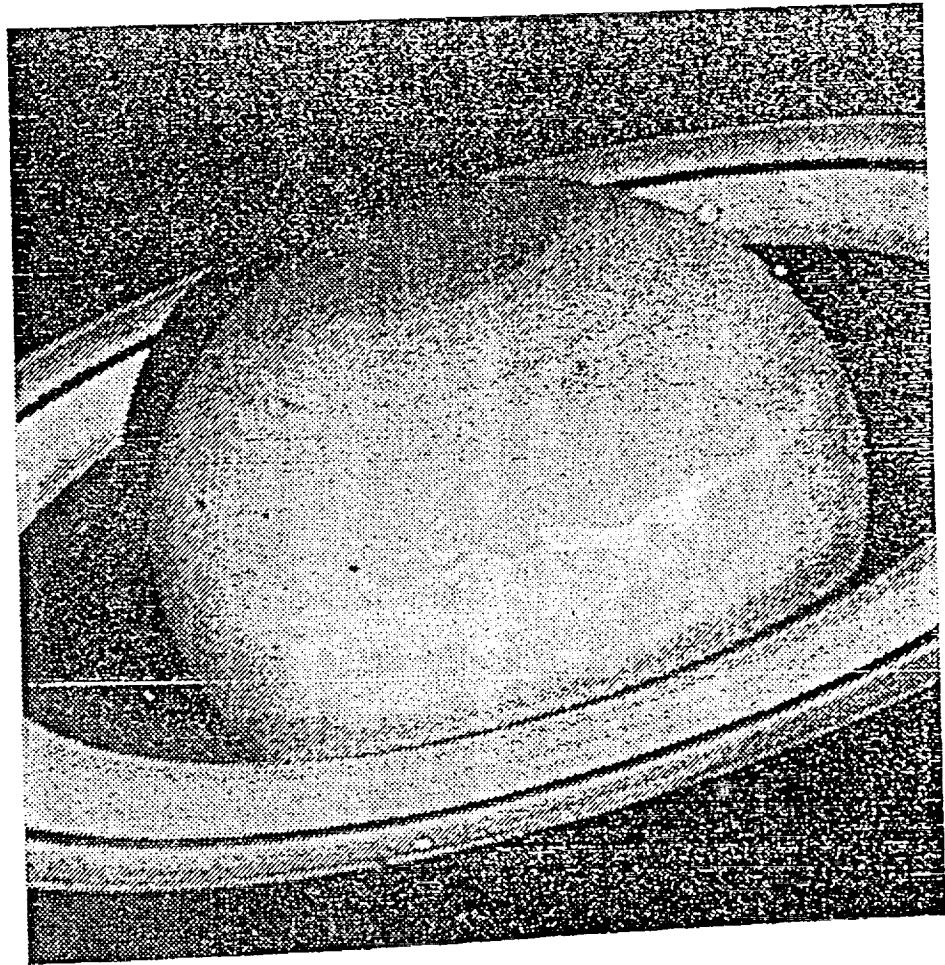


Fig. 1

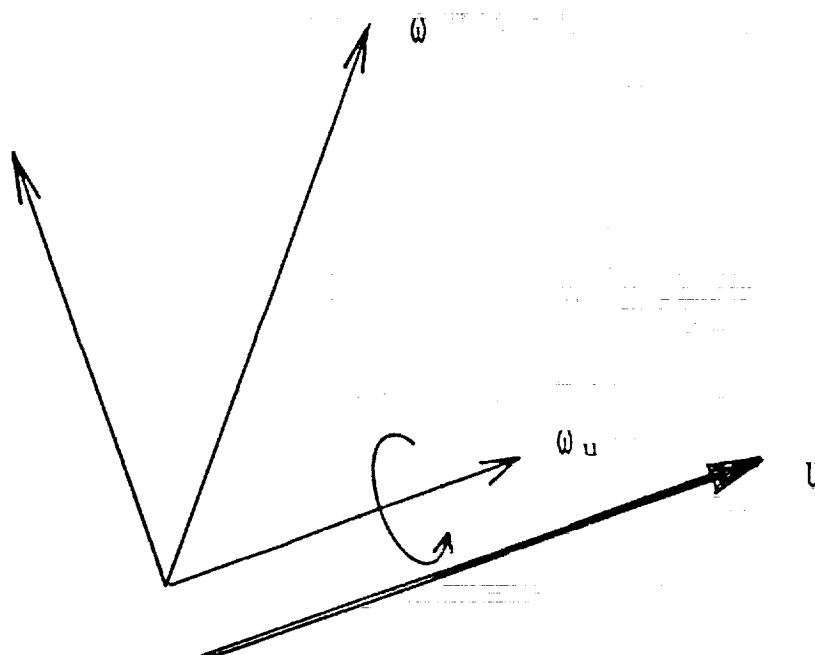


Fig. 2



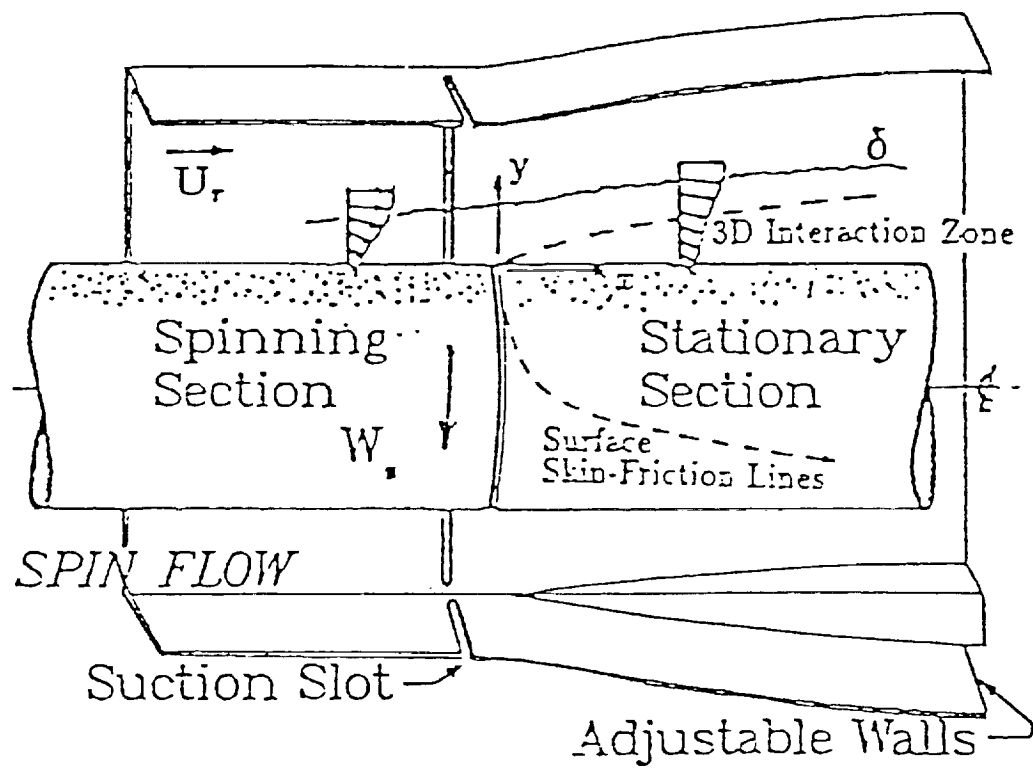


Fig. 3

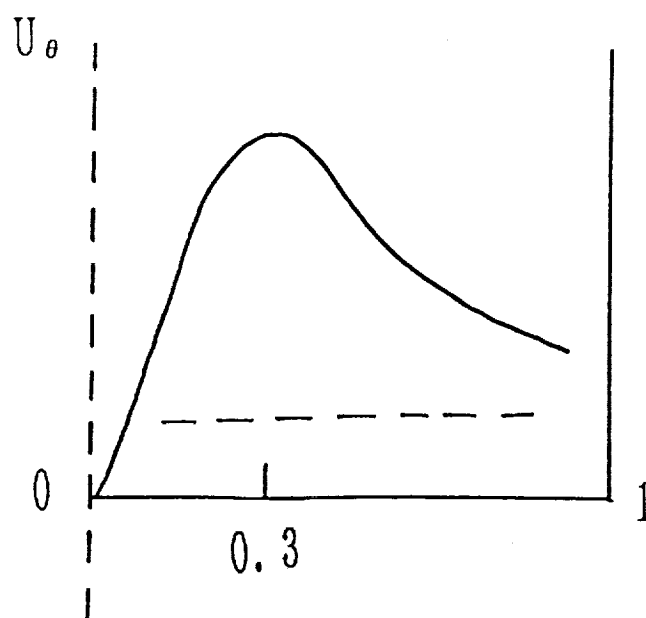
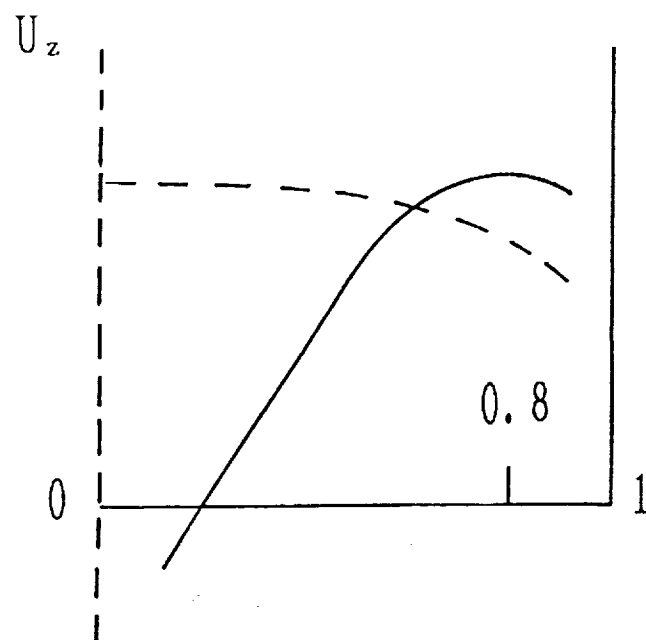


Fig. 4

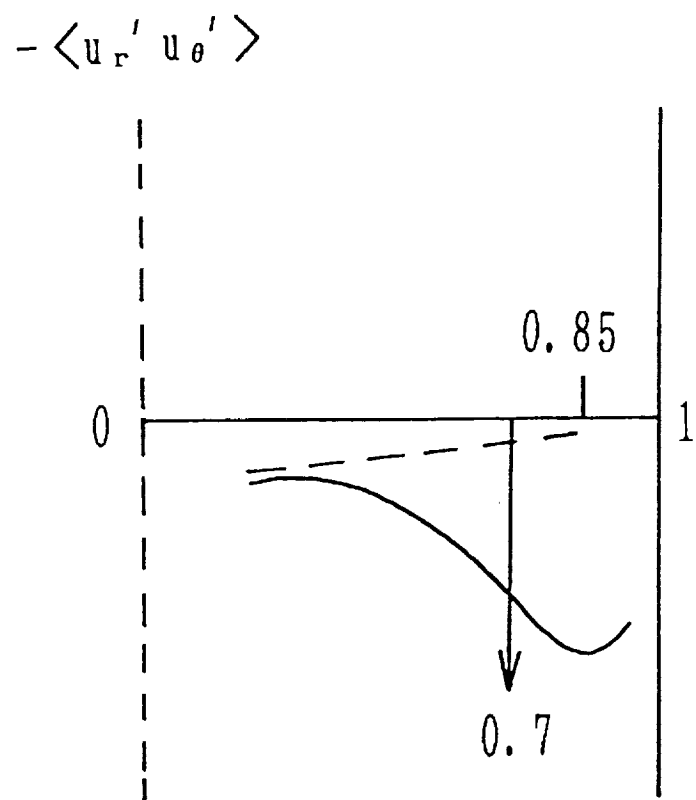


Fig. 5



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PRESENT STATE OF TWO-POINT CLOSURES AND THEIR  
IMPACT ON SINGLE-POINT CLOSURES

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BOULDER, COLORADO 80303

## CONTENTS

MOTIVATION

BACKGROUND OF TWO-POINT CLOSURES, BRIEFLY

VERY RECENT SIMPLIFICATION

RESULTS

APPLICATION TO SINGLE-POINT CLOSURES

"COMPLEXITY" ISSUE AND SIMPLIFICATION

PRESSURE - STRAIN RATE

DIFFUSION

DISSIPATION

DIRECTION FOR FUTURE WORK

## MOTIVATION, AND A VERY PERSONAL PERSPECTIVE

### CARDINAL PRINCIPLE

$$\langle u_i u_j \rangle = \int d\vec{k} S_{ij}(\vec{k}, t)$$

### ALL SCALES INCLUDED -- NOT ALL NEEDED

POTENTIAL FOR HIGH SUCCESS OF SINGLE-POINT CLOSURES

PARTNERSHIP

GOAL TO "ELIMINATE" EMPIRICAL COEFFICIENTS

○ KEY = ANALYTICAL SOLUTION

ARBITRARY REYNOLDS NUMBER

### DETAILS COMPLEX, BUT → MINIMIZES UNCERTAINTY

SHAPE OF SPECTRUM NOT NECESSARILY IMPORTANT

SIMPLIFICATION IS POSSIBLE

## STATUS OF 2-POINT CLOSURES - BACKGROUND

$$\frac{\partial S_k}{\partial t} = \underbrace{-L_0 S_k}_{\text{LINEAR (RDT)}} - \underbrace{(1+T_h) \langle u_k^* (\underline{u} \cdot \nabla) u_k \rangle}_{\text{NONLINEAR ADVECTION}} + \underbrace{\langle i k \frac{p_k}{\rho} u_k^* \rangle}_{\text{NONLINEAR PRESSURE-STRAIN RATE}} - \underbrace{2\nu k^2 S_k}_{\text{DISP.}}$$

$$\left( L_0 S_k = (1+T_h) S_k \cdot \nabla U \cdot \left( \underline{I} - \underline{\hat{k}} \underline{\hat{k}} \right) - (\nabla U \cdot \underline{\hat{k}}) \cdot \frac{\partial S_k}{\partial k} \right)$$

### TWO TASKS

DERIVE CLOSURE EQUATIONS

SOLVE CLOSURE EQUATIONS

- EDQNM, DIA,

- COMPLICATED

(HAS NOT BEEN REDUCED  
TO QUADRATURES)

### ANALYTICAL SOLUTION

RADICAL SIMPLIFICATION



# RADICAL SIMPLIFICATION

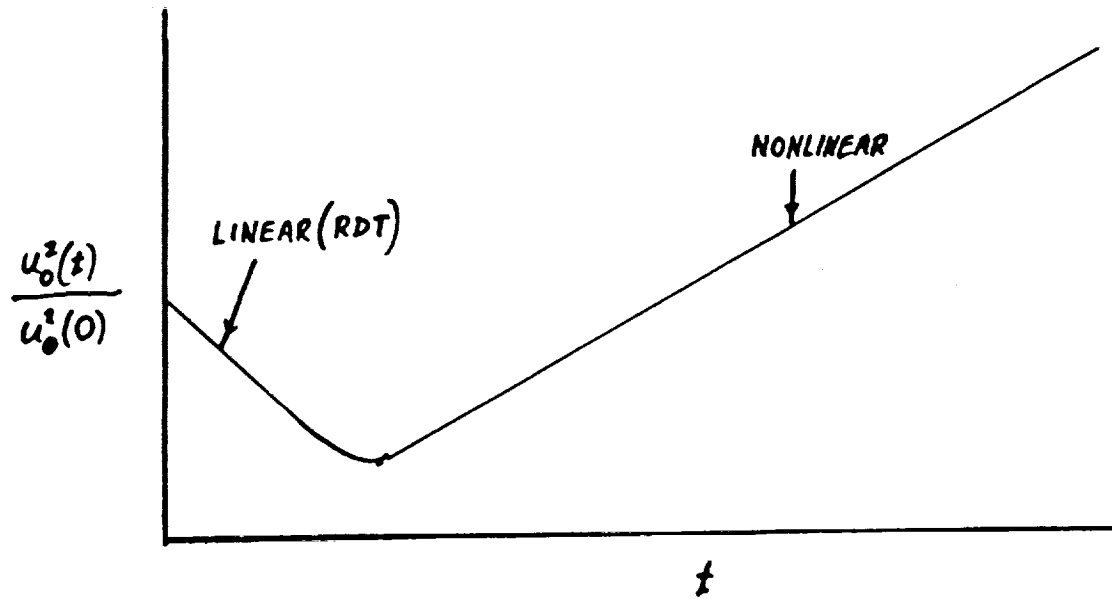
$$O \quad \frac{\text{NONLINEAR}}{\text{P.S.R.}} = -0.32 \frac{k^{7/3}}{k_0^{1/3} u_0} E(k) \left( S_{\underline{k}} - S_{\underline{k}}^I \right)$$

$$C \quad \frac{\text{NONLINEAR}}{\text{ADVECTION}} = -0.32 \frac{k^{7/3}}{k_0^{1/3} u_0} S_{\underline{k}}^I \left[ E(k) - \frac{(k \partial/\partial k)^2 E(k)}{(\frac{10}{3} + A)^2} \right]$$

$$\therefore \quad \frac{\partial S_{\underline{k}}}{\partial t} = -L_0 S_{\underline{k}} - 0.32 \frac{k^{7/3}}{k_0^{1/3} u_0} \left[ E(k) S_{\underline{k}} - \frac{(k \partial/\partial k)^2 E(k)}{(\frac{10}{3} + A)^2} S_{\underline{k}}^I \right]$$

DETERMINES  $S_{ij}(k, t)$  ALL  $i, j, k, t$  -- ANY  $\underline{U}_0$  AND  $R_\nu$

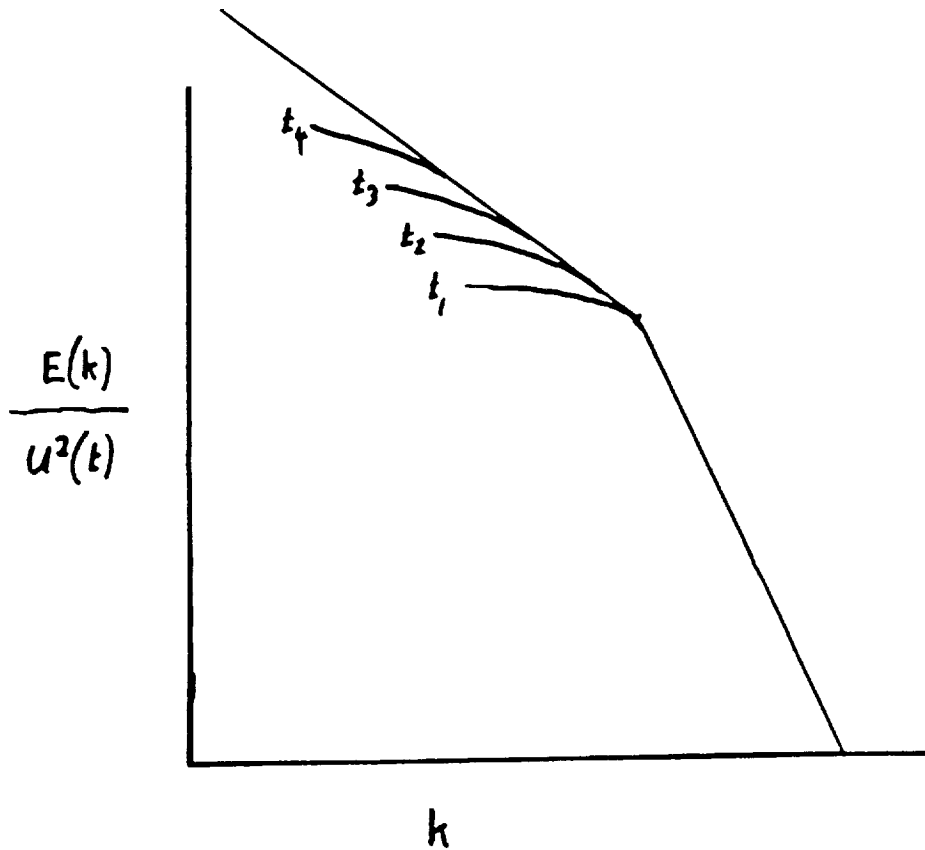
# RESULTS - GROWTH OF TURBULENCE



$u_0^2(t)$  DEPENDS ON  $t/\tau_0$  ( $\alpha \pm \frac{\partial u_0}{\partial x_2}$ ).

NO DISTINCTION BETWEEN WEAK & STRONG SHEAR

# ENERGY SPECTRUM AND DISSIPATION

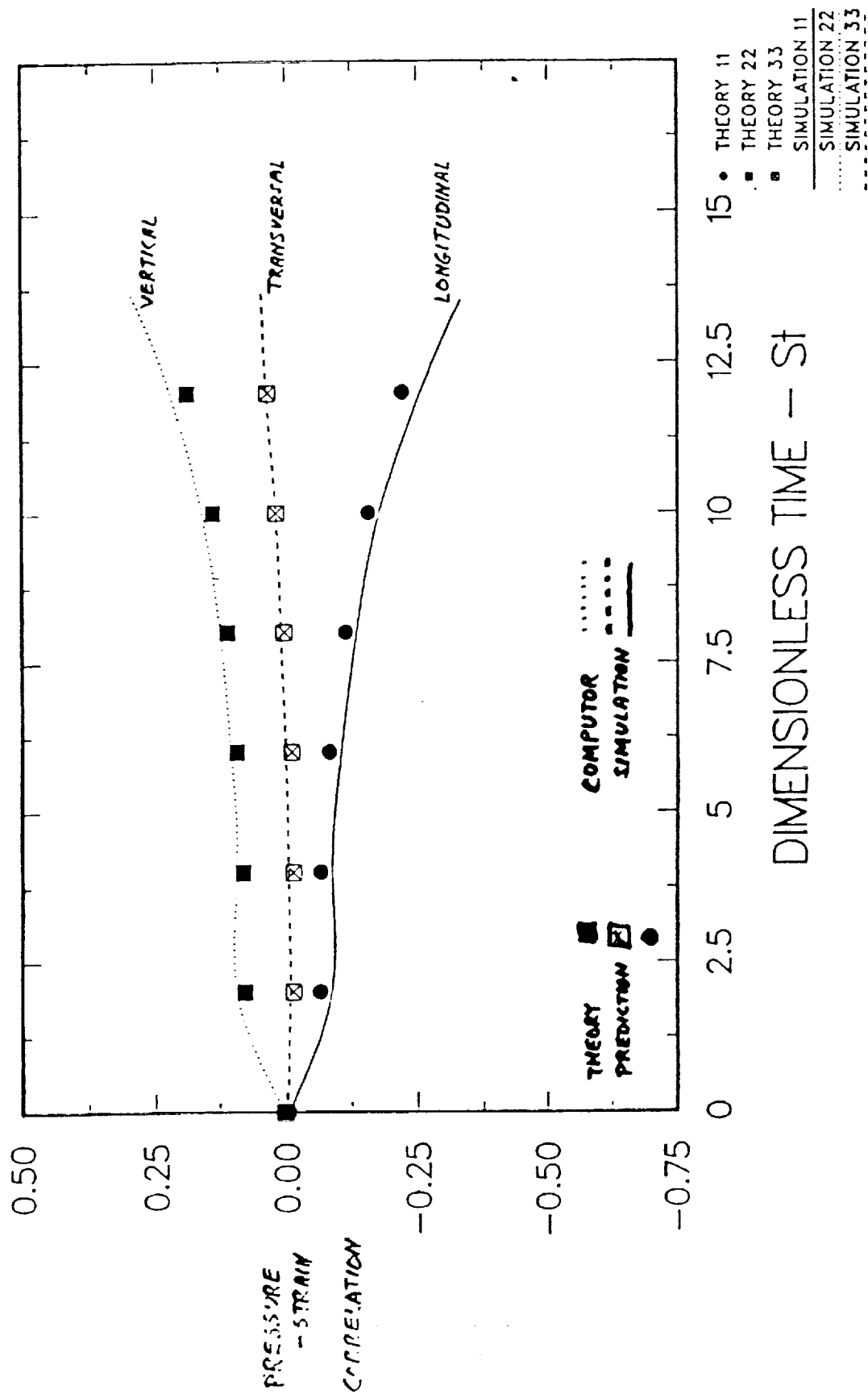


$$k_0 = \frac{1}{1.8} u^{-1} \partial u_0 / \partial x_3$$

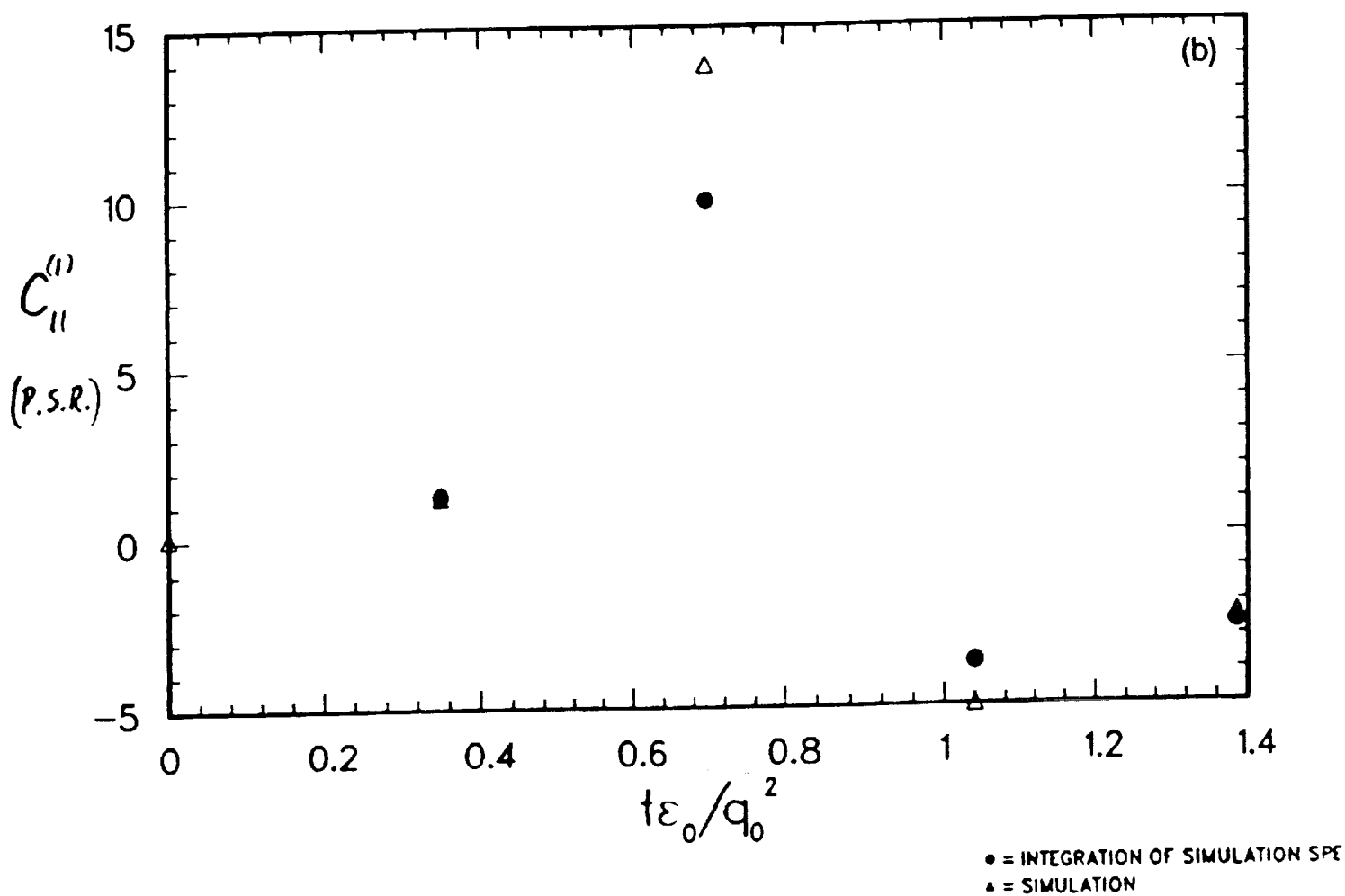
$$\epsilon = c(t) g^2 \partial u_0 / \partial x_3$$

$\epsilon_{ij}$  AND LOCAL ISOTROPY

COMPARISON OF THEORETICAL PREDICTION WITH  
NASA COMPUTER SIMULATION (AT AMES RES. CENTER)



# COMPARISON OF P.S.R. THEORY WITH SIMULATIONS



$$C_{II}^{(1)} \equiv \text{ROTTA "CONSTANT"} \equiv \frac{\phi_{II}^{(1)}}{\langle u_i^2 \rangle - u_0^2}$$

## APPLICATION TO SINGLE-POINT MODELING

$$\frac{D\langle u_i u_j \rangle}{Dt} = P_{ij} + \phi_{ij} + \frac{\partial}{\partial x_m} d_{ijm} - \epsilon_{ij}$$

PRESSURE                  DIFFUSION                  DISSIPATION  
STRAIN  
RATE

THEORY DETERMINES  $\phi_{ij}$   $d_{ijm}$   $\epsilon_{ij}$

### USE OF THEORY

TEST EXISTING MODELS

○ DETERMINE COEFFICIENTS

VARIATION OF COEFFICIENTS

DERIVE MODELS FROM "FIRST PRINCIPLES"

## COMPLEXITY ISSUE

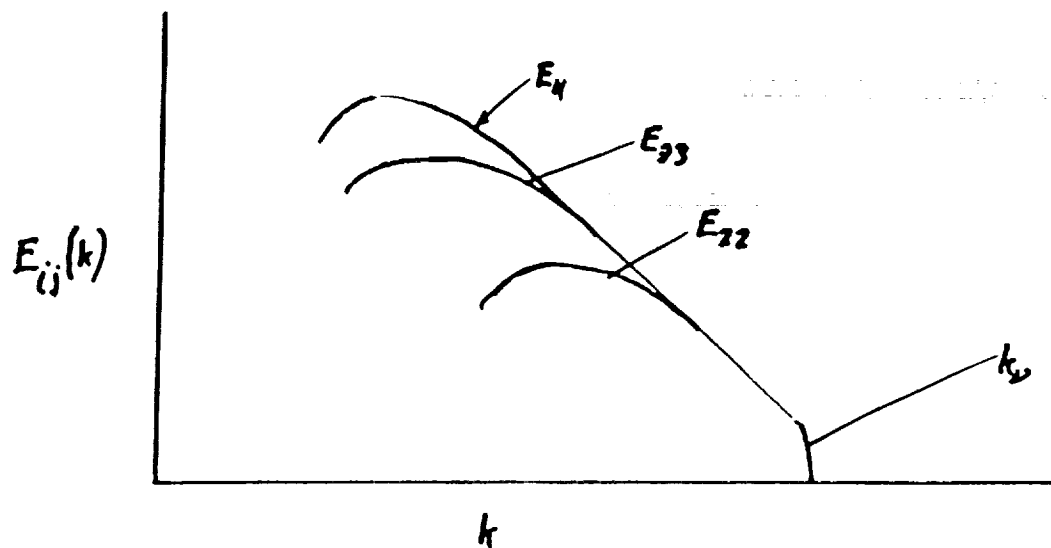
TOO MANY DETAILS - YES & NO

A SIMPLE "PHILOSOPHY"

A PRACTICAL SOLUTION

EXAMPLE - P.S.R. SLOW TERM

# SLOW TERM



$$\phi_{ij}^{(3)} = -\log \sqrt{\frac{2\pi}{3}} \int_0^\infty dk_2 \int_0^\infty dk_b \frac{k_a^2 k_b^2 E(k_b) [E_{ij}(k_a) - \frac{1}{3} E(k_a) \delta_{ij}]}{(k_a^2 + k_b^2)^{3/2}}$$

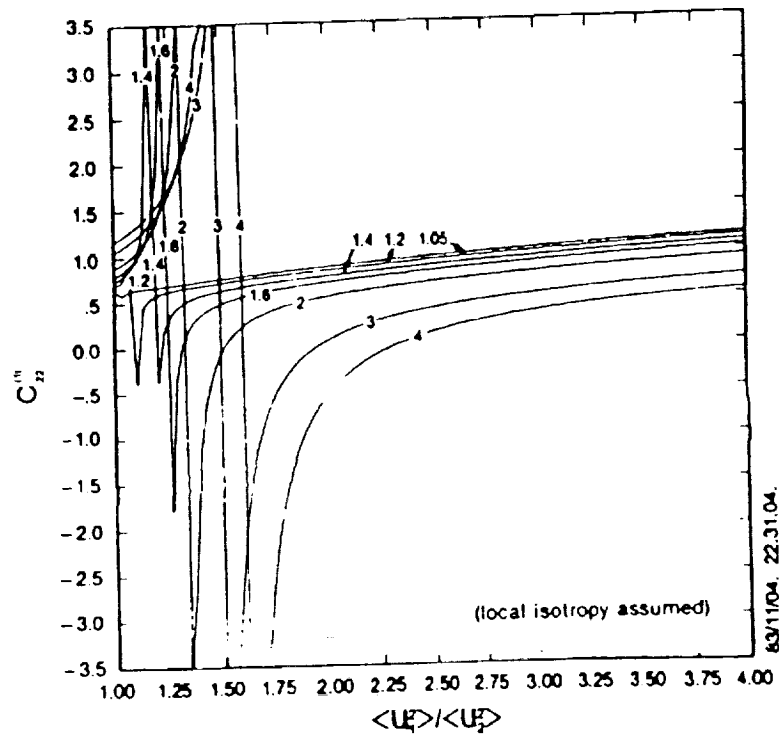
ONLY NEED  $\rightarrow k_1, k_2, k_3, k_v$

EMPIRICAL WITHOUT ALL  $k$ 'S.

STILL COMPLICATED.



# THEORETICAL $\phi_{12}$



DEFIES EMPIRICAL TREATMENT, BUT IS COMPLICATED :

C SIMPLIFICATION -  $R_\nu$  VARIATION

FIFTH SCALE BECAUSE  $E_{12} \propto k^{-5/3}$ ;  $C_{12} = 2 \langle C_{11} \rangle_{AVE}$

$$C_{12} = 1.7(1 - R_\nu^{-1/2})$$

UNIVERSALITY :

INSENSITIVITY TO SMALL  $k$  REGION OF  $E(k)$ ; PRODUCTION REGION  
 " "  $\nabla \bar{U}$

NEAR  
COMPLETED FOR SIMPLE MEAN SHEAR:

FUTURE = ARBITRARY MEAN STRAINING:

CONFIRM OR IMPROVE EXISTING MODEL

SHIH AND LUNDE, 1985

FU, LAUNDER & TSELEPIDAKIS, 1987

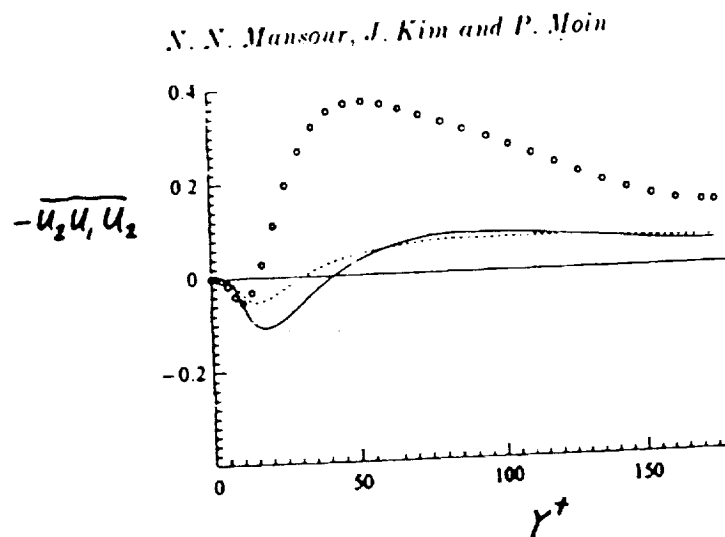
LOOK FOR UNIVERSALITY (GUIDED BY MODELS)

(RESOLVE) (HELP) QUESTIONS OF NONLINEARITY

## DIFFUSION

INFLUENCE OF BUOYANCY: LUMLEY, ZEMAN, SEISS (1978)  
(WEINSTOCK, 1989)

INFLUENCE OF MEAN SHEAR: MANSOUR, KIM, MOIN (1989); WYNGAARD (1980)



$$\begin{aligned} \langle u_2 \underline{\underline{u}} \underline{\underline{u}} \rangle &= -\tau_0 \left[ \langle u_2^2 \rangle \frac{\partial}{\partial x_2} \langle \underline{\underline{u}} \underline{\underline{u}} \rangle + (1+\Gamma) \langle u_2 \underline{\underline{u}} \rangle \frac{\partial}{\partial x_2} \langle u_2 \underline{\underline{u}} \rangle \right. \\ &\quad \left. + \frac{2 \langle u_2 \underline{\underline{u}} \nabla \phi \rangle}{\rho_c} + 2 \langle u_2 \underline{\underline{u}} \underline{\underline{u}} \rangle \cdot \nabla \bar{U} \right] \end{aligned}$$

# INHOMOGENEITY AND NON-STATIONARITY

WEAK INHOMOGENEITY: DONE

$$L \gg l_0$$

STRONG INHOMOGENEITY → FUTURE

CALCULATE  $\langle \underline{u}(t) \underline{u}(t) \underline{u}(t) \rangle$  FOR  $d_{ijm}$  AND  $\phi_{ij}$

SPECULATED METHOD:

$$\underline{u}(t) = \underline{u}(0) - \int_0^t [\underline{u} \cdot \nabla \underline{u} + \bar{\underline{U}} \cdot \nabla \underline{u} + \underline{u} \cdot \nabla \bar{\underline{U}} + \frac{\nabla p}{\rho_0} + \frac{\underline{g}}{\theta_0} + \nu \nabla^2 \underline{u}] dt$$

$$\therefore \langle \underline{u}(t) \underline{u}(t) \underline{u}(t) \rangle = - \int_0^t \langle \underline{u}(t) \underline{u}(t) \underline{u}(t_1) \cdot \nabla \underline{u}(t_1) \rangle dt_1 + \dots$$

$$= - \int_0^t \langle G(t, t_1) \underline{u}(t) \underline{u}(t) \underline{u}(t_1) \cdot \nabla \underline{u}(t_1) \rangle dt_1 + \dots$$

$$= - \int_0^t \langle G(t, t_1) \rangle \langle \underline{u}(t) \underline{u}(t) \rangle \cdot \nabla \langle \underline{u}(t_1) \underline{u}(t_1) \rangle dt_1 + \dots$$

$\swarrow$   
 PRESENT

$\searrow$   
 FUTURE

## SUMMARY

RADICAL SIMPLIFICATION

SOLVES HOMOGENEOUS SHEAR FLOWS

PROVIDES  $\phi_{ij}$ ,  $d_{ijm}$ ,  $\epsilon_{ij}$

TEST AND/OR IMPROVE EXISTING MODELS.

GOAL  $\Rightarrow$  AVOID EMPIRICAL COEFFICIENTS

FUTURE

INCORPORATE HOMOGENEOUS THEORY INTO 1-POINT MODELS

INFLUENCE OF VARIOUS STRAINING ON FAST TERM

STRONG INHOMOGENEITY



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# The Present State of RNG

+

## Its Impact on One Point Closure

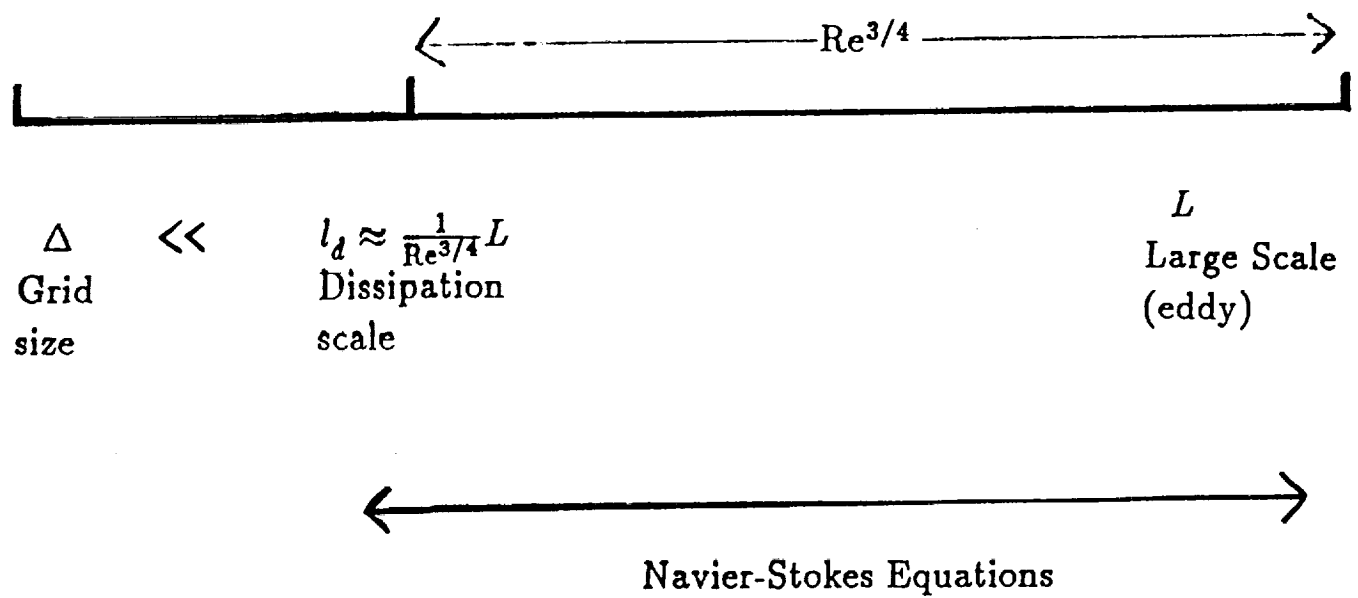
- Introduction to renormalization group methods for one-point closure
- Strain rate expansion  
V. Yakhot, S. Thangam, T. Gatski, + C. Speziale

S. Orszag

Princeton University

# DIRECT NUMERICAL SIMULATION

## DNS

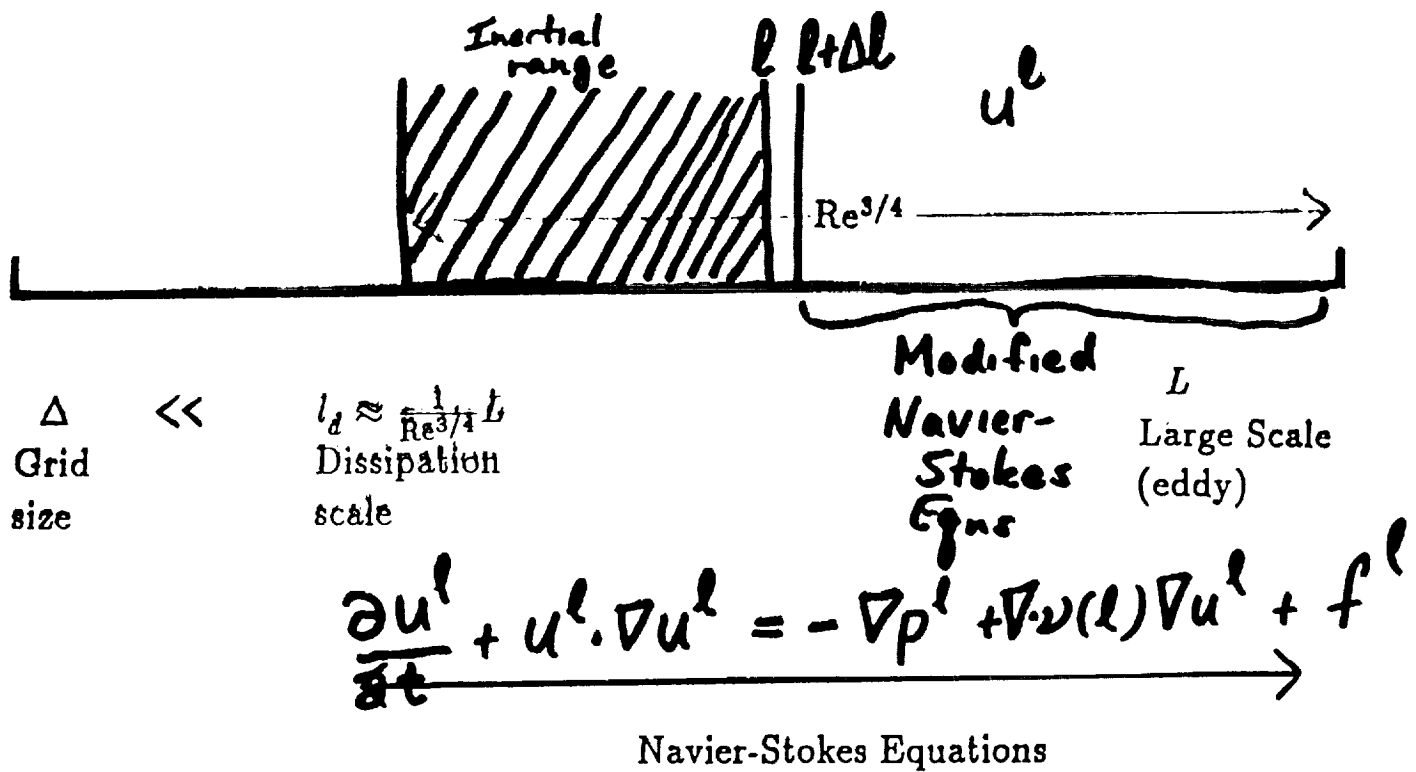




# DIRECT NUMERICAL SIMULATION

DNS

Renormalization Group



- Construct calculus of recursion relations to compute  $\nu(l)$ ,  $f^l$ , ...

# Langevin Model

$$\frac{\partial u^l}{\partial t} = - \underbrace{\nu(l)}_{\text{Nonlocal convolution operator}} u^l l^{-2} + f^l$$

Assume

$$\frac{d\nu(l)}{dl} \propto \varepsilon \quad (\text{rate of energy dissipation})$$

Dimensional analysis  $\Rightarrow$

$$\frac{d\nu}{dl} = \frac{A\varepsilon l^3}{\nu^2}, \quad \nu(l_d) = \nu_0$$

Solution:

$$\nu(l) = \nu_0 \left[ 1 + \frac{3}{4} \frac{A\varepsilon}{\nu_0^3} (l^4 - l_d^4) \right]^{1/3} \quad (l > l_d)$$

$$\nu(l) \sim \left( \frac{3}{4} A\varepsilon \right)^{1/3} l^{4/3} \quad (l \gg l_d)$$

# Intrinsic Stirring Force in Turbulence and the $\varepsilon$ -Expansion

Large-scale force to model the effects of *initial* and *boundary* conditions

$$\left( -i(\omega + \nu_0 k^2) u_\alpha(\mathbf{k}, t) \right) = -\frac{1}{2} i P_{\alpha\beta\gamma}(\mathbf{k}) \int u_\beta(\mathbf{p}, \Omega) u_\gamma(\mathbf{k} - \mathbf{p}, \omega - \Omega) d\mathbf{p} d\Omega + f_\alpha(\mathbf{k}, \omega)$$

$$\left\langle f_\alpha(\hat{\mathbf{k}}) f_\beta(\hat{\mathbf{k}}') \right\rangle \propto D_0 P_{\alpha\beta}(\mathbf{k}) \delta(\mathbf{k}) \delta(\mathbf{k} + \mathbf{k}') \delta(\omega + \omega')$$

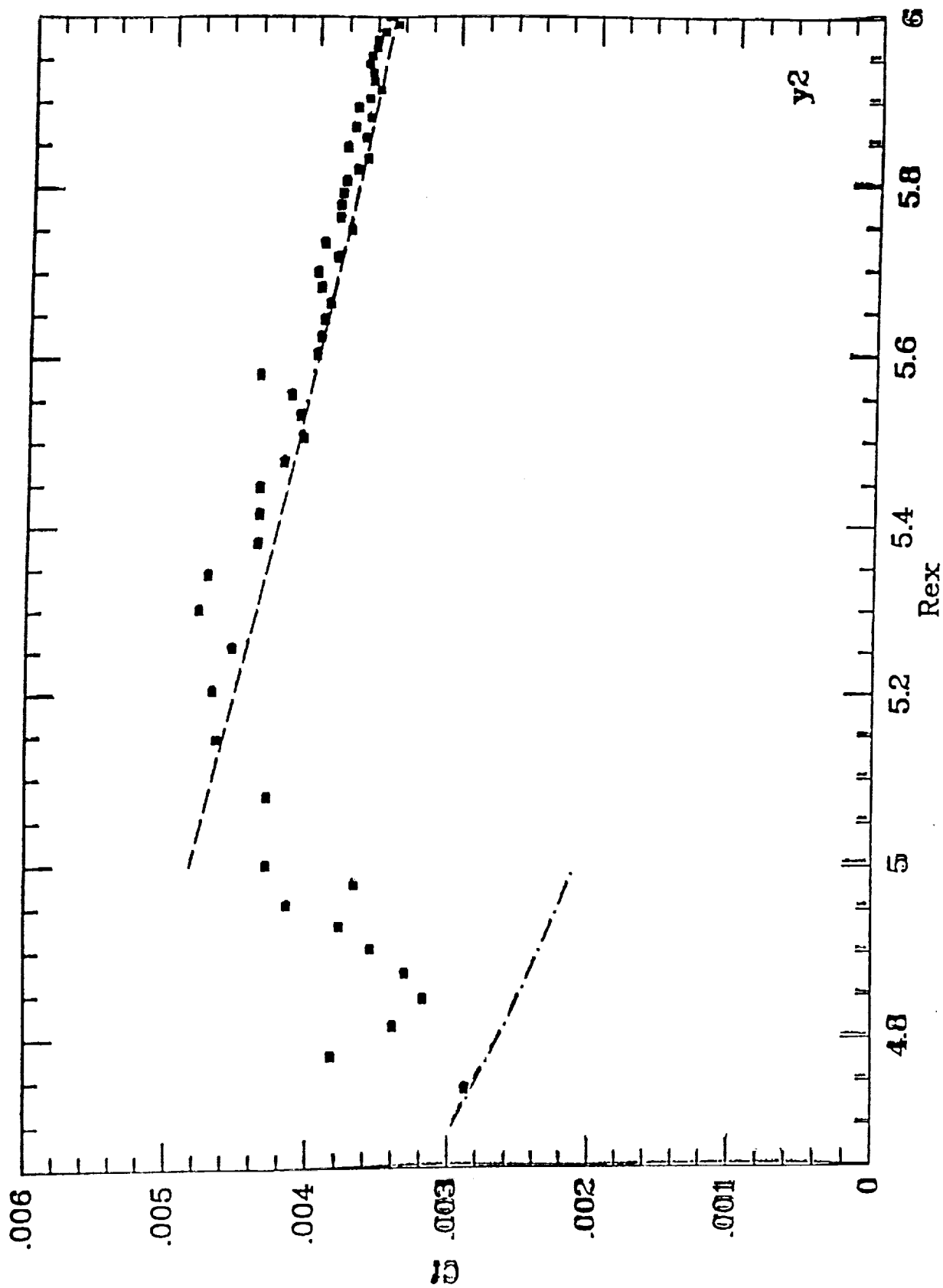
**DIFFICULTY:** Nonlinear solutions of the Navier-Stokes equations involve an 'infinity' of interacting f's to produce  $u(\mathbf{k})$  [ $\mathbf{k}$  finite]

**Correspondence Principle** (Yakhot & Orszag 1986)

$$\left\langle f_\alpha(\hat{\mathbf{k}}) f_\beta(\hat{\mathbf{k}}') \right\rangle \propto D_1 P_{\alpha\beta}(\mathbf{k}) k^{1-\varepsilon} \delta(\mathbf{k} + \mathbf{k}') \delta(\omega + \omega')$$

$$\delta(\mathbf{k}) = \lim_{\varepsilon \rightarrow 4} \frac{(4 - \varepsilon) k^{1-\varepsilon}}{4\pi}$$

Gel'fand

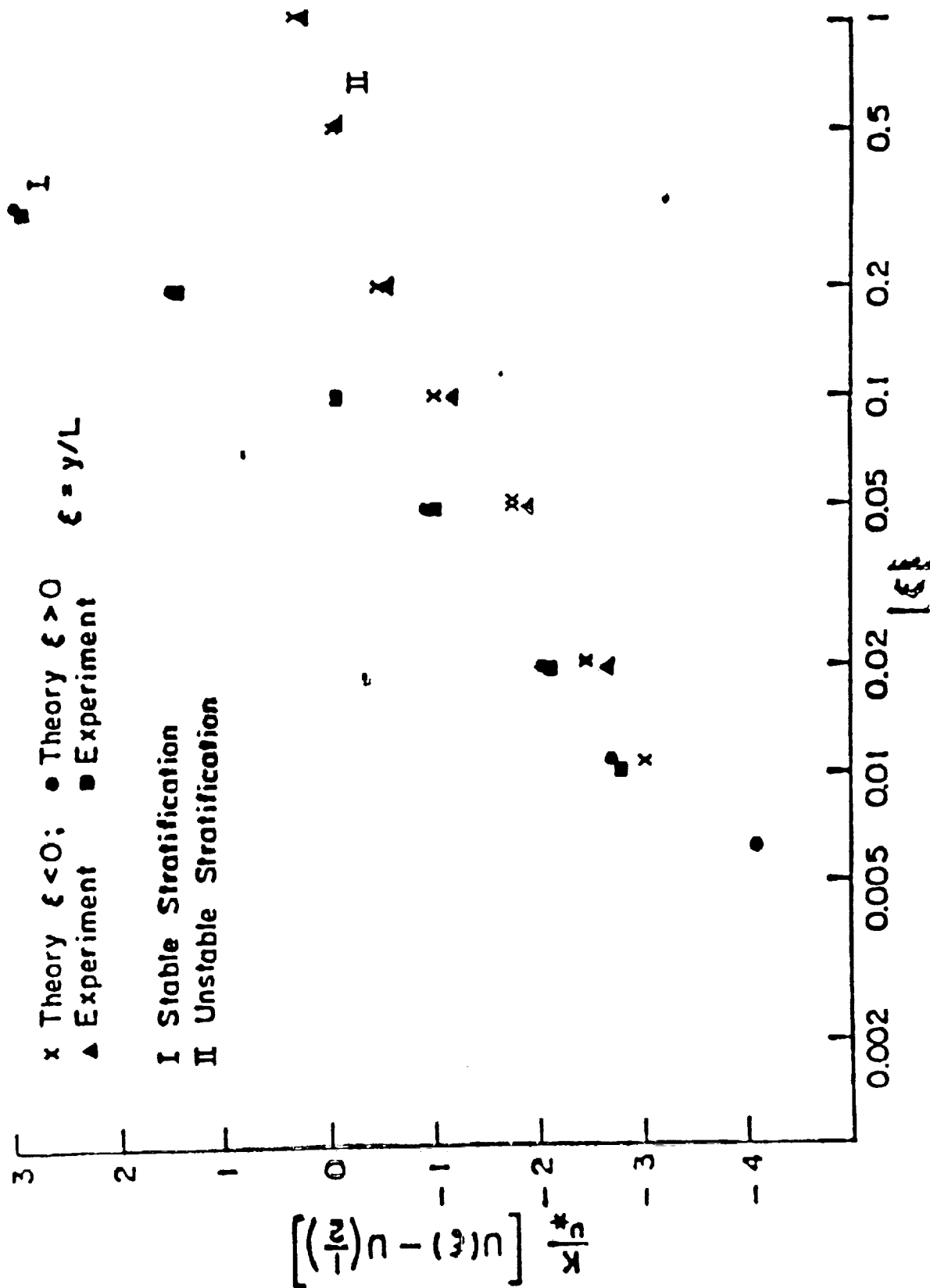


## Additional Physics

Example: Stratified Shear Flow

$$\nu_{\text{eddy}} = \nu_0 \left[ 1 + \frac{3}{4} \frac{A}{\nu_0^2} (\epsilon - g \frac{\partial \bar{T}}{\partial z}) (l^4 - l_d^4) \right]^{1/3}$$

# RNG CLOSURE FOR STRATIFIED TURBULENCE



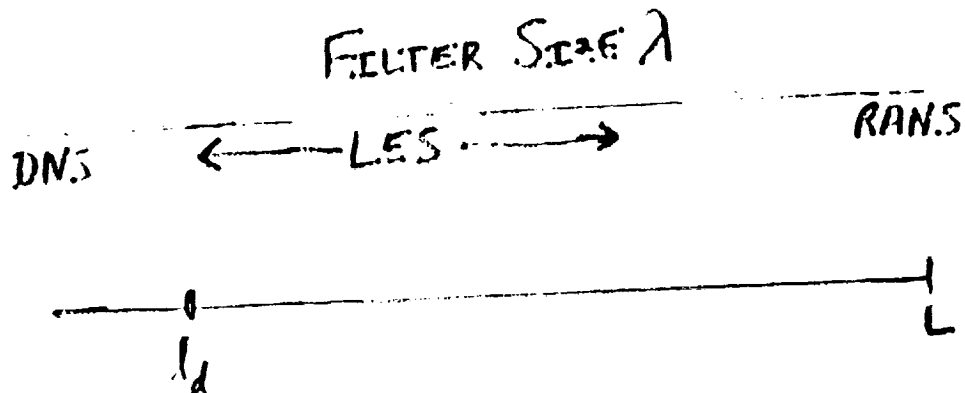
# Renormalization Group Analysis of Compressible Turbulence

- $c(L) \sim L^{4/3}$  : Effective sound speed is scale dependent
- $Ma_{\text{eff}} \sim \frac{1}{\sqrt{3}}$  : Universal equation of state at large scales
- Equipartition of energy  

$$E_{\text{incompressible}} \sim 3 E_{\text{compressible}}$$
- As  $Ma \rightarrow \infty$ ,  $\nu_{\text{eddy}} \propto 1/|\frac{\partial \bar{U}}{\partial y}|$   
not  $\nu_{\text{eddy}} \propto |\frac{\partial \bar{U}}{\partial y}|$  as in Prandtl mixing length theory  

$$\frac{\partial \bar{U}}{\partial t} = \frac{\partial}{\partial y} \left( \frac{1}{|\frac{\partial \bar{U}}{\partial y}|} \frac{\partial \bar{U}}{\partial y} \right) = 0 \quad (\text{Self-focusing of jets})$$

# TURBULENCE SIMULATIONS



NUMERICAL ACCURACY:  $|\Delta \ll \lambda|$

DNS     $\Delta \approx l_d$

LES     $l_d \ll \Delta \approx \frac{1}{2} \lambda \ll L$

RANS     $l_d \ll \Delta < \lambda \approx L$

ROBUSTNESS:  $|\lambda - \text{INDEPENDENCE}|$

$u_\lambda$  —  $\lambda$ -filtered velocity

$R_\lambda$  —  $\lambda$ -sub-grid model

$\mathcal{F}_\lambda$  —  $\lambda$  filter     $R_\lambda \approx u - \mathcal{F}_\lambda u$

Test: If  $\lambda < \mu$ , is

$$\overline{F(u_\lambda)} \stackrel{?}{\approx} \overline{F(u_\mu)} + \{ \overline{F(R_\mu)} - \overline{F(R_\lambda)} \}$$



# Renormalization Group Subgrid Model for LES

$$K \propto 1/\Delta$$

$$\nu_{\text{eddy}} = \nu_{\text{mol}} \left[ 1 + \frac{3}{4} \frac{A E}{(2\pi)^4 \nu_{\text{mol}}^3} (\Delta^4 - \eta_d^4) \right]^{1/3}$$

$$E = \nu_{\text{eddy}} |\nabla \bar{U}|^2$$

High turbulence limit

$$\nu_{\text{eddy}} \sim \left[ \frac{3}{4} \frac{A}{(2\pi)^4} \right]^{1/2} \Delta^2 |\nabla \bar{U}|$$

Transitional regimes

$$\nu_{\text{eddy}}^3 = \nu_{\text{mol}}^3 + \frac{3}{4} \frac{A}{(2\pi)^4} (\Delta^4 - \eta_d^4) \nu_{\text{eddy}} |\nabla \bar{U}|^2$$

Alternate Formulation — unphysical roots can be a problem

$$E = \frac{S^2}{\nu_{\text{eddy}}} \quad S = \nu_{\text{eddy}} |\nabla \bar{U}| \quad \left[ \begin{array}{l} \text{eddy SGS} \\ \text{stress} \end{array} \right]$$

$$\nu_{\text{eddy}}^4 = \nu_{\text{mol}}^3 \nu_{\text{eddy}} + \underbrace{\frac{3}{4} \frac{A S^2}{(2\pi)^4} (\Delta^4 - \eta_d^4)}_{\geq 0}$$

Only 1 real root with  $\nu_{\text{eddy}} \geq \nu_{\text{mol}}$

# RNG K- $\epsilon$ Eddy Viscosity

$$\nu(l) \sim \left(\frac{3}{4} A \epsilon\right)^{1/3} l^{4/3}$$

$$\begin{aligned} K &= \int_0^\infty C_{K0} \epsilon^{2/3} k^{-5/3} dk \\ &= \frac{3}{2} C_{K0} \epsilon^{2/3} l^{2/3} \end{aligned}$$

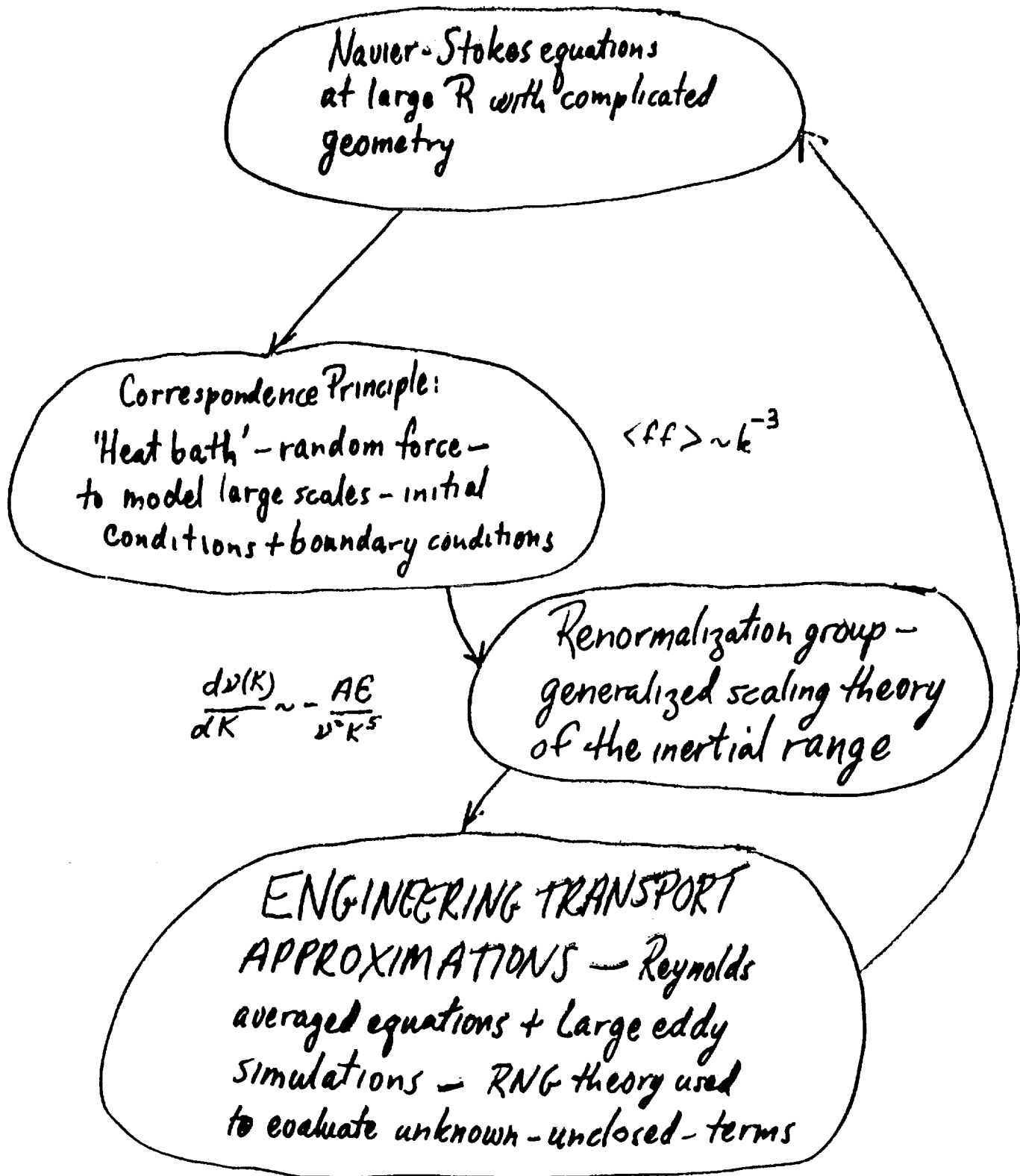
$$\nu = \frac{\left(\frac{3}{4} A\right)^{1/3}}{\left(\frac{3}{2} C_{K0}\right)^2} \frac{K^2}{\epsilon}$$

$$C_\mu = \frac{\left(\frac{3}{4} A\right)^{1/3}}{\left(\frac{3}{2} C_{K0}\right)^2} \stackrel{\text{RNG}}{=} 0.0845$$

Actually —  $\nu = C_\mu K^2/\epsilon$  only if  $\nu \gg \nu_0$   
RNG Differential Relation

$$\nu = \nu_0 \left[ 1 + \sqrt{\frac{C_\mu}{\nu_0}} \frac{K}{\sqrt{\epsilon}} \right]^2$$

# Practical Turbulence Theory



# RNG K-ε Modelling

$$\frac{D\bar{\epsilon}}{Dt} = \underbrace{-2\nu_0 \nabla_j v_i \nabla_j v_i \nabla_i v_i}_{T_1} - \underbrace{2\nu_0 (\nabla_j \nabla_i v_i)^2}_{T_2} - 2\nu_0 \nabla_j v_i \nabla_i \nabla_j p + \nu_0 \nabla^2 \bar{\epsilon}$$

$$T_{1,2} = O(\sqrt{Re_\epsilon}) \quad \text{where } Re_\epsilon = \bar{K}^2 / \nu_0 \bar{\epsilon}$$

$$\text{but } \bar{T}_1 + \bar{T}_2 = C_{\epsilon 2} \frac{\bar{\epsilon}^2}{\bar{K}} = O(1) \text{ as } Re_\epsilon \rightarrow \infty$$

$C_{\epsilon 2} = 1.68$  by RNG (Smith + Reynolds 1991, Yakhot + Smith 1991)

Final result (Yakhot + Smith 1991):  $Re_\epsilon \rightarrow \infty$

$$\frac{\partial \bar{\epsilon}}{\partial t} + \bar{U} \cdot \nabla \bar{\epsilon} = -C_{\epsilon 1} \frac{\bar{\epsilon}}{\bar{K}} \bar{\tau}_{ij} S_{ij} - C_{\epsilon 2} \frac{\bar{\epsilon}^2}{\bar{K}} - \mathcal{R} + \nu_0 \nabla^2 \bar{\epsilon}$$

$$\text{High } Re \text{ limit: } C_{\epsilon 1} = 1.42 \quad C_{\epsilon 2} = 1.68 \quad \alpha_\epsilon = 1.39$$

$$\mathcal{R} = 2\nu_0 S_{ij} \frac{\partial u_i}{\partial x_i} \frac{\partial u_j}{\partial x_j}$$

$\mathcal{R}$  not yet evaluated by RNG methods

$$\frac{\partial \bar{U}}{\partial t} + \bar{U} \cdot \nabla \bar{U} = -\nabla \bar{p} + \nabla \cdot [\nu (\nabla \bar{U} + \nabla \bar{U}^T)]$$

$$\frac{\partial \bar{K}}{\partial t} + \bar{U} \cdot \nabla \bar{K} = -\bar{E} - \bar{\tau}_{ij} \bar{S}_{ij} + \nabla \cdot \alpha_k \nu \nabla K$$

$$\alpha_k \rightarrow 1.39 \text{ as } Re \rightarrow \infty$$

$$\bar{\tau}_{ij} = -2 C_\mu K \eta_{ij}$$

$$\eta_{ij} = S_{ij} \bar{K} / \bar{E}$$

# Generalized K- $\epsilon$ RNG Model

Yakhot, Thangam, Gatski, Orszag, Speziale  
1991

$$\mathcal{R} = 2\nu_0 S_{ij} \overline{\frac{\partial u_i}{\partial x_i} \frac{\partial u_j}{\partial x_j}}$$

-  $\mathcal{R} = 0$  if  $\eta = S\bar{K}/\bar{\epsilon} = 0$

-  $\eta = 0$  Homogeneous turbulence

$\eta \approx 4$  Homogeneous shear

$\eta \approx 3$  Log layer (channel)

-  $\eta \rightarrow 0$   $\mathcal{R} = \nu_T S^3 \ll \frac{\bar{\epsilon}}{K} \nu_T S^2$  so  $\mathcal{R}$  negligible

-  $\eta \rightarrow \infty$   $\mathcal{R} = O\left(\frac{\bar{\epsilon}}{K} \eta\right)$

Expansion in powers of  $\eta$ :

$$\mathcal{R} = \nu_T S^3 [1 + r_1 \eta + r_2 \eta^2 + \dots]$$

where  $r_n$  are constants.....

Truncations of this series after a finite # of terms violate  $\mathcal{R} = O(\eta)$  and realizability!

## Evaluation of $\mathcal{R}$

- Consistency with weakly sheared turbulence  $\eta \rightarrow 0$   
 $\mathcal{R} \sim \nu S^3$
- Consistency with strongly sheared (rapid distortion, turbulence)  $\mathcal{R} = O(\eta) \quad \eta \rightarrow \infty$

## Padé Approximant

$$\mathcal{R} = \frac{\nu S^3 (1 - \eta/\eta_0)}{1 + \beta \eta^3}$$

( $\sum r_n \eta^n$  is a geometrical series)

I.e. 
$$\mathcal{R} = \frac{C_\mu \eta^3 (1 - \eta/\eta_0)}{1 + \beta \eta^3} \frac{\bar{\epsilon}^2}{K}$$

## Diffusion by Random Narrow-Band Velocity Field

Kraichnan 1970

$$E(k) = \frac{3}{2} v_0^2 \delta(k - k_0) \quad \text{Not scale invariant}$$

RNG (Differential model with  $\epsilon$ -expansion to lowest order)

$$\kappa_{\text{RNG}} = \frac{v_0}{k_0}$$

RNG (To all orders in  $\epsilon$  with generalized Wilson rule)

$$\kappa_{\text{RNG}} = \sqrt{\frac{2}{\pi}} \frac{v_0}{k_0} \approx 0.80 \frac{v_0}{k_0}$$

Direct-interaction approximation

$$\kappa_{\text{DIA}} \approx 1.1 \frac{v_0}{k_0}$$

Numerical (Kraichnan)

$$\kappa_{\text{numerical}} \approx 0.95 \frac{v_0}{k_0}$$

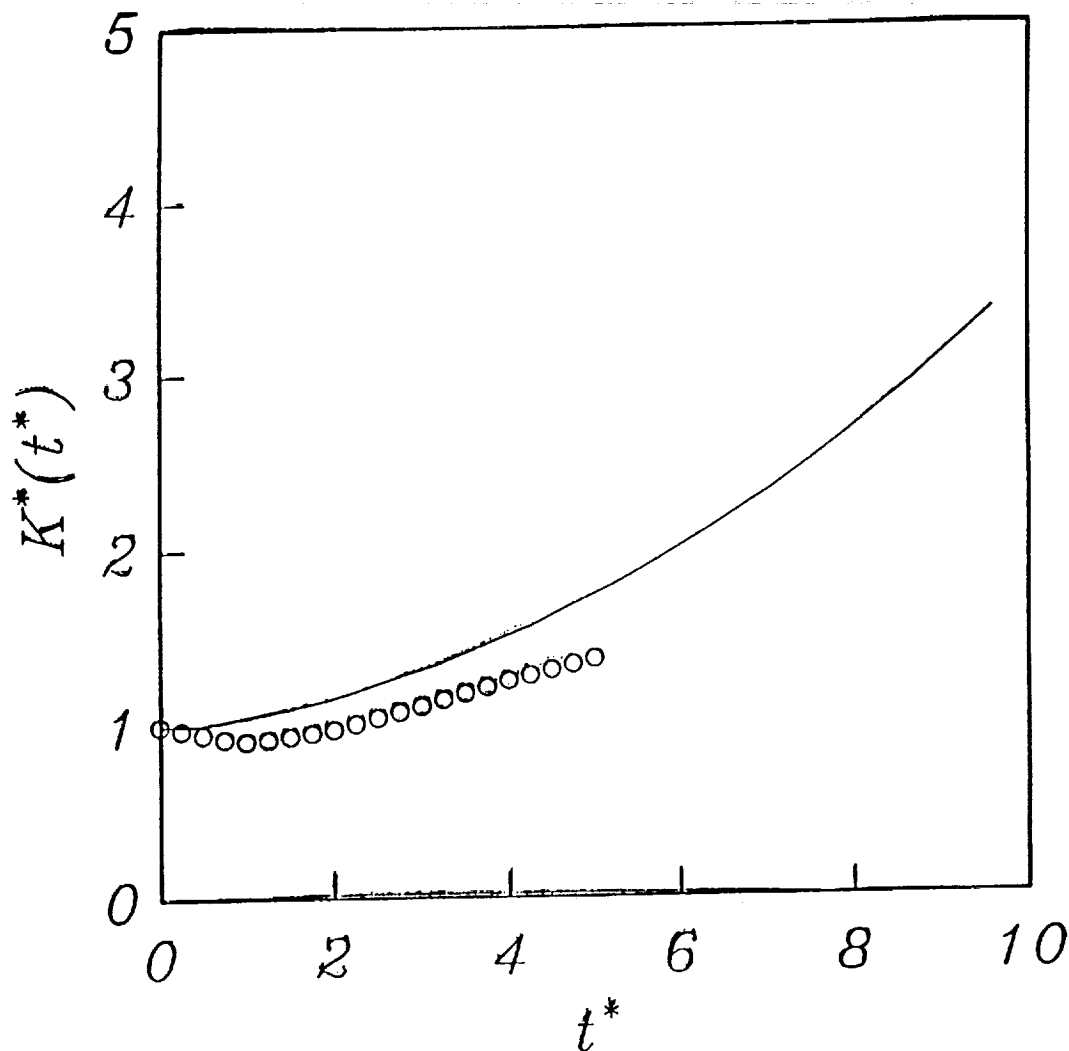


Two Constants:  $\eta_0 + \beta$  not yet determined

- Karman constant  $K = 0.4$
- Growth rate of homogeneous shear turbulence

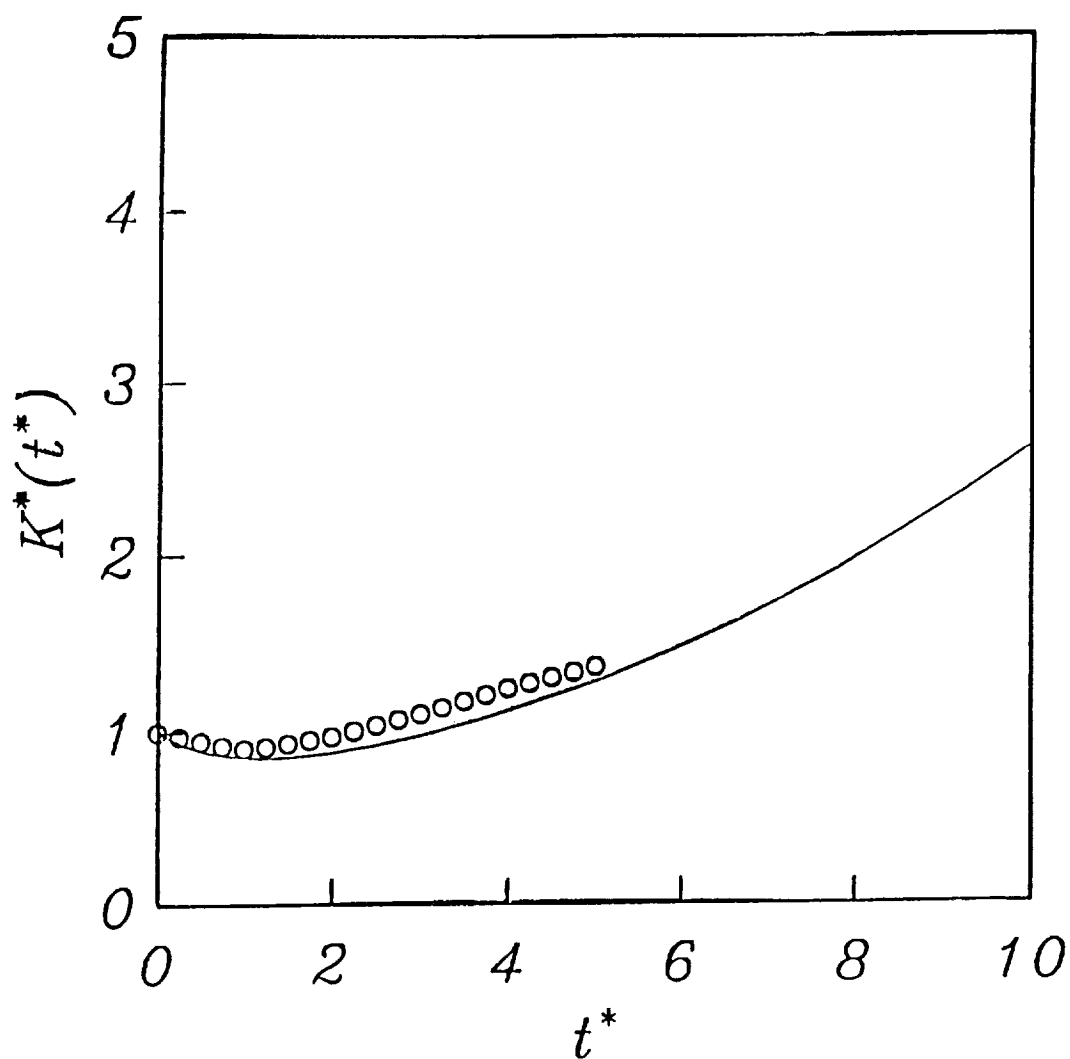
$$\frac{K}{K_0} \propto e^{0.14 t^*}$$

$$\Rightarrow \eta_0 \approx 4.38 \quad \beta = 0.012$$



Time evolution of the turbulent kinetic energy in homogeneous shear flow.  
 —  $K$ - $\epsilon$  model; o Large-eddy simulation of Bardina *et al*<sup>14</sup> for  $\epsilon_0/SK_0 = 0.296$

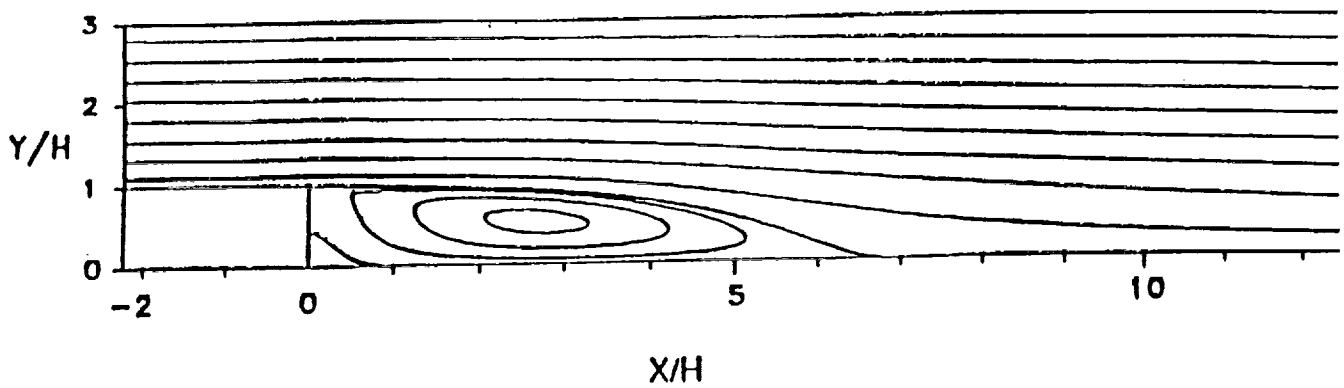
Figure 1



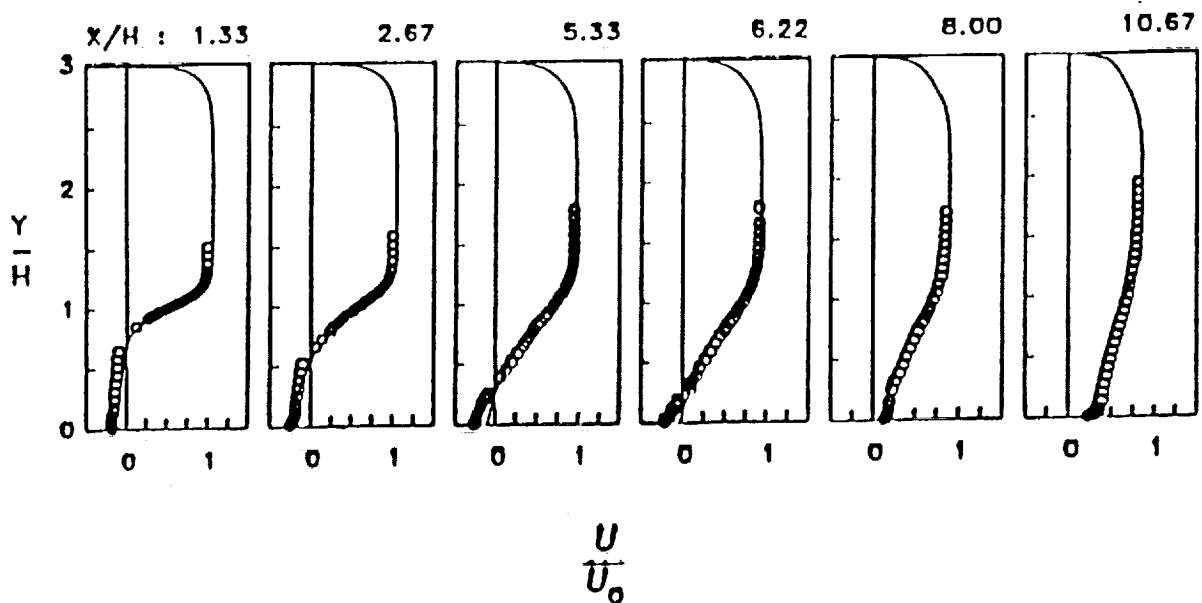
Time evolution of the turbulent kinetic energy in homogeneous shear flow.  
 — Relaxation model; o Large-eddy simulation of Bardina *et al.*<sup>14</sup> for  $\epsilon_0/SK_0=0.296$

Figure 2

# **BACKWARD-FACING STEP:**



**(a) Streamlines**



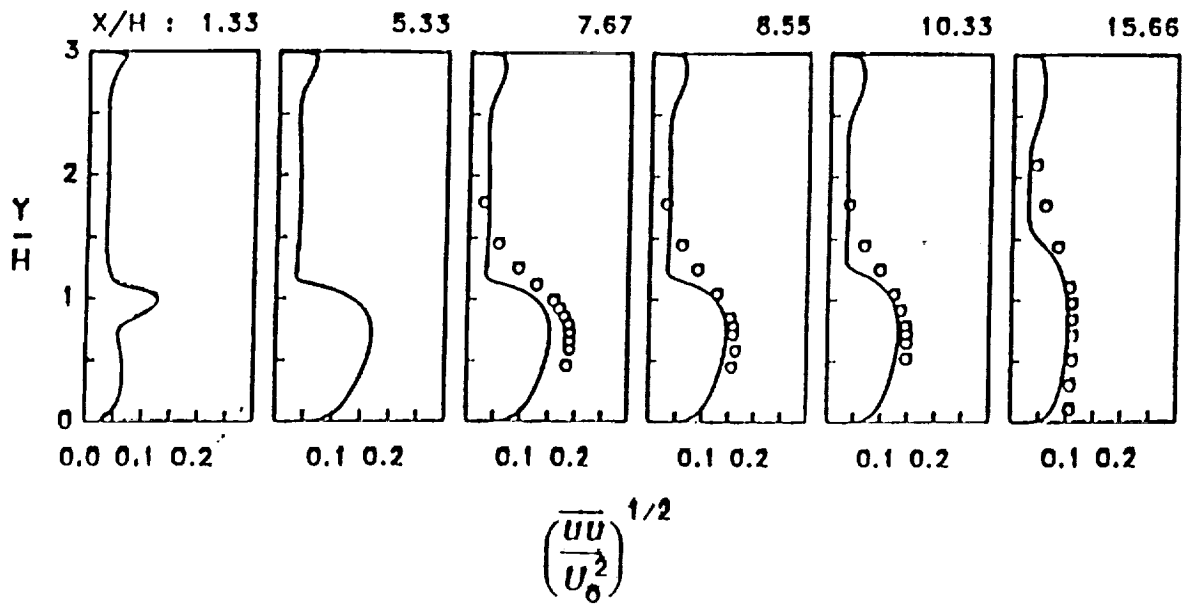
**(b) Dimensionless mean velocity profile**

(— Computations with Isotropic eddy viscosity;  
 o Experiments of Kim *et al*, 1980; Eaton & Johnston, 1981)

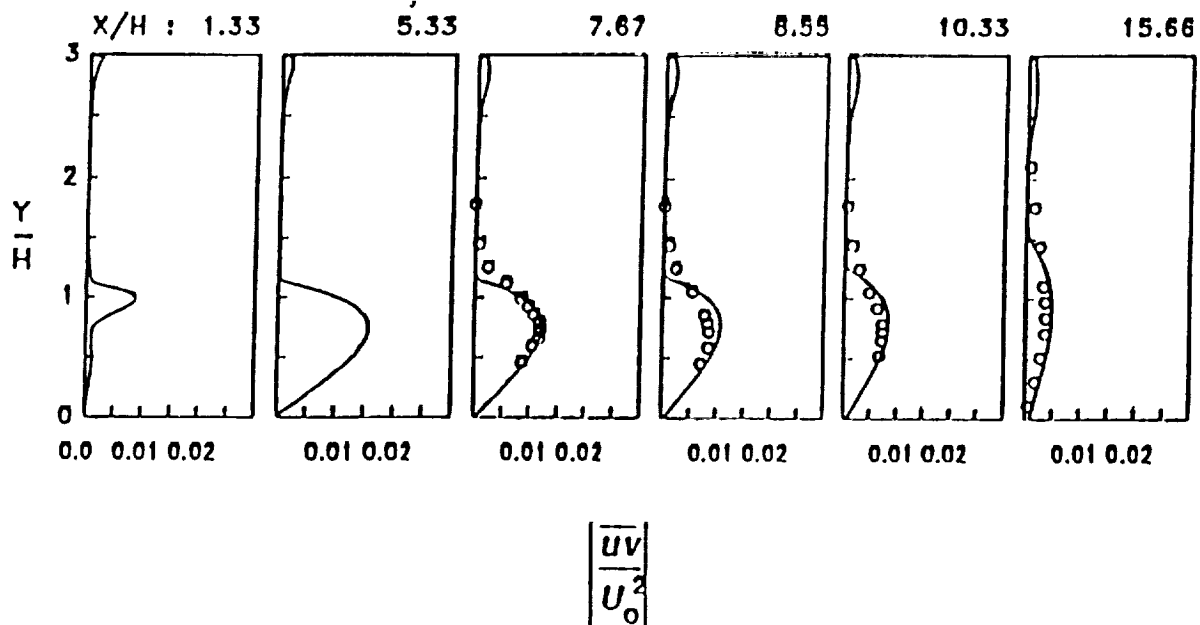
Computed mean flowfield for the new RNG  $K-\epsilon$  model  
 [ $E = 1:3$ ;  $Re = 132,000$ ;  $200 \times 100$  mesh]

**Figure 4**

# BACKWARD-FACING STEP:



(a) Turbulence Intensity

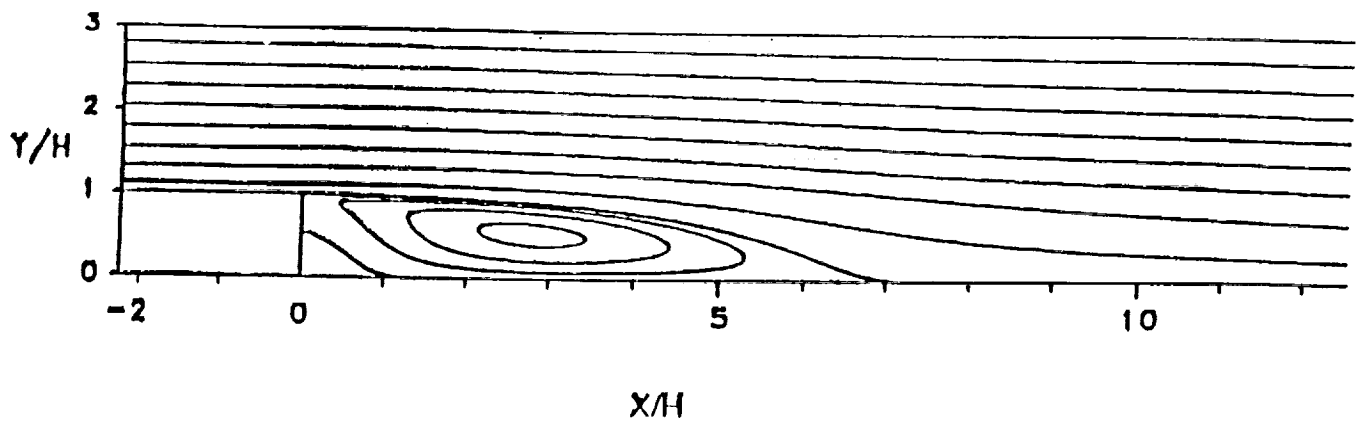


(b) Turbulence shear stress

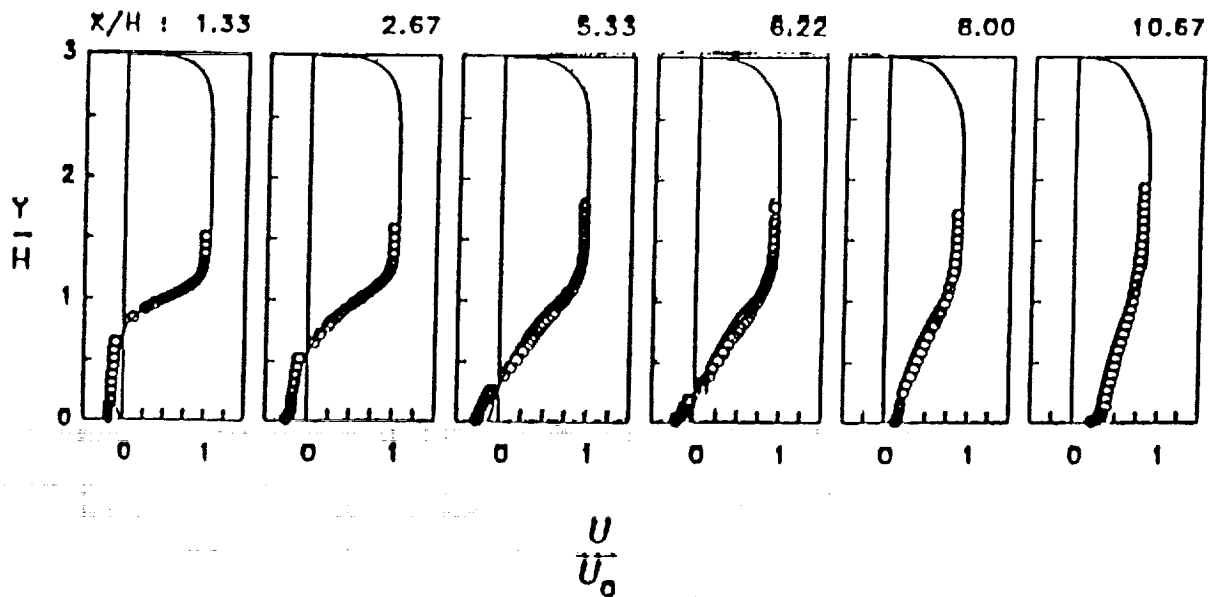
Computed turbulence stresses for the new RNG  $K-\epsilon$  model [ $E=1:3$ ;  $Re = 132,000$ ;  $200 \times 100$  mesh; — computations with isotropic eddy viscosity; o experiments of Kim *et al*, 1980; Eaton & Johnston, 1981]

Figure 5

# BACKWARD-FACING STEP:



(a) Streamlines



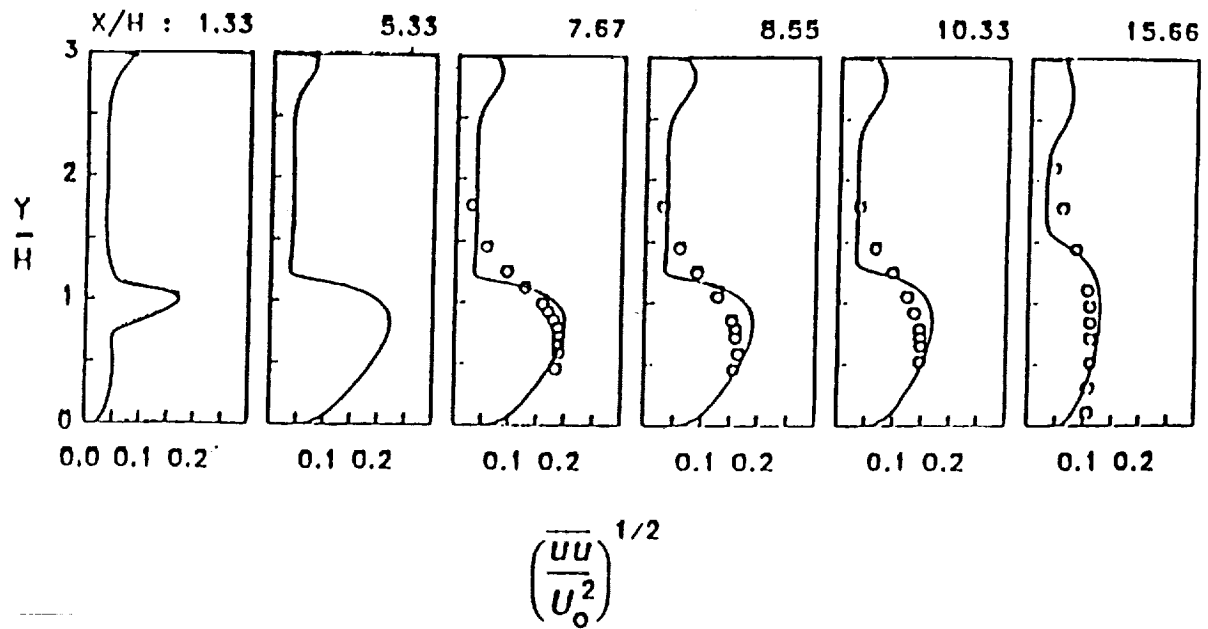
(b) Dimensionless mean velocity profile

(— Computations with anisotropic eddy viscosity;  
 o Experiments of Kim *et al*, 1980; Eaton & Johnston, 1981)

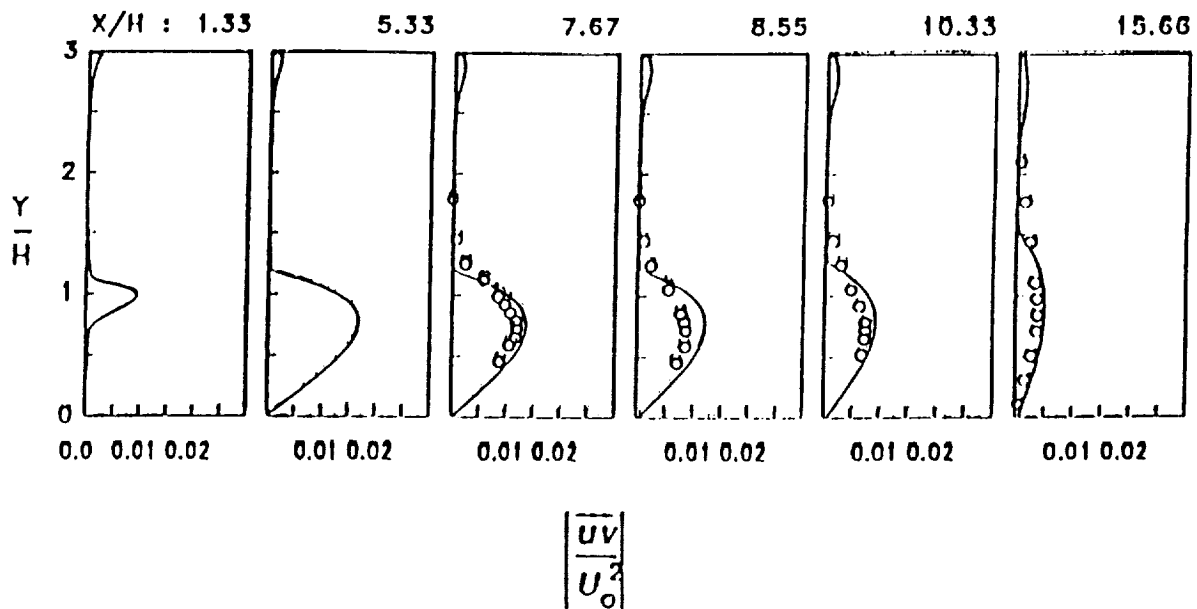
Computed mean flowfield for the new RNG  $K-\epsilon$  model  
 [ $E = 1:3$ ;  $Re = 132,000$ ;  $200 \times 100$  mesh]

Figure 6

# BACKWARD-FACING STEP:



(a) Turbulence Intensity



(b) Turbulence shear stress

Computed turbulence stresses for the new RNG K-ε model [ $E=1:3$ ;  $Re = 132,000$ ;  $200 \times 100$  mesh; — computations with anisotropic eddy viscosity; o experiments of Kim *et al*, 1980; Eaton & Johnston, 1981]

Figure 7





THE PRESENT STATE OF USING RDT IN  
TURBULENCE MODELLING OF UNSTEADY FLOWS

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## **OUTLINE**

- 1. TURBULENCE RESPONSE TO OSCILLATORY DISTURBANCE IN THE BASIC (MEAN) FLOW**
- 2. QUASI-STEADY APPROACH TO TURBULENCE MODELLING**
- 3. RAPID-DISTORTION THEORY**
- 4. OSCILLATORY TURBULENT BOUNDARY LAYER**
- 5. OPEN QUESTIONS, ONGOING ACTIVITIES**

THE DISSIPATION TRANSPORT EQUATION SHOULD NOT BE BASED ON LOCAL-EQUILIBRIUM HYPOTHESIS BUT ON RAPID-DISTORTION HYPOTHESIS. NO SUCH DIFFERENTIAL EQUATION IS CURRENTLY AVAILABLE. BUT HUNT ET AL. 1986 AND NEWLEY (1985), BASED ON RAPID-DISTORTION ARGUMENTS, PROPOSED AN ALGEBRAIC EXPRESSION FOR THE DISSIPATION LENGTH SCALE:

$$1/l_e = (1/0.09\delta + 1/\kappa y) + \beta \left( \frac{\partial U / \partial z}{k^{1/2}} - \left\langle \frac{\partial U / \partial z}{k^{1/2}} \right\rangle \right) + 1/L_0,$$

WHERE  $\langle \rangle$  DENOTES A STREAMWISE AVERAGE. THE FIRST PART OF THIS EQUATION RELATES  $L_e$  TO THE MIXING LENGTH VIA A SMOOTHED VERSION OF THE NORMAL RAMP FUNCTION, WHEREAS THE SECOND PART REPRESENTS THE INFLUENCE OF THE MEAN-STRAIN-RATE DEVELOPMENT ON  $L_e$  AND THE THIRD ACCOUNTS FOR ANY IMPOSED LENGTH SCALE.

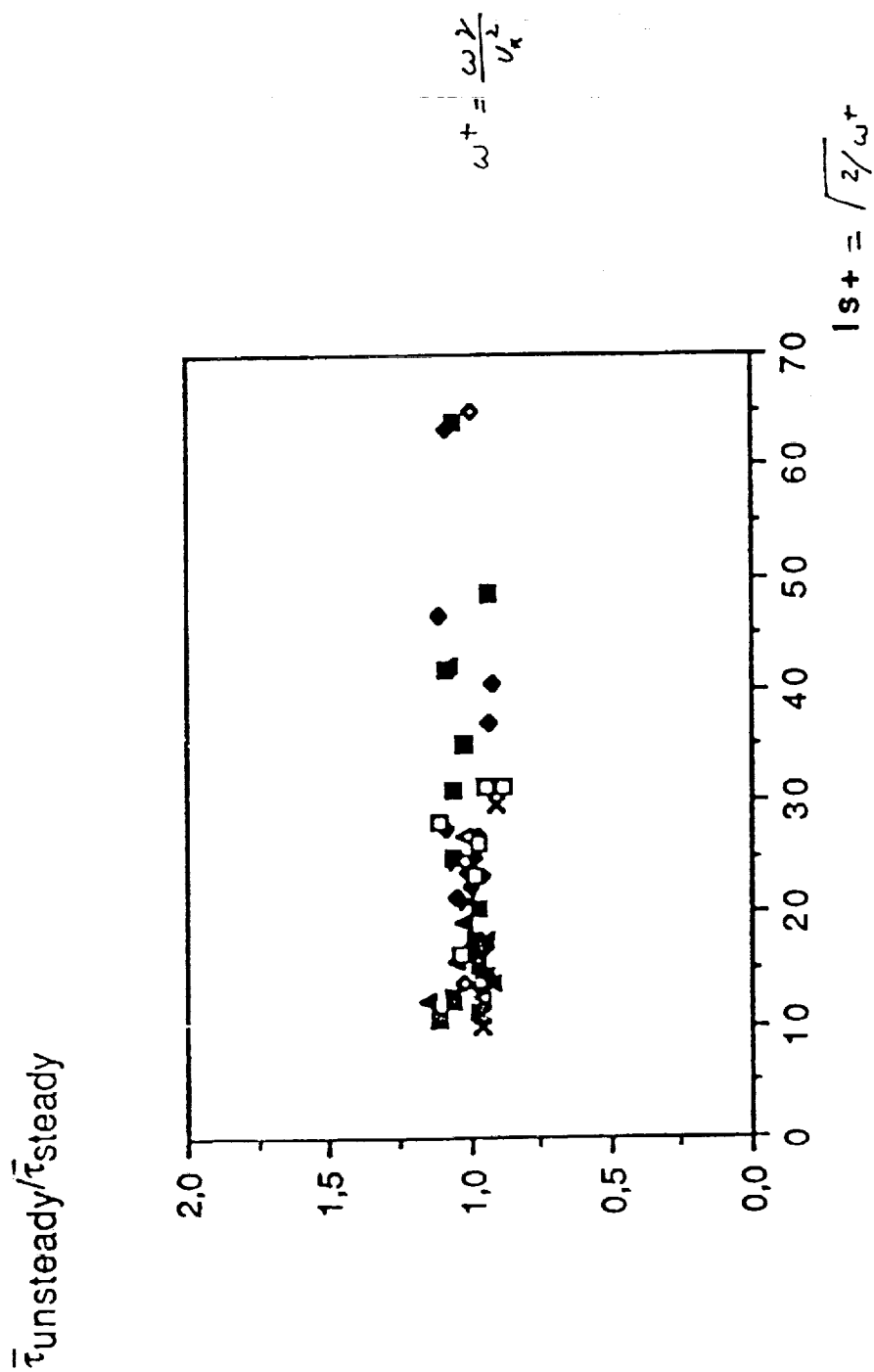
## TURBULENCE RESPONSE TO IMPOSED OSCILLATORY DISTURBANCES

\* SEVERAL EXPERIMENTS ON TURBULENT BOUNDARY LAYERS,  
CHANNEL , OR PIPE FLOW SUBJECTED PERIODIC  
DISTURBANCES:

ACHARYA & REYNOLDS 1975  
BRERETON, REYNOLDS & JAYARAMAN 1990,  
BINDER ET AL. 1981, 1991,  
COUSTEIX & HOUEVILLE 1977, 1985  
HANARATY ET AL. 1986, 1988  
KARLSON 1959, RAMAPRIAN & TU 1983  
REYNOLDS ET AL. 1982, 1987  
RONNEBERGER ET AL. 1977, 1985

**\* THESE EXPERIMENTS INDICATE THAT:**

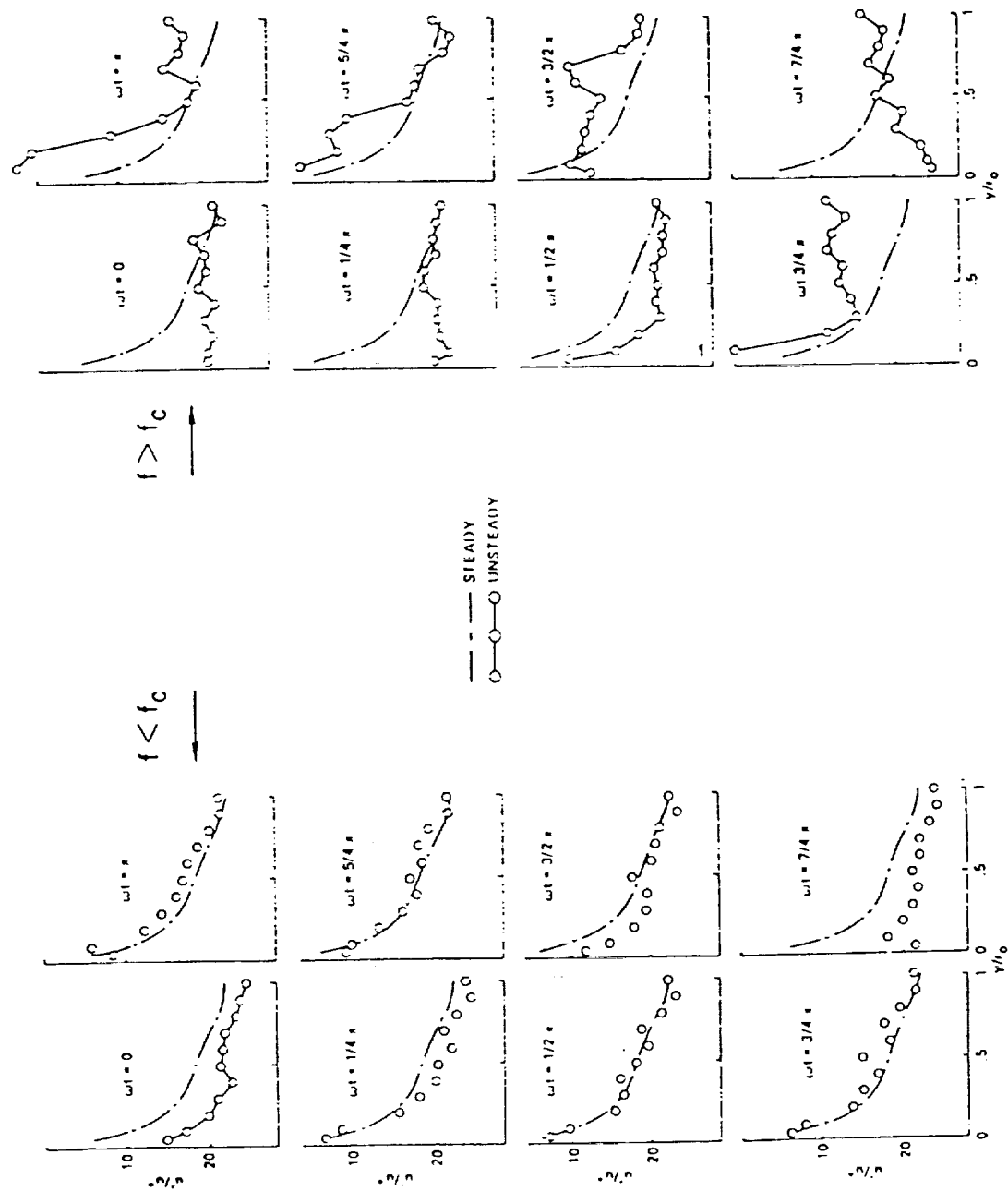
- 1. TIME-AVERAGED QUANTITIES ARE PRACTICALLY NOT INFLUENCED BY UNSTEADINESS**
- 2. OSCILLATING QUANTITIES BEHAVE EITHER IN A QUASI-STEADY MANNER OR IN A TRULY UNSTEADY MANNER, DEPENDING ON THE FREQUENCY OF IMPOSED OSCILLATIONS.**

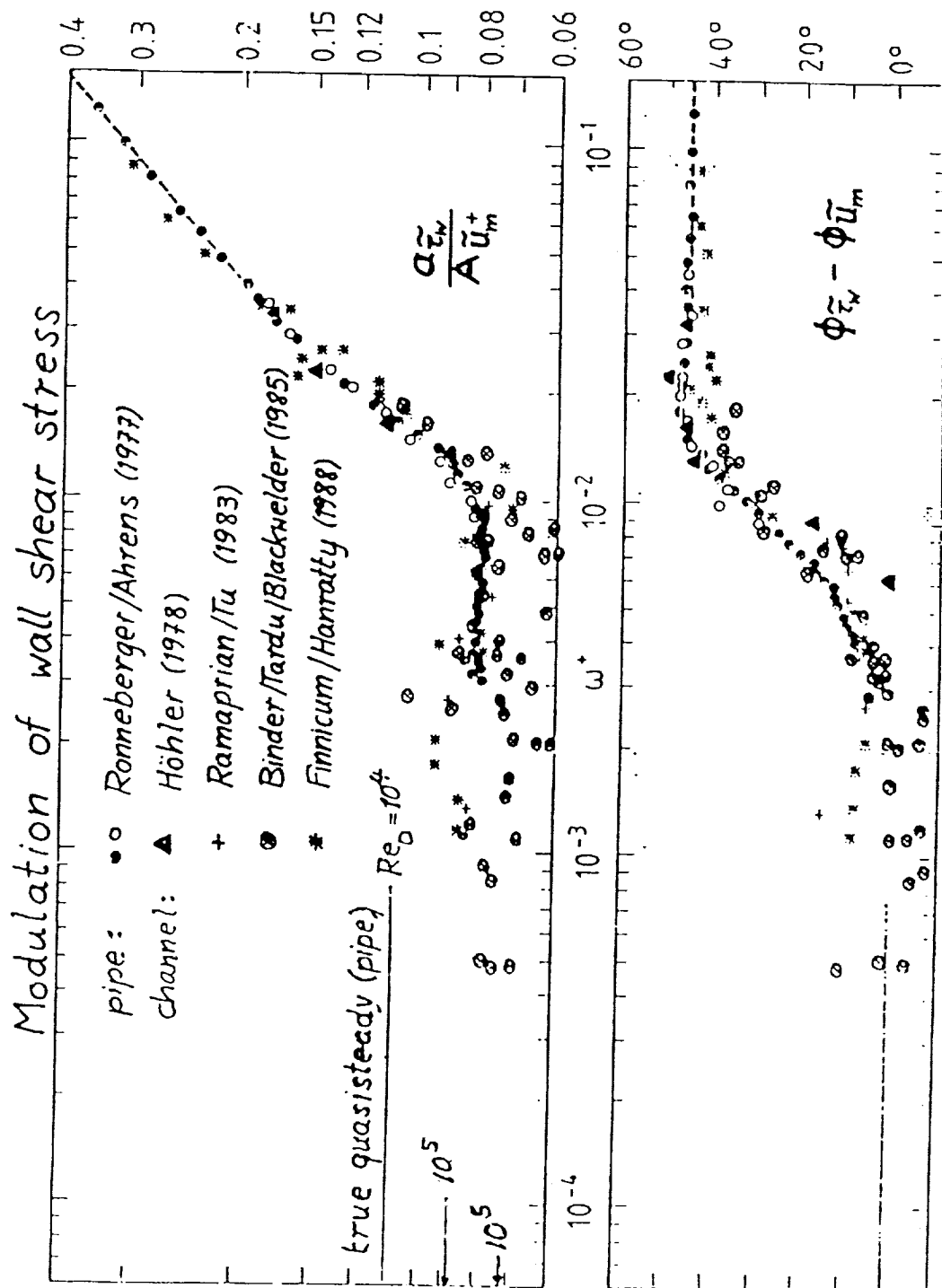


Ratio of unsteady to steady time mean wall shear stress.

(Tradu et al. 1991)

# Effect of Frequency on Turbulence Intensity - From Mizushima et al. (1973)





(Ronneberger & Binder 1991)

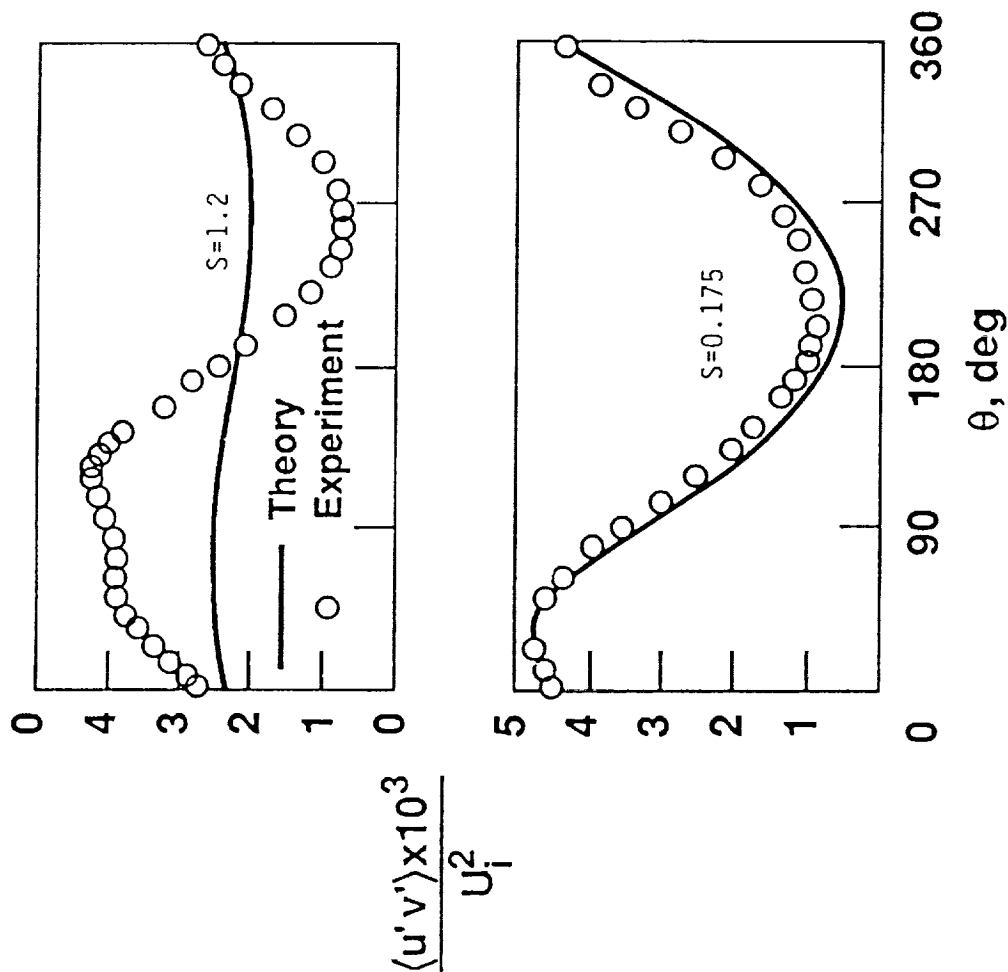


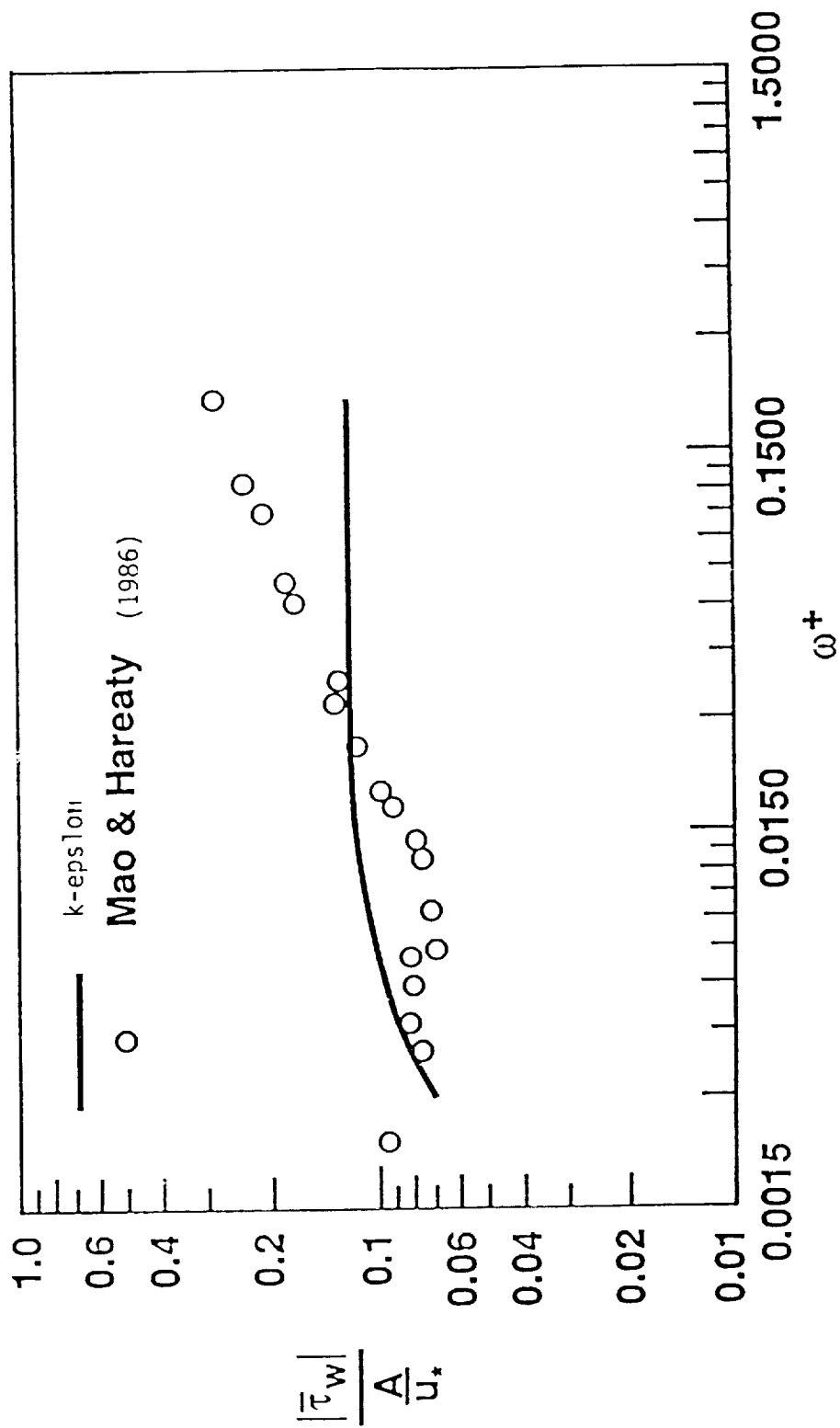
## QUASI-STEADY APPROACH TO MODELLING

EXTENDS STEADY-FLOW MODELS TO THE UNSTEADY CASE BY ACCOUNTING FOR ONLY THE UNSTEADY DERIVATIVES IN THE TRANSPORT EQUATIONS.

1. STANDARD  $K-\epsilon$  MODEL (E.G. COUSTEIX ET AL. 1981, TU & RAMAPRIAN 1983, MOBARK & MANKBADI 1989)
2. LOW-REYNOLDS NUMBER  $K-\epsilon$  MODEL (E.G. HANJALIC & STOSIC 1983, JUSTESEN & SPALART 1990, IBRAHIM & AHN 1991, KIM 1991)
3. REYNOLDS STRESS MODEL (KEBDE, LAUNDER & YOUNIS 1985, HA MINH ET AL. 1989)

# The Calculated Cycle-Variations in the Reynolds Shear Stress at $Re = 50,000$ in Comparison with Tu and Ramaprian's (1983) Data for a Pipe Flow at $y/R = 0.035$





## QUASI-STEADY MODELS ARE:

- \* SUCCESSFUL AT THE QUASI-STEADY DOMAIN (LOW-FREQUENCY, OSCILLATIONS).
- \* FAIL AT THE HIGHLY UNSTEADY DOMAIN (HIGH-FREQUENCY OSCILLATIONS).

\* REYNOLDS-STRESS TRANSPORT MODELS ARE VALID WHEN LOCAL EQUILIBRIUM HOLDS ( $T_d > T_L$ ). MAY NOT RESPOND PROPERLY WHEN THEY ARE USED TO PREDICT MORE COMPLEX STRAIN FIELDS.

\* EDDY-VISCOSITY MODELS MAY NOT BE SUITABLE FOR UNSTEADY FLOWS SINCE THEY ASSUME THE ALIGNMENT OF REYNOLDS STRESSES WITH THEIR GENERATING RATES OF STRAIN (NO MEMORY EFFECTS).

\* RAPID-DISTORTION THEORY IS ONLY VALID ON THE LIMIT  $T_d < T_L$ , BUT IT IS NOT RESTRICTED BY LOCAL APPROXIMATIONS AND DESCRIBES THE CONTINUUM INFLUENCE OF STRAIN HISTORY.

SINCE IT DOES NOT ASSUME LOCAL TIME DEPENDENCE, IT MAY BE PARTICULARLY SUITED FOR OSCILLATING TURBULENT FLOWS WHERE AT HIGHER FREQUENCIES THE OSCILLATION PERIOD IS OF SIMILAR MAGNITUDE AS THE LARGE-EDDY TIME SCALE.

## **RAPID-DISTORTION THEORY (RDT)**

**\* BASIC DEVELOPMENT BY BACHELOR & PROUDMAN (1954),  
TOWNSEND (1970, 1976).**

**\* RECENT APPLICATIONS BY:**

**BALAKUMAR & WIDNALL (1986)  
GARSHORE ET AL. (1983)  
GATSKI (1987)  
GOLDSTEIN & ATTASI (1976)  
GOLDSTEIN (1979)  
GOLDSTEIN & DURBIN (1980)  
MANSOUR, SHIH & REYNOLDS (1991)  
STRECHER & BRITTER (1985)**

## BASIC RDT

- \* TURBULENCE MOTION ARE MADE UP OF MANY INDIVIDUAL STRUCTURES. FOURIER SERIES IS DEFINED FOR THE VELOCITY FLUCTUATION AND SUBSTITUTED IN THE EQUATIONS OF MOTION.
- \* THE RESULTING EQUATIONS ARE LINEARIZED ASSUMING THAT THE TURBULENCE IS WEAK.
- \* THE TURBULENT-REYNOLDS-NUMBER IS CONSIDERED TO BE HIGH AND THE FLOW IS LOCALLY HOMOGENEOUS.



\* IT FOLLOWS THAT THE VARIATION OF TIME OF A SINGLE COMPONENT OF THE FLUCTUATION FIELD IS DESCRIBED BY AN EQUATION FOR ITS AMPLITUDE  $a_i(k)$

$$\frac{da_i(k)}{dt} = -\frac{\partial U_i}{\partial x_i} a_i(k) + 2 \frac{k_i k_l}{k^2} \frac{\partial U_l}{\partial x_m} a_m(k)$$

AND BY AN EQUATION FOR THE RATE OF CHANGE OF THE WAVENUMBER  $k$  OF THE COMPONENT

$$\frac{dk_i}{dt} = -\frac{\partial U_l}{\partial x_i} k_l$$

WHICH DESCRIBES THE ROTATION AND DISTORTION OF THE VELOCITY PATTERN BY MEAN-FLOW GRADIENT.

\* TO OBTAIN DEFINITE PREDICTIONS IT IS NECESSARY TO ASSUME THE INITIAL STATE OF THE TURBULENCE BEFORE DISTORTION. USUALLY AN INITIALLY ISOTROPIC TURBULENCE IS ASSUMED.

\* BUT MAXEY (1982) HAVE SHOWN THAT ASSUMING THE FLOW TO BE INITIAL AXISYMMETRIC INSTEAD OF ISOTROPIC PRODUCES RESULTS IN BETTER AGREEMENT WITH EXPERIMENTAL DATA FOR SHEAR FLOWS.

\* GIVEN THE INITIAL SPECTRUM AND THE APPLIED SHEAR, THE SOLUTION OF THE RAPID-DISTORTION EQUATIONS IS OBTAINED IN TERMS OF THE DISTORTION STRAIN PARAMETER, WHICH CHARACTERIZES THE EFFECT OF THE MEAN SHEAR ON THE TURBULENCE STRUCTURE AND IS DEFINED BY

$$\alpha = \int_0^t dt' \frac{\partial U_1}{\partial z},$$

\* MATHIEU (1971) AND HUNT & MAXEY (1978) HAVE POINTED OUT THAT THE EFFECTIVE STRAIN IS LIMITED BY THE FINITE DISTORTION TIME. THUS AT LARGER TIMES THERE WILL ALSO BE AN EQUILIBRIUM VALUE OF STRAIN

$$\alpha = T_D \frac{\partial U_1}{\partial z},$$

WHERE  $T_D$  IS SOME EDDY TIME SCALE.

**\* THE TWO LIMITS OF RAPIDLY EVOLVING  
FLOWS AND EQUILIBRIUM FLOWS ARE  
RELATED THROUGH**

$$\frac{\partial \alpha_{\text{eff}}}{\partial t} = \frac{\partial U_1}{\partial x_3} - \frac{\alpha_{\text{eff}}}{T_D}.$$

RDT AS AN AID IN MODELLING:

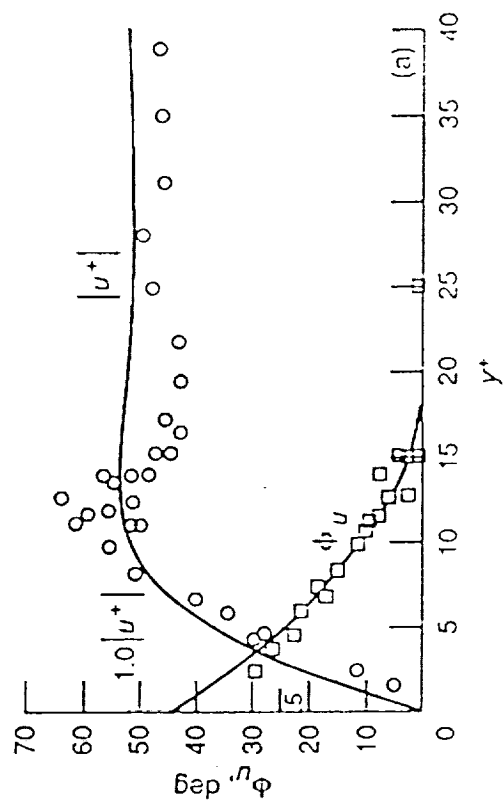
WORK IN TERMS OF STRUCTURAL  
PARAMETERS  $a_{ij}$ . MATHIEU (1971) PROPOSED  
THAT:

$\dot{u}\dot{v}/k = F(\text{ACCUMULATED STRAIN RATE}$   
 $\text{ALONG A STREAMLINE})$

**SIMPLE MODEL FOR OSCILLATORY TURBULENT  
BOUNDARY LAYERS: (MANKBADI & LIU, 1991, JFM)**

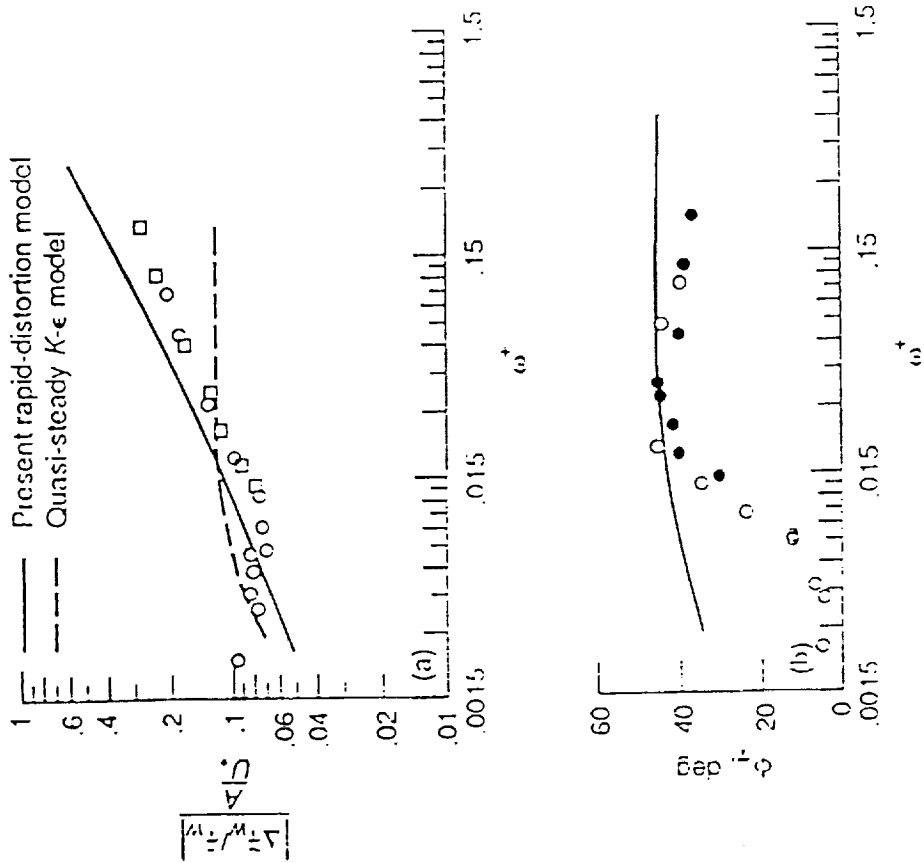
**SIMPLIFICATIONS:**

- 1. THE OSCILLATIONS ARE OF SMALL AMPLITUDE AND REMAINS HARMONIC SUCH THAT THE GOVERNING EQUATION CAN BE LINEARIZED.**
- 2. THE MEAN FLOW IS ASSUMED TO BE GIVEN AND NOT INFLUENCED BY THE UNSTEADINESS.**
- 3. THE DISSIPATION IS ESTIMATED BASED ON LOCAL EQUILIBRIUM HYPOTHESIS.**
- 4. NO DIFFUSION TERM IN THE EFFECTIVE STRAIN EQUATION.**

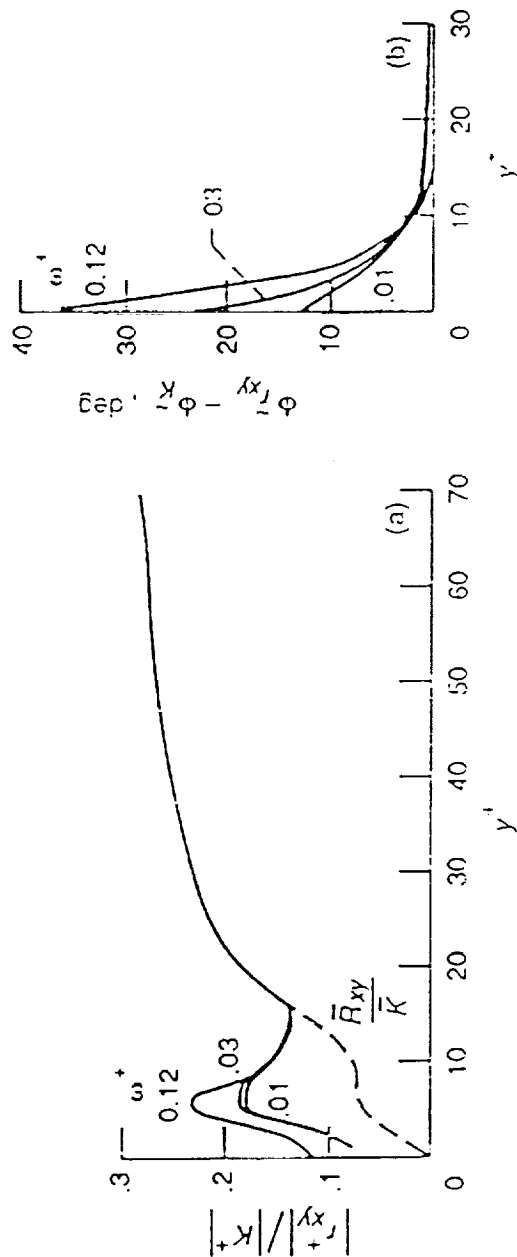


Computed phase-averaged streamwise velocity compared with data of Binder & Kueny (1981), in terms of magnitude of  $u'$  and phase angle  $\phi_u$  relative to their respective imposed values.  $w' = 0.0637 \left( \epsilon_n^+ = 5.6 \right)$ .





Computed phase-averaged wall shear stress compared with data of Mao & Hanratty (1986), O, and Ramaprian & Tu (1983), □. (a) Amplitude  $\left( |\bar{\tau}_w| / \bar{\tau}_w \right) / (\rho u_*^2)$ . (b) phase angle.



Ratio of modulated Reynolds shear stress to modulated turbulent kinetic energy. (a) Amplitude  $|r'_{xy}| / |K'|$ . (b) Phase difference.

## ONGOING WORK

THE EFFECTIVE STRAIN EQUATION FOR INHOMOGENEOUS FLOW SHOULD BE MODIFIED TO ALLOW FOR THE TRANSPORT OF EFFECTIVE STRAIN BY THE ADVECTION OF TURBULENT EDDIES BY THE TURBULENCE ITSELF (TOWNSEND 1970, MAXEY 1982).

$$\frac{\partial \alpha_{\text{eff}}}{\partial t} = \frac{\partial u_1}{\partial x_3} + \frac{\partial}{\partial x_3} \left( E_{13} \frac{\partial \alpha_{\text{eff}}}{\partial x_3} \right) - \frac{\alpha_{\text{eff}}}{T_D}.$$



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THE ROLE OF EXPERIMENTS

IN

TURBULENCE MODELLING

by

William K. George  
Turbulence Research Laboratory  
State University of New York at Buffalo

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### RECENT REFERENCES

1. W. K. George "Governing Equations, Experiments, and the Experimentalist"  
Experimental Fluid Thermal Science - 1990
2. W. K. George "Designing Experiments to Test Closure Hypothesis"  
& D. B. Taulbee to appear Experimental Fluid Thermal Science - 1992  
(Also Proceedings Symposium on Turbulence Modelling  
Dubrovnik 1990).

### Some Experimental Dilemmas -- just a few of many

- . the Round Jet - what is its growth rate?
- . the Turbulent Boundary Layer
- . the Plane Wake
- . the Dissipation

### THE ROUND JET

$$\delta_{1/2} \approx 0.095 X$$

not 0.085 X

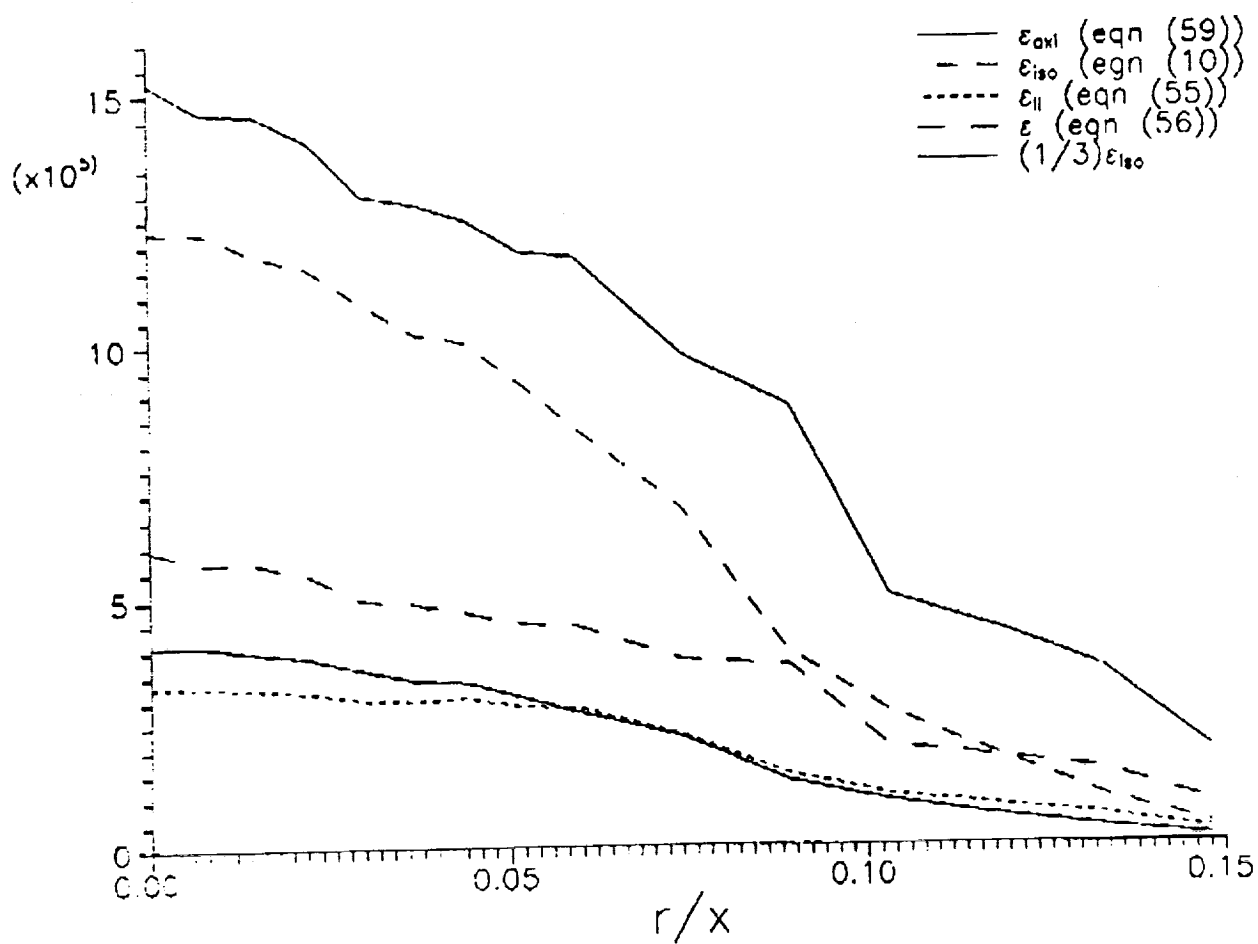
Illustrates one of the biggest problems for modeller - Who to believe?

- . Problem for experimentalists is
  - How to overturn old results?
    - unpublished since disagree with earlier results
    - unaccepted since unpublished
    - unused since unknown
- . Problem for everyone - Isn't there better way
  - to get results out
  - to purge old results

### LOCAL ISOTROPY ???

- Local Axisymmetry?
- Equi-partition of Dissipation?

# Axisymmetric Jet ( $Re_D = 10^5$ )





L. W. B. Browne, R. A. Antonia and D. A. Shah

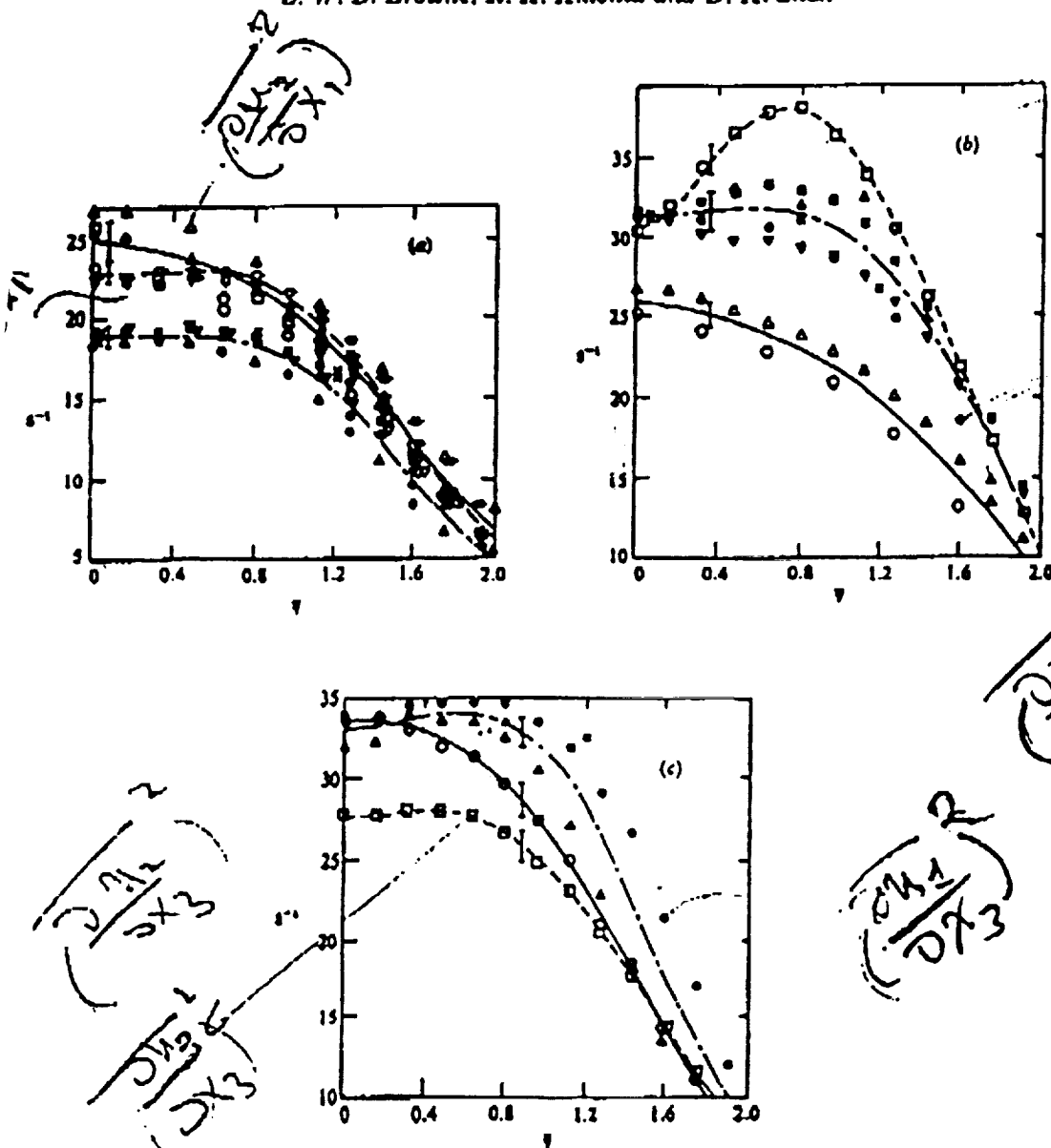


FIGURE 4. Distribution across the wake of  $(\delta p / \delta y)^2$ , the r.m.s. of derivatives of velocity fluctuations. Best fit lines: ---,  $\beta = u$ ; —,  $v$ ; ---,  $w$ . (a)  $\gamma = z$ :  $\bullet$ ,  $\beta = u$  (figure 1a);  $\blacktriangle$ ,  $\beta = u$  (figure 1a, repeated);  $\nabla$ ,  $\beta = u$  (figure 1b);  $\blacksquare$ ,  $\beta = u$  (figure 1c);  $\circ$ ,  $\beta = v$  (figure 1a, lower X-wire);  $\triangle$ ,  $\beta = v$  (figure 1a, upper X-wire);  $\square$ ,  $\beta = v$  (figure 1d, X-wire at  $z = 0$ );  $\nabla$ ,  $\beta = v$  (figure 1d, X-wire at  $z = 1.6$  mm);  $\circ$ ,  $\beta = w$  (figure 1b, lower X-wire);  $\square$ ,  $\beta = w$  (figure 1b, upper X-wire);  $\phi$ ,  $\beta = w$  (figure 1c, X-wire at  $z = 0$ );  $\circ$ ,  $\beta = w$  (figure 1c, X-wire at  $z = 1.6$  mm).  $\times$ ,  $\beta = u$ , using two single wires. (b)  $\gamma = y$ :  $\bullet$ ,  $\beta = u$  (figure 1a);  $\blacktriangle$ ,  $\beta = u$  (figure 1a, repeated);  $\nabla$ ,  $\beta = u$  (figure 1a, repeated);  $\blacksquare$ ,  $\beta = u$  (figure 1b);  $\circ$ ,  $\beta = v$  (figure 1a);  $\triangle$ ,  $\beta = v$  (figure 1a, repeated);  $\square$ ,  $\beta = w$  (figure 1b);  $\star$ ,  $\beta = u$ , using two single wires. (c)  $\gamma = x$ :  $\bullet$ ,  $\beta = u$  (figure 1c);  $\blacktriangle$ ,  $\beta = u$  (figure 1d);  $\circ$ ,  $\beta = v$  (figure 1d);  $\square$ ,  $\beta = w$  (figure 1c);  $\star$ ,  $\beta = u$ , using two single wires.

suggest that for reasonable overs, the ratios  $K_s$  to

With the exce, are all greater th departures from, of validating (3)

The results of large and the re of the same expt differences betw correlation  $(\partial u / \partial t)$  to [9] of (4). Bec assumed the isat of the 'expected terms in (4), this, are shown in fig underestimates centreline to abo Laufer (1954), derivatives with

## THE PLANE WAKE

— there is no wake which is independent of generator

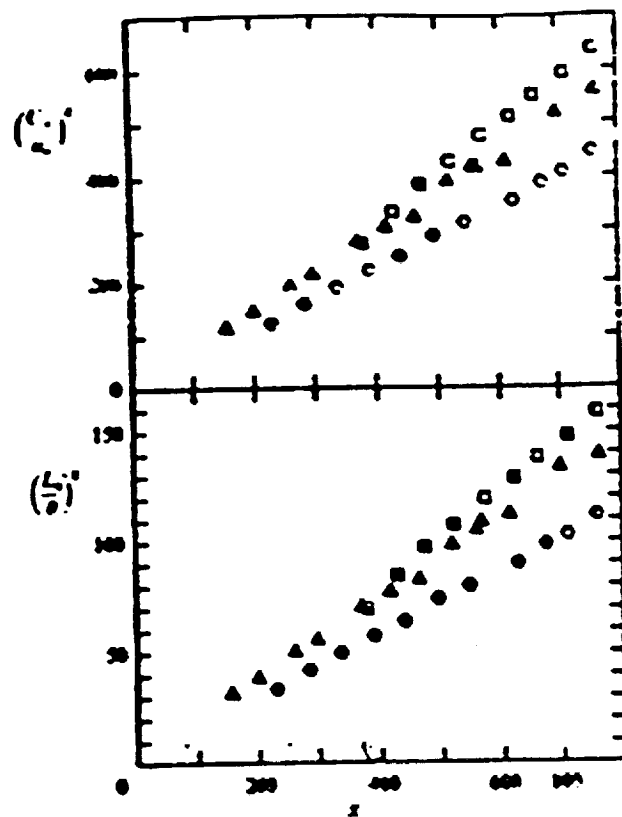


Figure 5. Variation centerline velocity deficit and half-width with distance for three wake generators;  $\square$ , airfoil;  $\Delta$ , 70% solidity screen;  $\circ$ , solid strip (from Wagnanski et al. 1986).

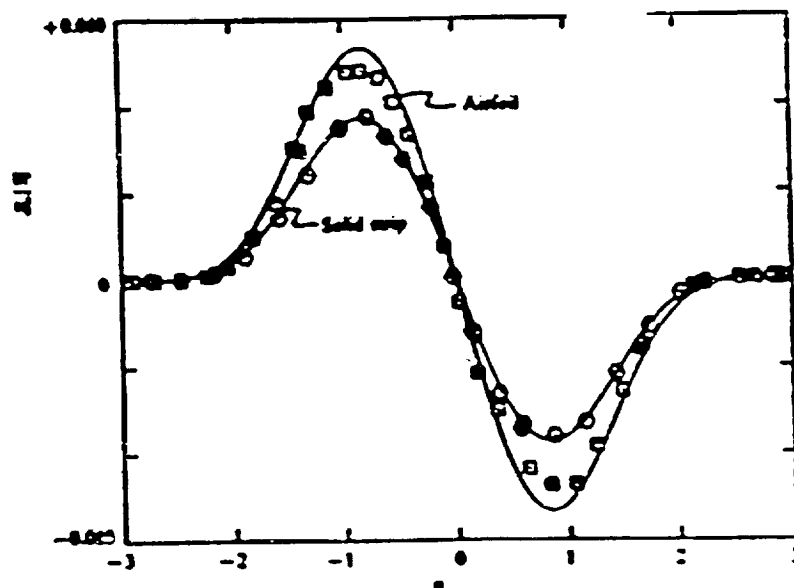


Figure 6. Reynolds stress normalized by centerline velocity deficit for the solid strip and airfoil (Wagnanski et al. 1986).

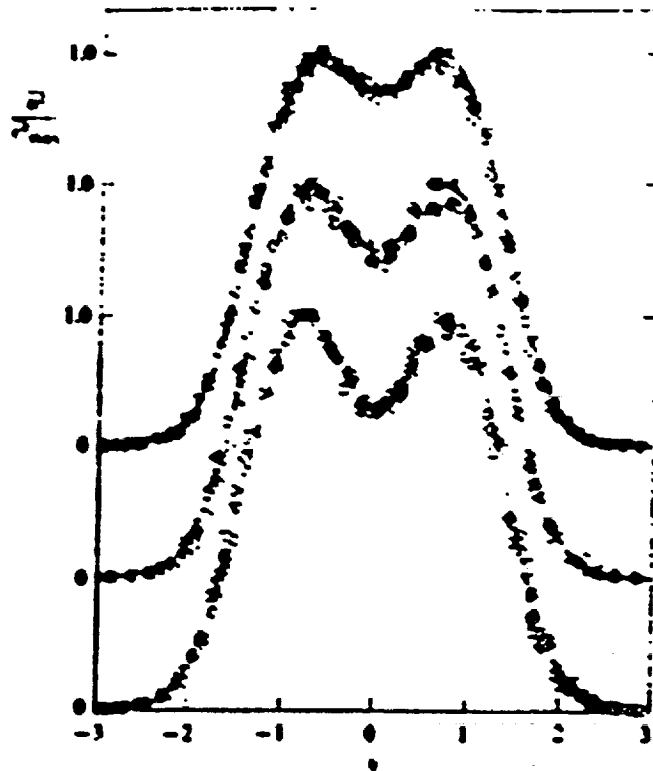


Figure 7. Normalized turbulence intensity profiles for the three generators of Figure 5 (Wynanski et al. 1986).

### THE AXISYMMETRIC WAKE: A FLOW WHICH DOES NOT EVOLVE AT CONSTANT REYNOLDS NUMBER

The axisymmetric wake presents an interesting contrast to the axisymmetric jet and plane wake flows described above in that it does not evolve at constant Reynolds number (as will be seen). As a consequence, the nature of the assumptions regarding the dissipation will be seen to predict two quite different asymptotic developments. There appears to be experimental evidence for both forms in different experiments, which raises an interesting question as to how the flow chooses one form or another. An interesting possibility is that the flow evolves from one state to another as the Reynolds number changes. These possibilities will be discussed in more detail below following the analysis.

The equations of motion describing the axisymmetric wake to first order can be shown to reduce to,

$$U_\infty \frac{\partial}{\partial x} (U - U_\infty) = - \frac{1}{r} \frac{\partial}{\partial r} r \overline{uv} \quad (78)$$

where  $U_\infty$  is the undisturbed speed of the free stream. This can be integrated across the flow to yield the integral constraint,

$$2\pi \int_0^\infty U_\infty (U - U_\infty) r dr = \pi U_\infty^2 \theta^2 = M/\rho \quad (79)$$

where  $\theta$  is defined to be the momentum thickness.

This persistent effect of initial conditions may be widespread!

Theory also predicts that spectral shape is determined by initial conditions. Note different shapes of 1" & 2" grid spectra.

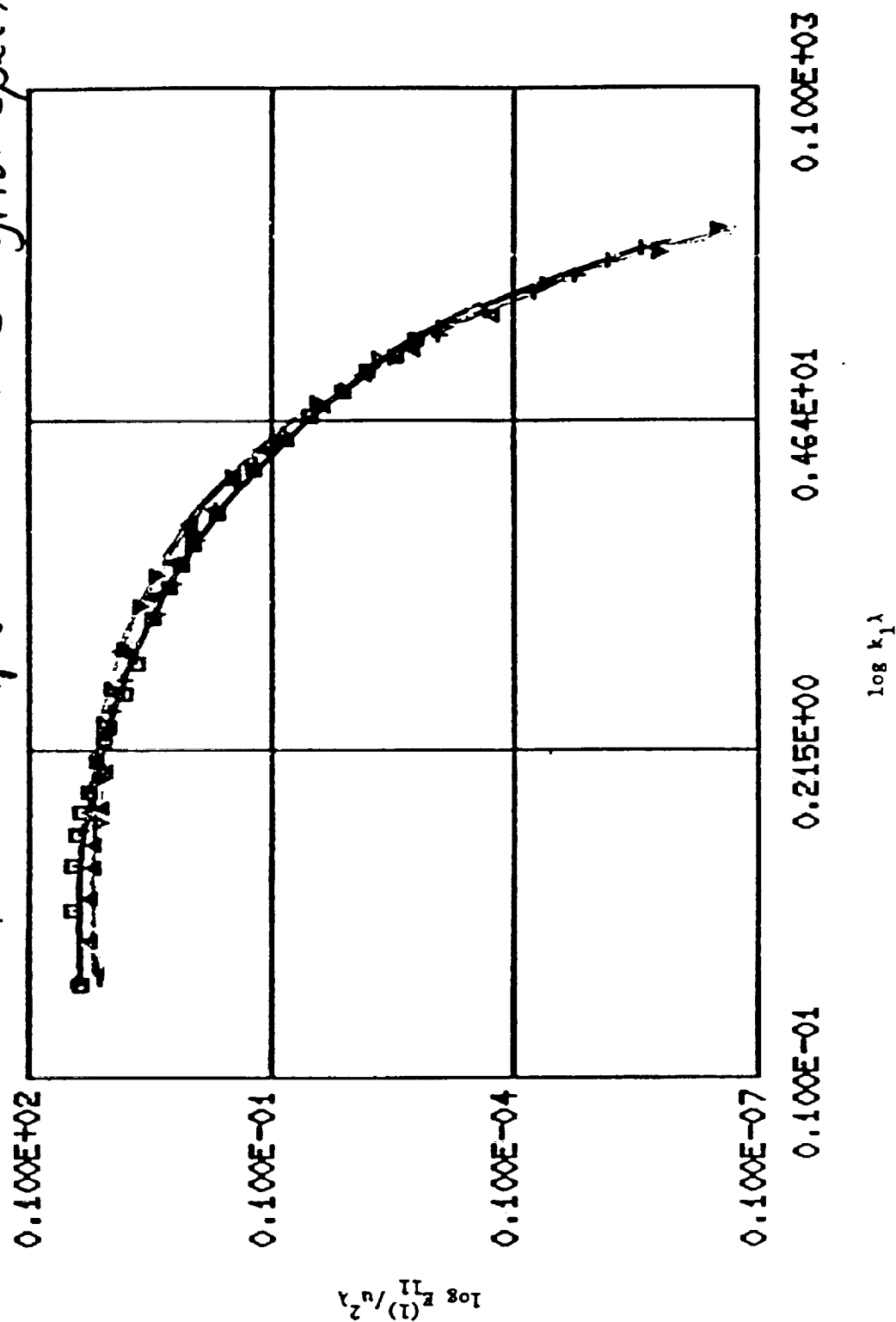


Fig. 7

The 'Well-established' Turbulent Boundary Layer

- . Is there really a log layer?
- . Should comparisons be made with these correlations?

Fig. 1.1.1 On the determination of the friction coefficient.

from KLINE et al. (1967).

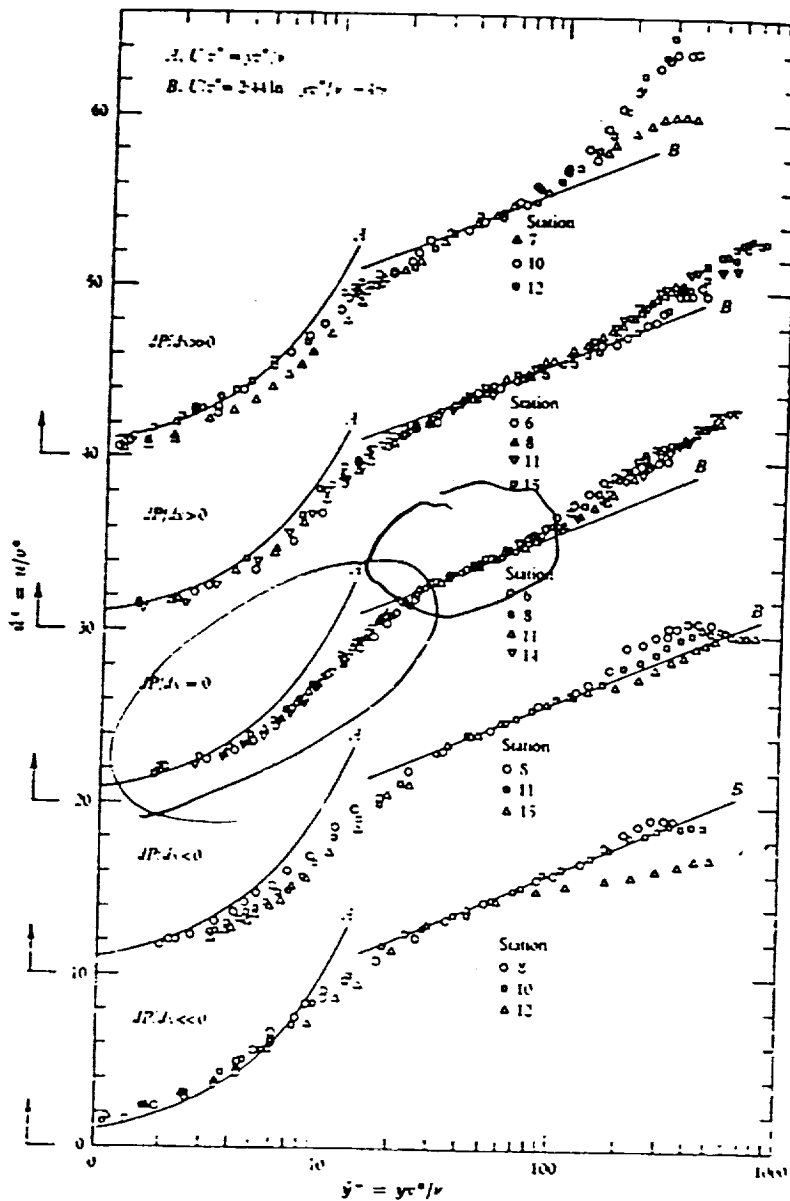


FIGURE 9a.  $u^+-y^+$  profiles using  $u^*$  (note shifted origins).

Clauser plot?

Bottom line: If <sup>78</sup> you assume log layer and use it to calculate  $\tau_w$  you get log layer — Not very comforting!



Fig. 1.1.2 On the determination of the friction coefficient:  
from KLINE et al. (1967).

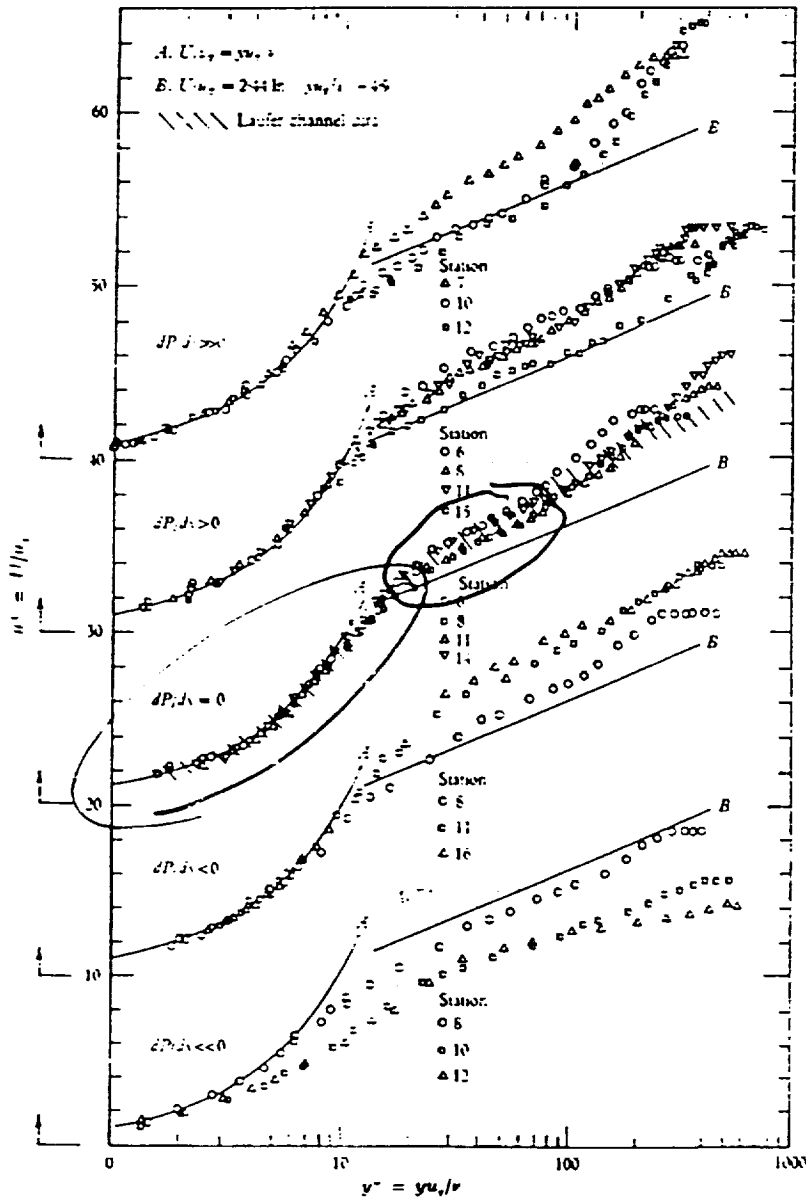


FIGURE 9b.  $u^+ - y^+$  profiles using  $u_\tau$  (note shifted origins).

Fig. 2.1.2.1.1 Velocity in *inner* variables with log law:

Purtell et al..  $R_{th} = 465, 498, 700, 1000, 1340, 1370, 1840, 2840, 3480,$   
 $4090, 5100.$

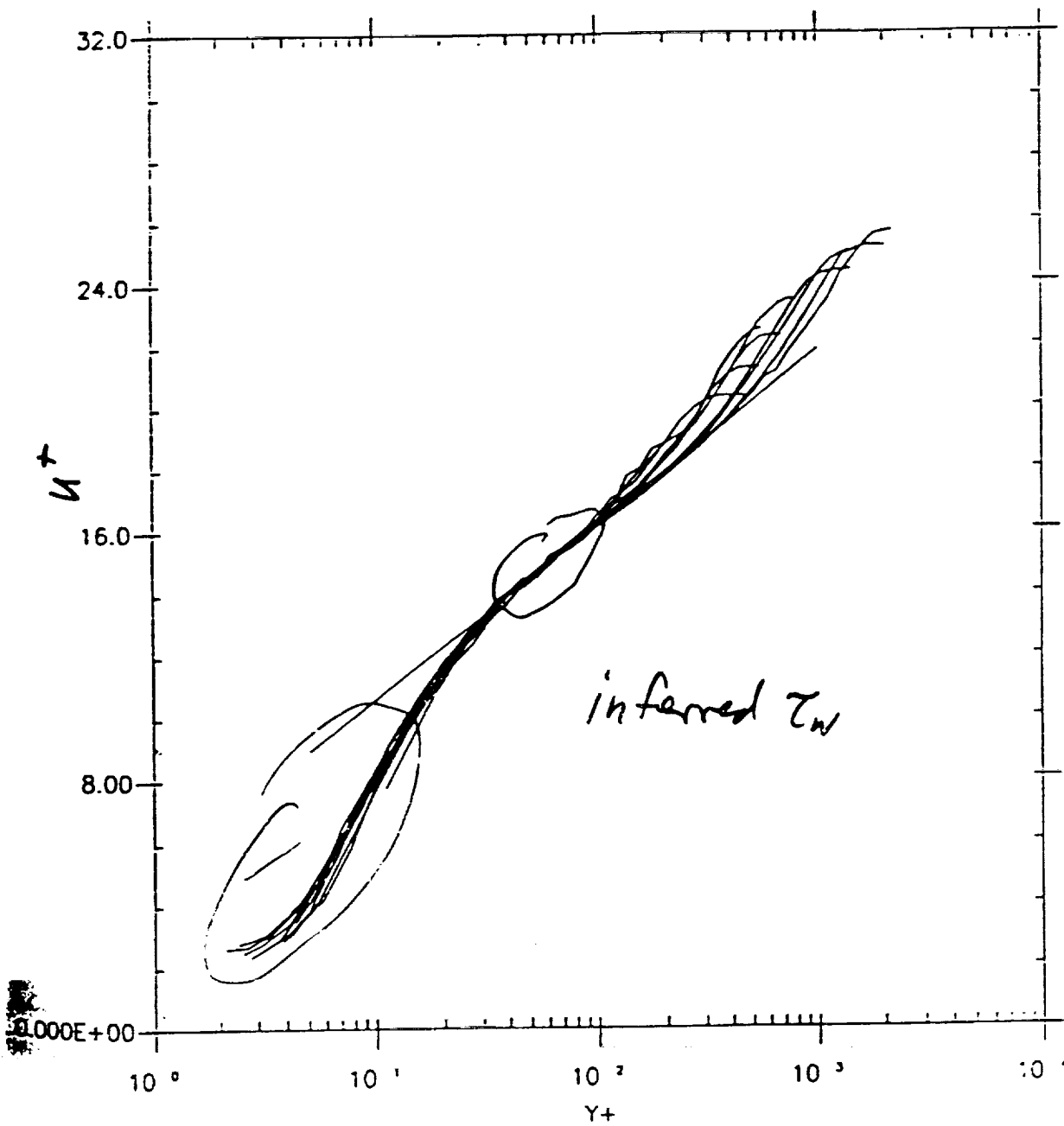


Fig. 1.4.1 Velocity profiles obtained with a wall slope determined shear stress:

Purtell et al..  $R_{th} = 465, 498, 700, 1000,$   
 $1340, 1370, 1840, 2840, 3480, 4090, 5100.$

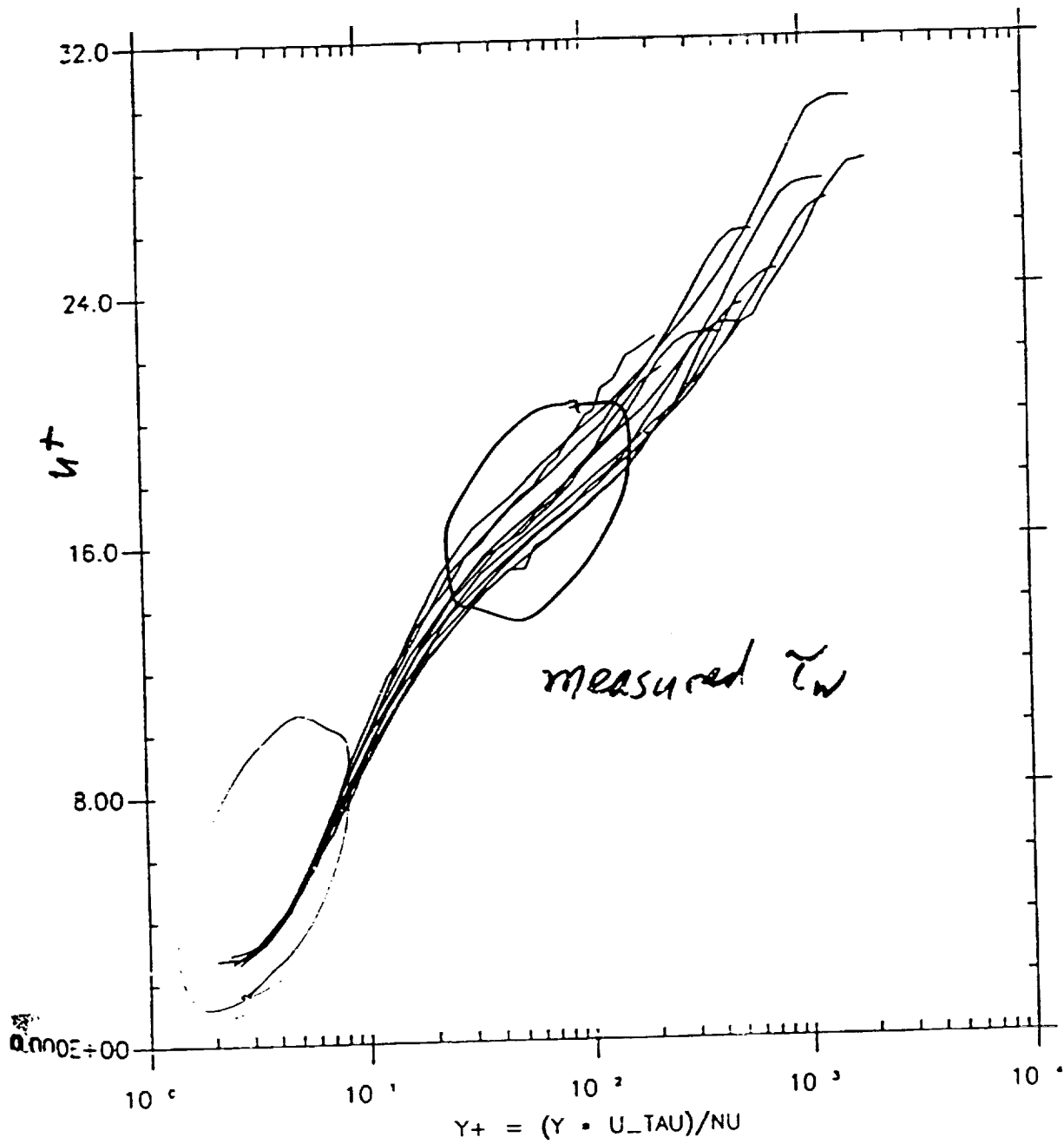


Fig. 1.2 Velocity derivative from Direct Simulation:  
from SPALART (1988)

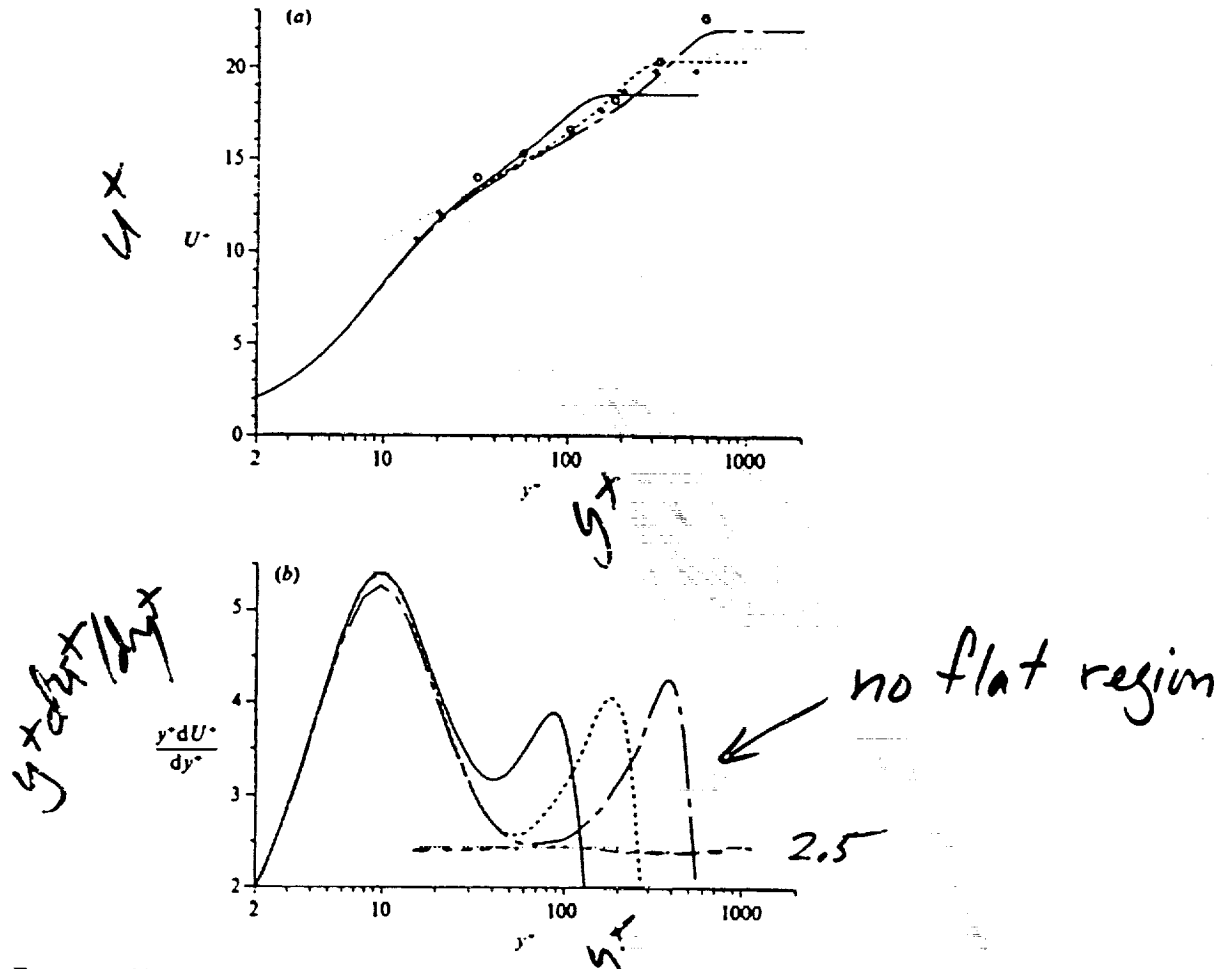


FIGURE 5. Mean-velocity profile and its derivative. —,  $R_\tau = 300$ ; ---,  $R_\tau = 670$ ; - - - -,  $R_\tau = 1410$ ; ···, log law  $U^+ = \log(y^+)/0.41 + 5$ ; +,  $R_\tau = 617$  (Erm *et al.* 1985); ○,  $R_\tau = 1368$  (Murlis *et al.* 1982). (a)  $U^+$ ; (b)  $y^+ dU^+/dy^+$ .

Fig. 3.1.2.2.11 Outer variables:

Purcell et al.,  $R_{th} = 5100$ .

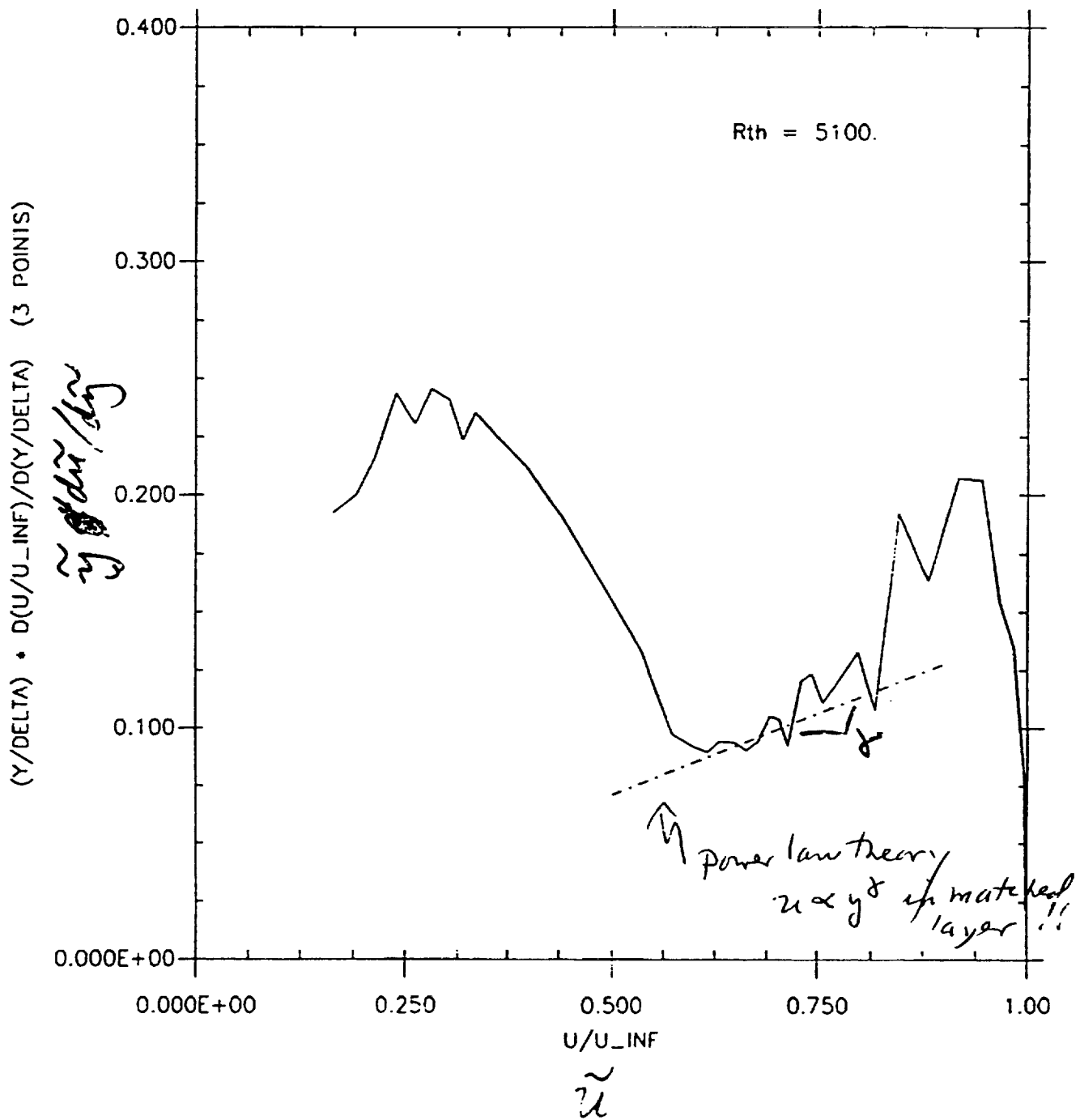


Fig. 4.4.3.1.1 Shape factor.

Smith and Walker.  $x = 15.75$  in,  $27.75$  in,  $39.75$  in,  $51.75$  in.

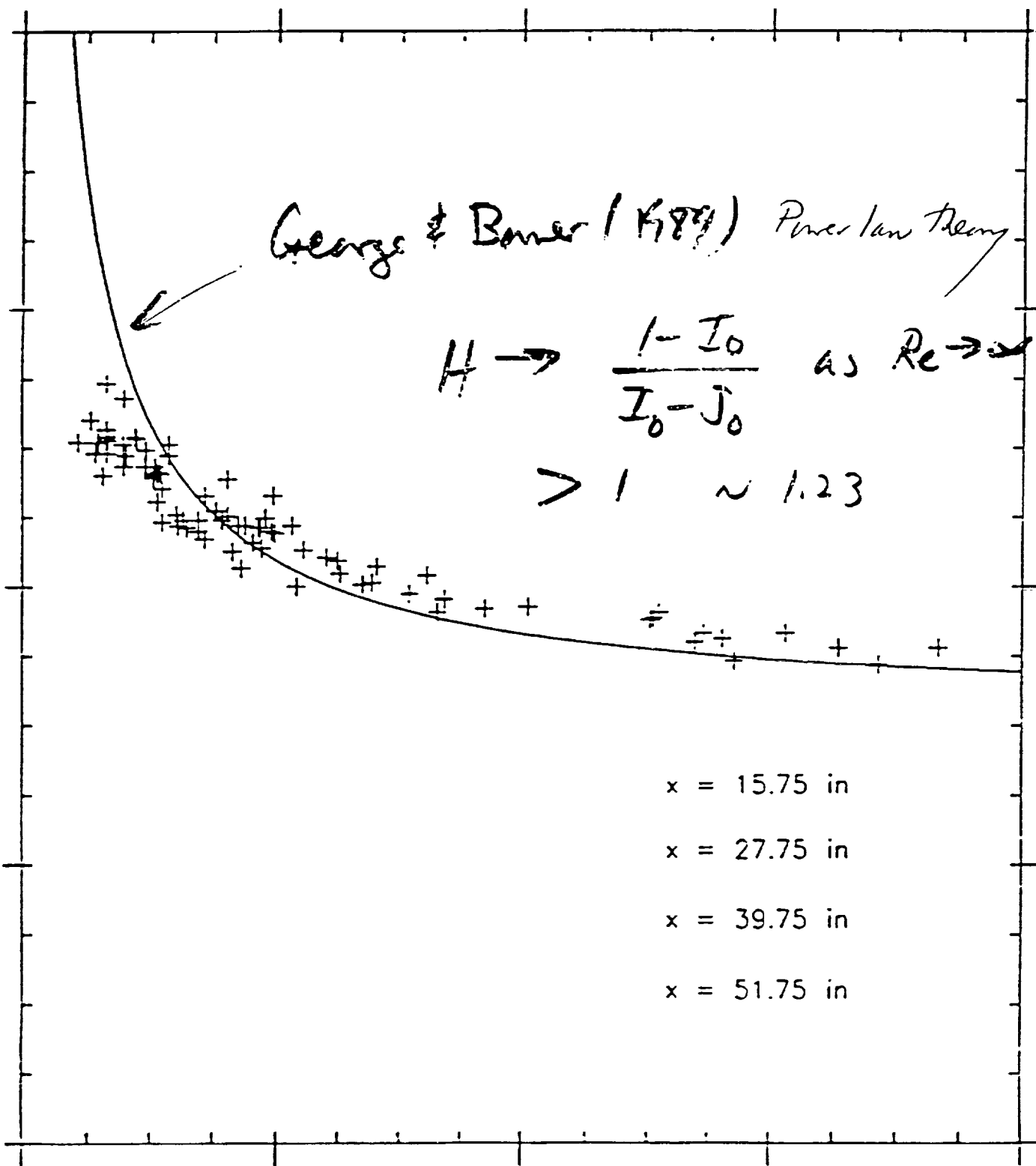
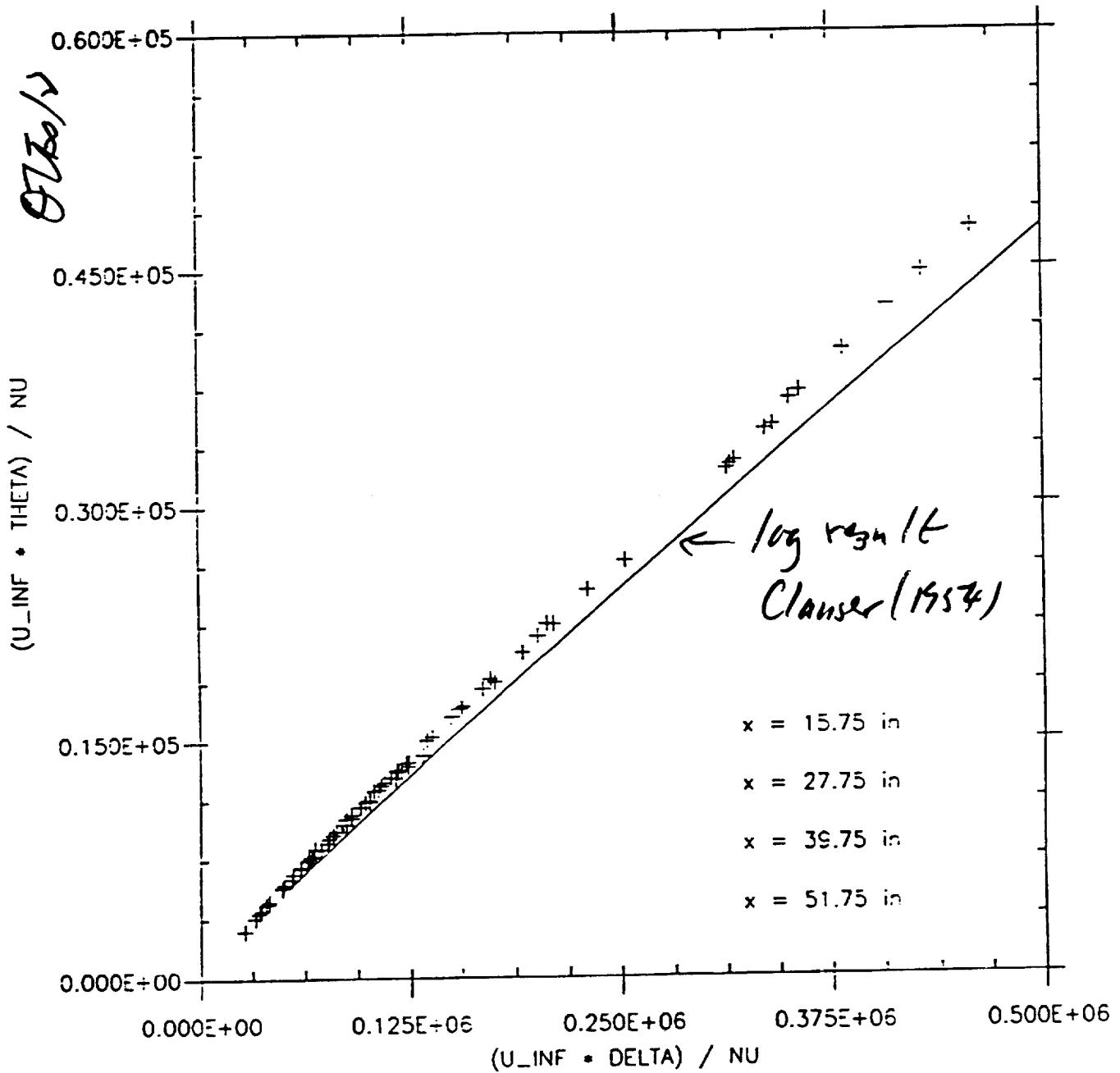


Fig. 4.2.2.1.1 Momentum thickness:

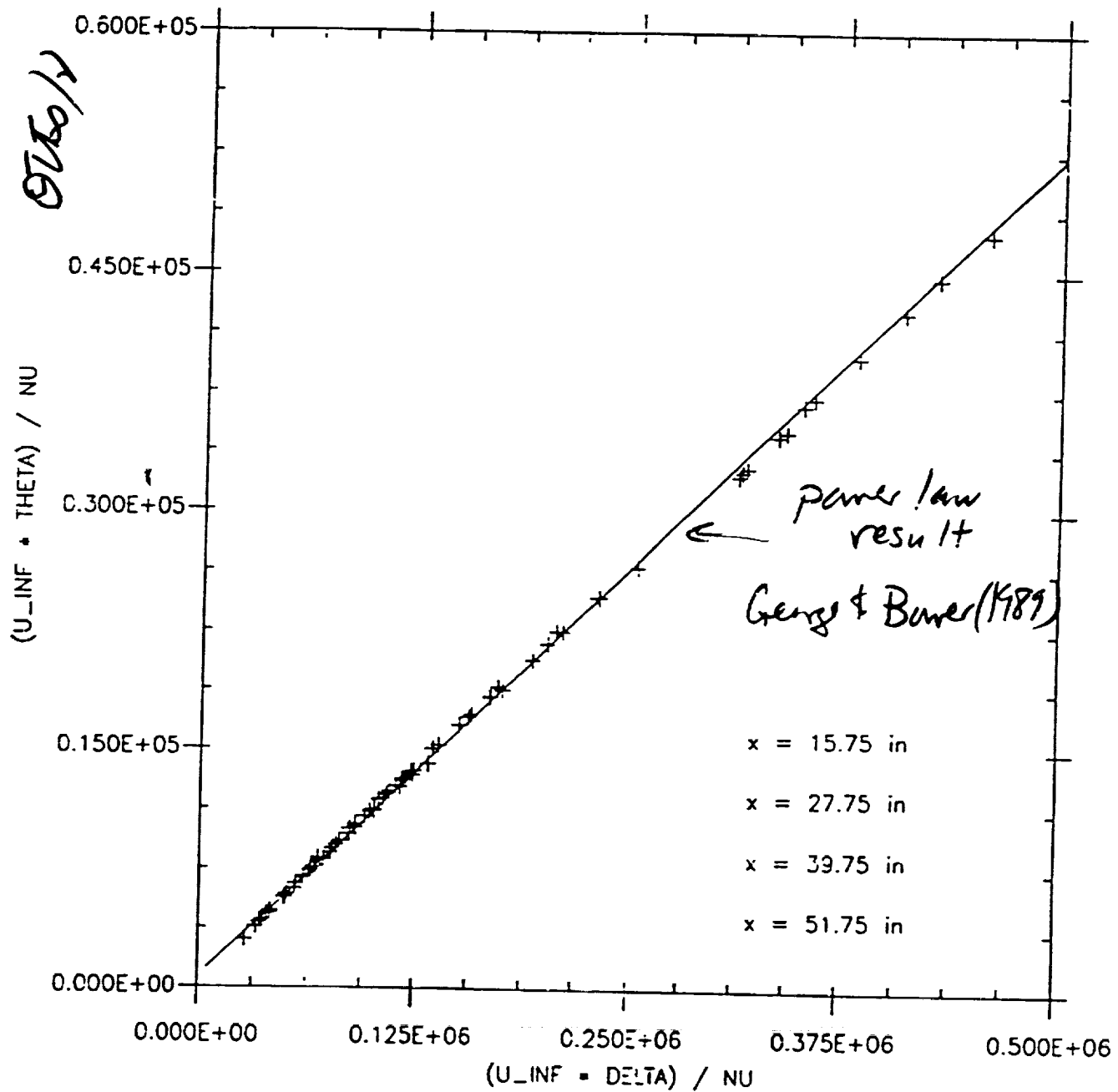
Smith and Walker.  $x = 15.75$  in,  $27.75$  in,  $39.75$  in,  $51.75$  in.



$St_{\infty} / \nu$

Fig. 4.4.2.1.1 *Momentum thickness:*

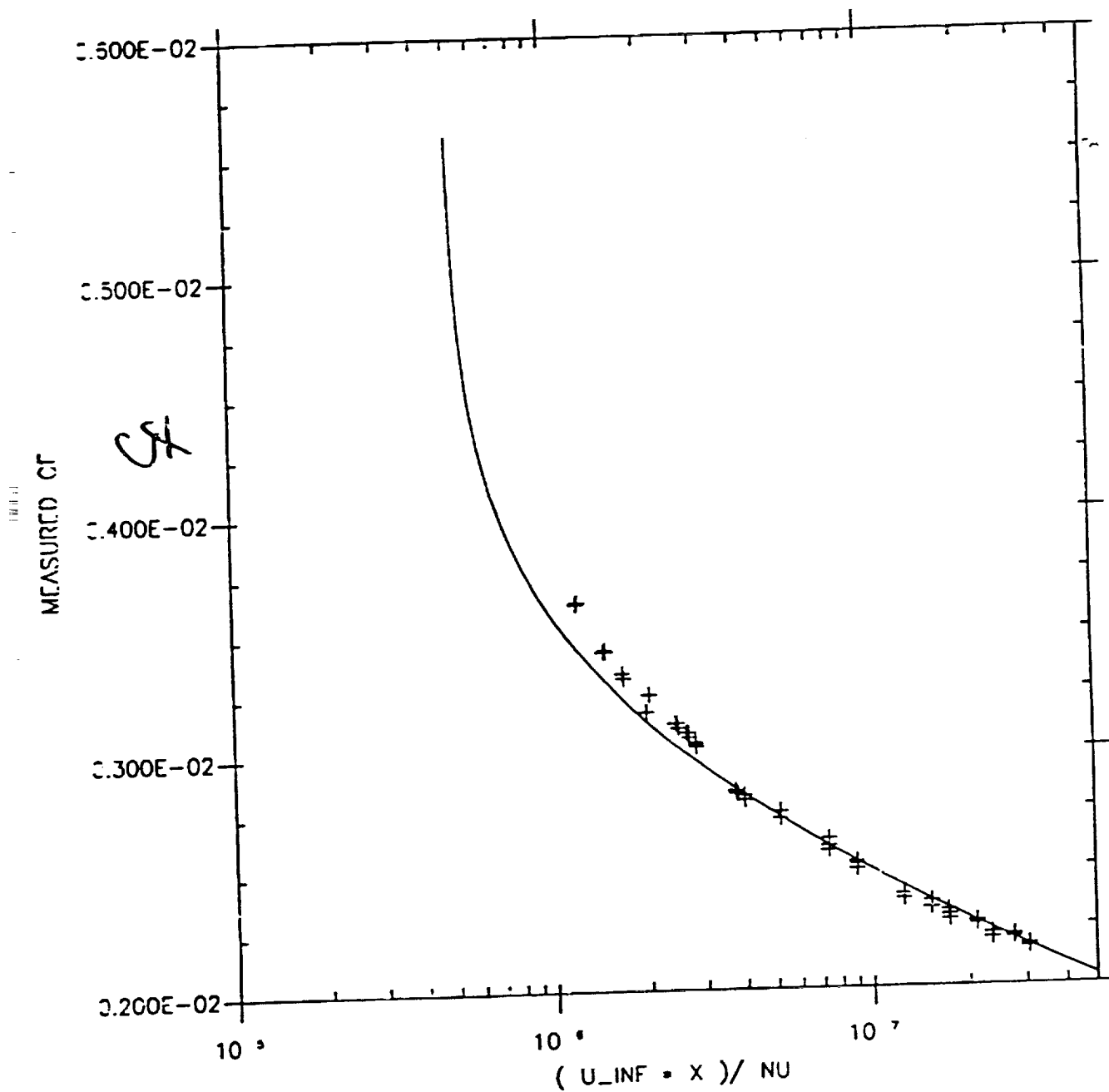
Smith and Walker.  $x = 15.75$  in,  $27.75$  in,  $39.75$  in,  $51.75$  in.



*SL50/2*



Fig. 4.4.6.1.1 Friction coefficient as a function of  $x$ .  
Smith and Walker (53 points).



## The Past

- . Without experiments, there would be no single-point turbulence models.
- . DNS, LES, etc. have helped augment data base in recent years. Contributions have been important, but relatively minor when taken as part of the whole.

## The Future

- . Question: Is this balance likely to change?
- . No question simulations will play increasingly important role
  - ability to produce difficult quantities (for expts) ( $p v_i$ , pressure strainrate, dissipation, etc.)
  - increasing Reynolds number (still small)
  - National economic agenda of U.S. - we will do what is necessary to maintain market position
  - indirect subsidy of CFD
- . What role will this leave for expts?
- . Experiments of value to modelling community are very difficult to do
- . Most efforts, however well-intended, do not measure up!

Reasons complicated but... high on the list....

- . Inexperience of investigator.
- . Ignorance of goals.
- . Money usually runs out before experimenter learns to do it right.
- . These will be increasing problems in next decade.

## Biggest Problem for Experimentalist:

- . Few sponsors have patience to see an experiment through to its completion - none in my experience!
- . As complexity of flows to be investigated increases (Bushnell comment), this problem will be exacerbated!

### FINAL THOUGHT

However successful our closure efforts may be, we are not solving the turbulence problem - we are only being responsible engineers!

We must therefore be careful not to demean the efforts of those who are trying to grapple with the real turbulence problem - there may be an underlying physical principle which will very much affect what we do!



**NUMERICAL SIMULATION:  
ITS CONTRIBUTIONS TO TURBULENCE MODELING**

**Joel H. Ferziger  
Stanford University**

523-34  
86904  
p 35  
N92-24537

# **TWO TYPES OF NUMERICAL SIMULATION**

## **Direct Numerical Simulation (DNS)**

**Exact realizations**

**Good for studying physics**

**Limits: Low Re, Simple geometry**

## **Large Eddy Simulation (LES)**

**Small scales modeled**

**More complex flows**

**Less accurate**

# **FLOWS SIMULATED**

**Homogeneous turbulence**

**Isotropic**

**Strained (various)**

**Sheared**

**Compressed**

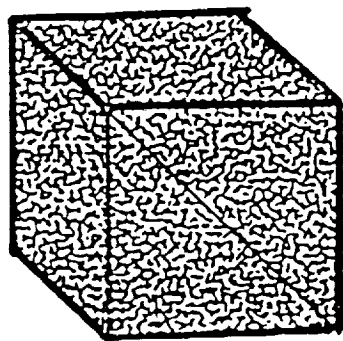
**Compressible**

**Rotating**

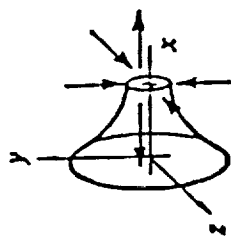
**Combinations**

# Homogeneous Incompressible Turbulence

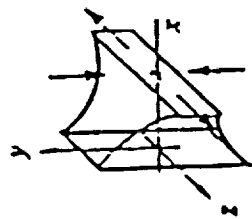
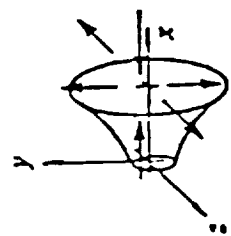
Isotropic decay



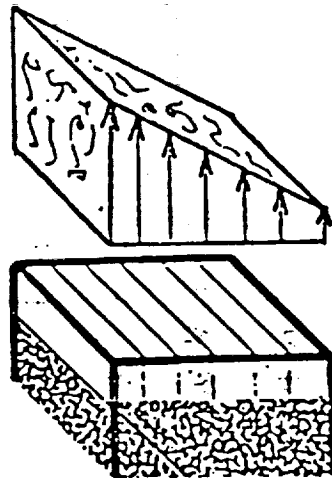
Strain



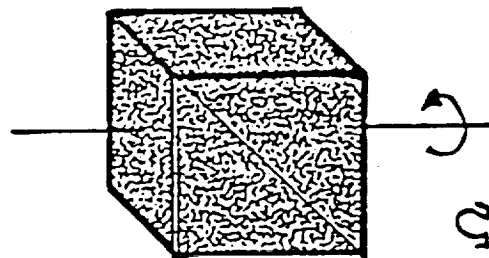
Relaxation from strain



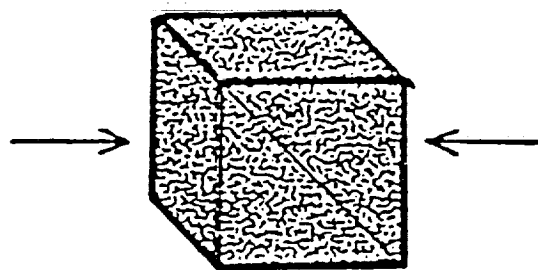
Shear



Rotation



Compression





$$\overline{k - \varepsilon}:$$

$$\begin{aligned} \partial_y(\nu + \nu_T / \sigma_k) \partial_y k &= \varepsilon - \nu_T (\partial_y U)^2 \\ \partial_y(\nu + \nu_T / \sigma_\varepsilon) \partial_y \varepsilon &= \frac{[C_{\varepsilon_2} \varepsilon - C_{\varepsilon_1} \nu_T (\partial_y U)^2]}{T} \end{aligned}$$

$$\overline{v^2 - \rho_{22}}:$$

$$\begin{aligned} \partial_y(\nu + \nu_T / \sigma_k) \partial_y \overline{v^2} &= \frac{\overline{v^2}}{k} \varepsilon - \rho_{22} \\ L^2 \partial_y^2 f_{22} - f_{22} &= -\Pi_{22} \\ \rho_{22} &= k f_{22} \\ \Pi_{22} &= \frac{C_1}{T} \left[ \frac{2}{3} \overline{v^2} - \frac{1}{k} \right] + C_2 \nu_T (\partial_y U)^2 \end{aligned}$$

# Mean velocity

- DNS,  $Re_{\tau}=395$
- DNS (+6),  $Re_{\tau}=180$

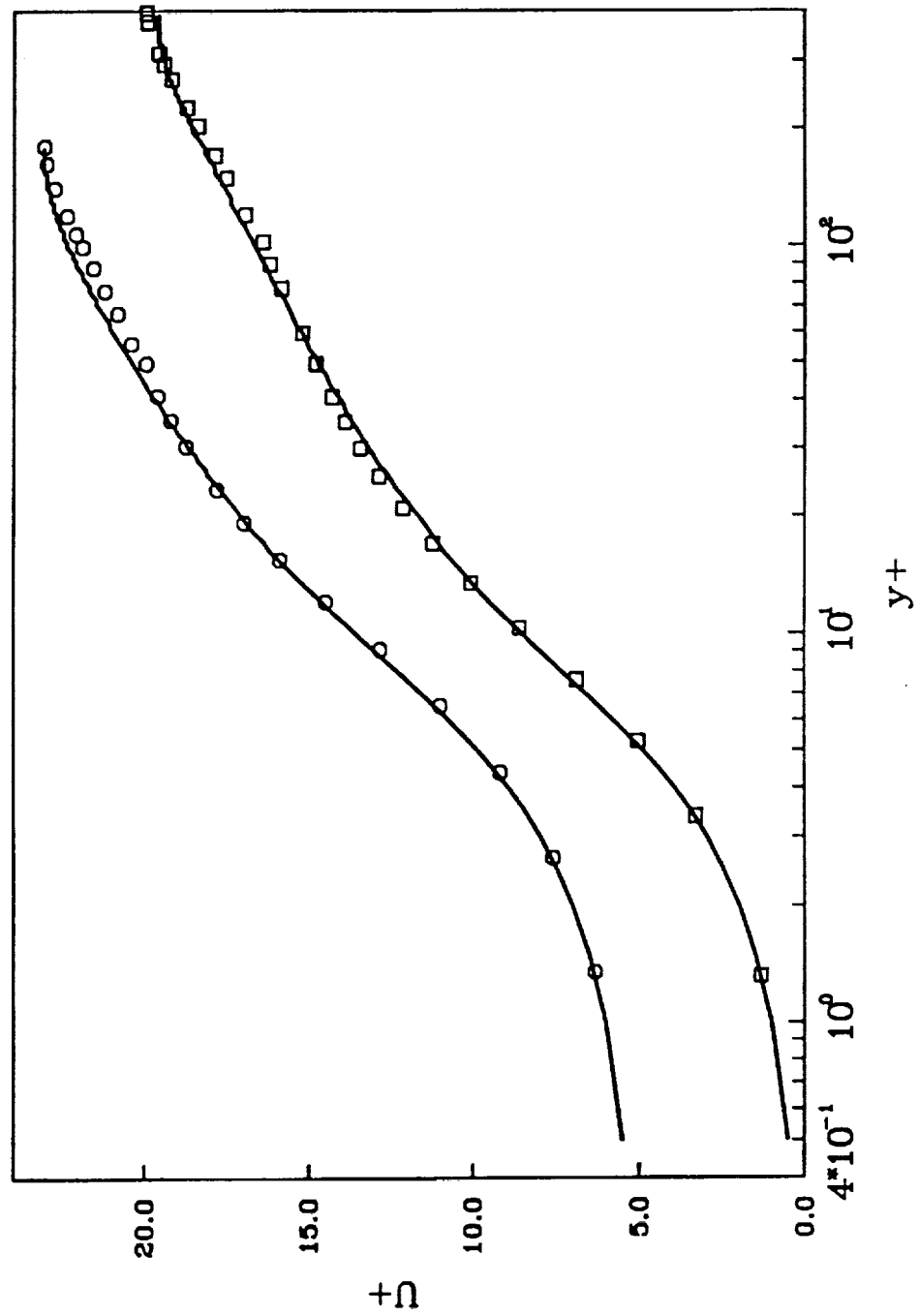


Fig. 2. Mean flow profiles

variances  
k and  $v_2$

	$\frac{k}{v_2}$
$\square$	DNS, $R=365$
$\circ$	DNS, $R=395$

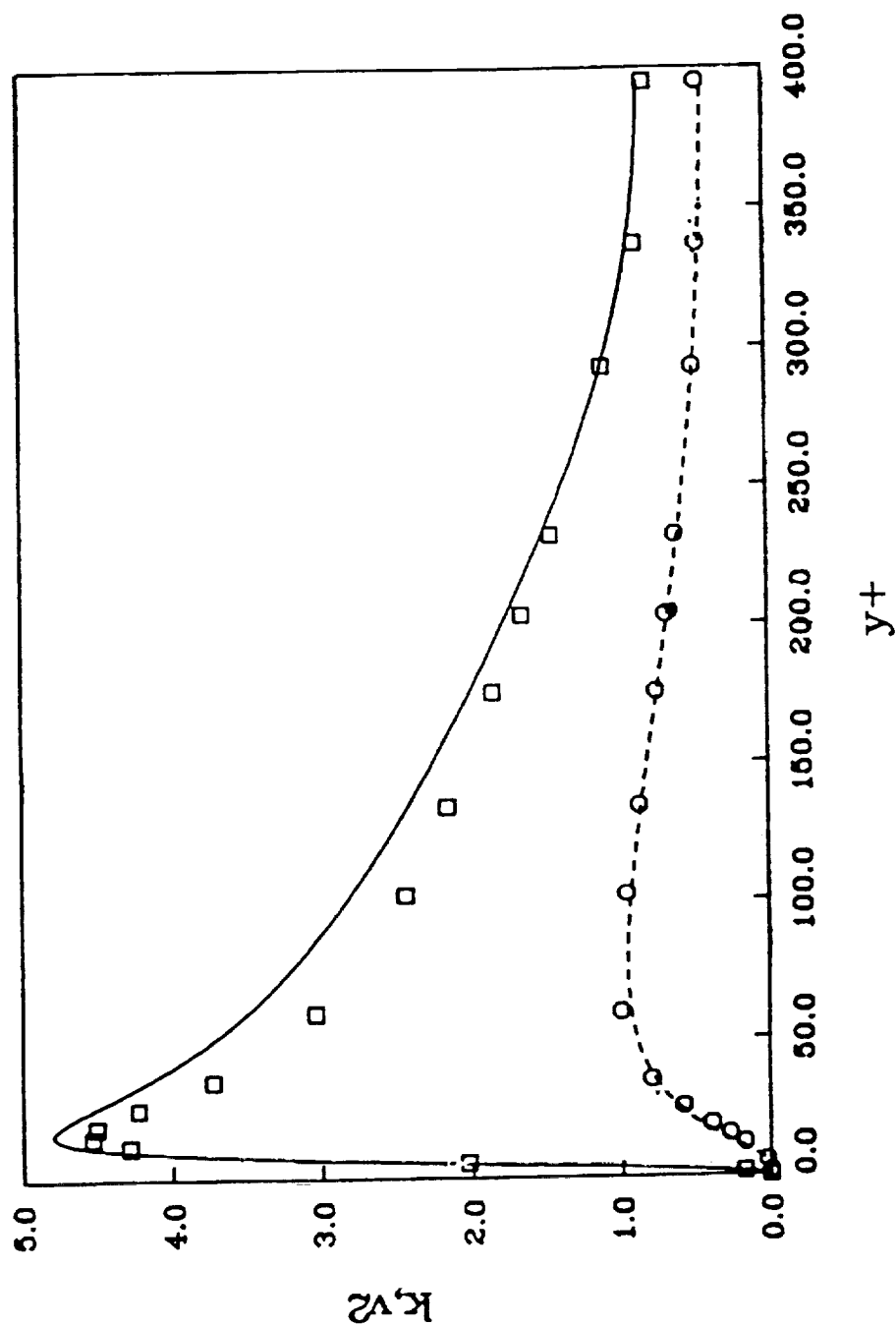


Fig. 1b.  $k$  (solid line) and  $v_2$  (dashed line) from the model compared to DNS data (symbols).

# **COMPRESSIBLE TURBULENCE**

## **New Physics:**

**Faster decay at high Mach number**

**Mixing layer growth rate reduced**

**Eddy shocklets**

**Model by Zeman**

**Introduced dilation dissipation concept**

**Model based on concept**

**Isotropic turbulence decay—well predicted**

**Growth of mixing layer—well predicted**

**Boundary layer questionable**

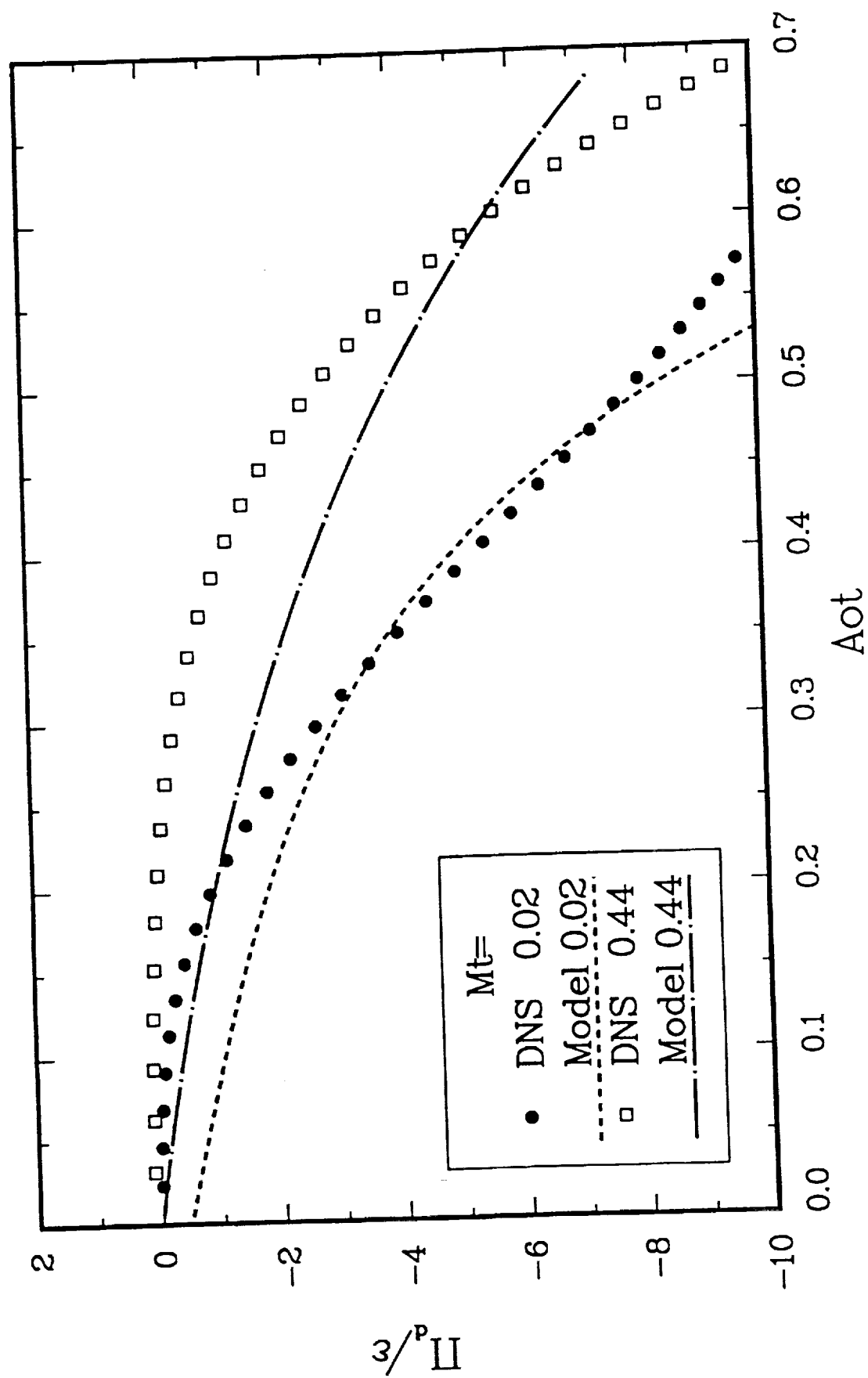


Fig. 1 Pressure dilatation during 1D rapid compression

# STOCHASTIC SHOCKLET DISSIPATION MODEL.

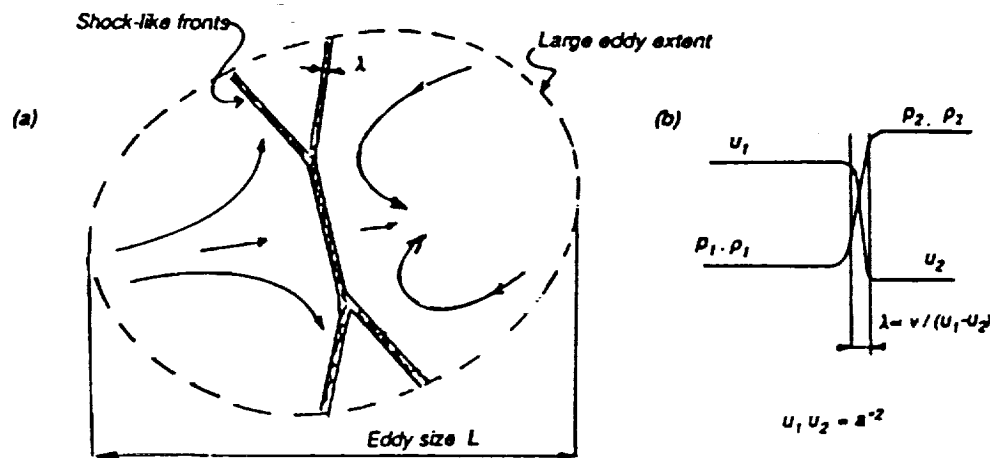


FIGURE 1. (a) Sketch of shock-like structure in a turbulent eddy; (b) normal shock relations.

Average dilatation dissipation:

$$\epsilon_d \propto \frac{q^3}{L} \left[ \frac{1}{M_t^{*4}} \int_1^\infty \left( \frac{m^2 - 1}{m} \right)^3 p(m, K) dm \right]$$

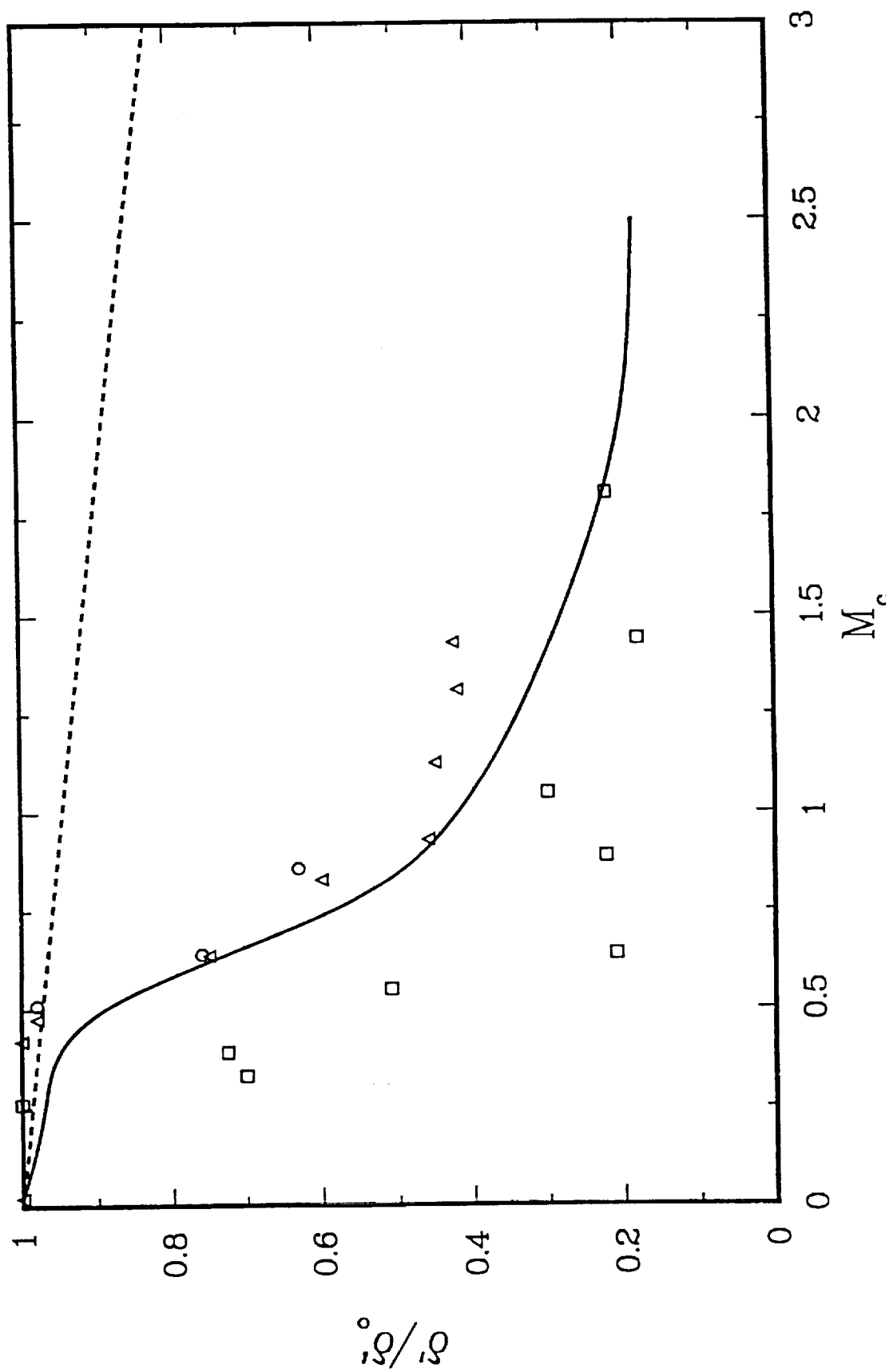
and

$$\epsilon_d = \epsilon_s c_d F(M_t, K)$$

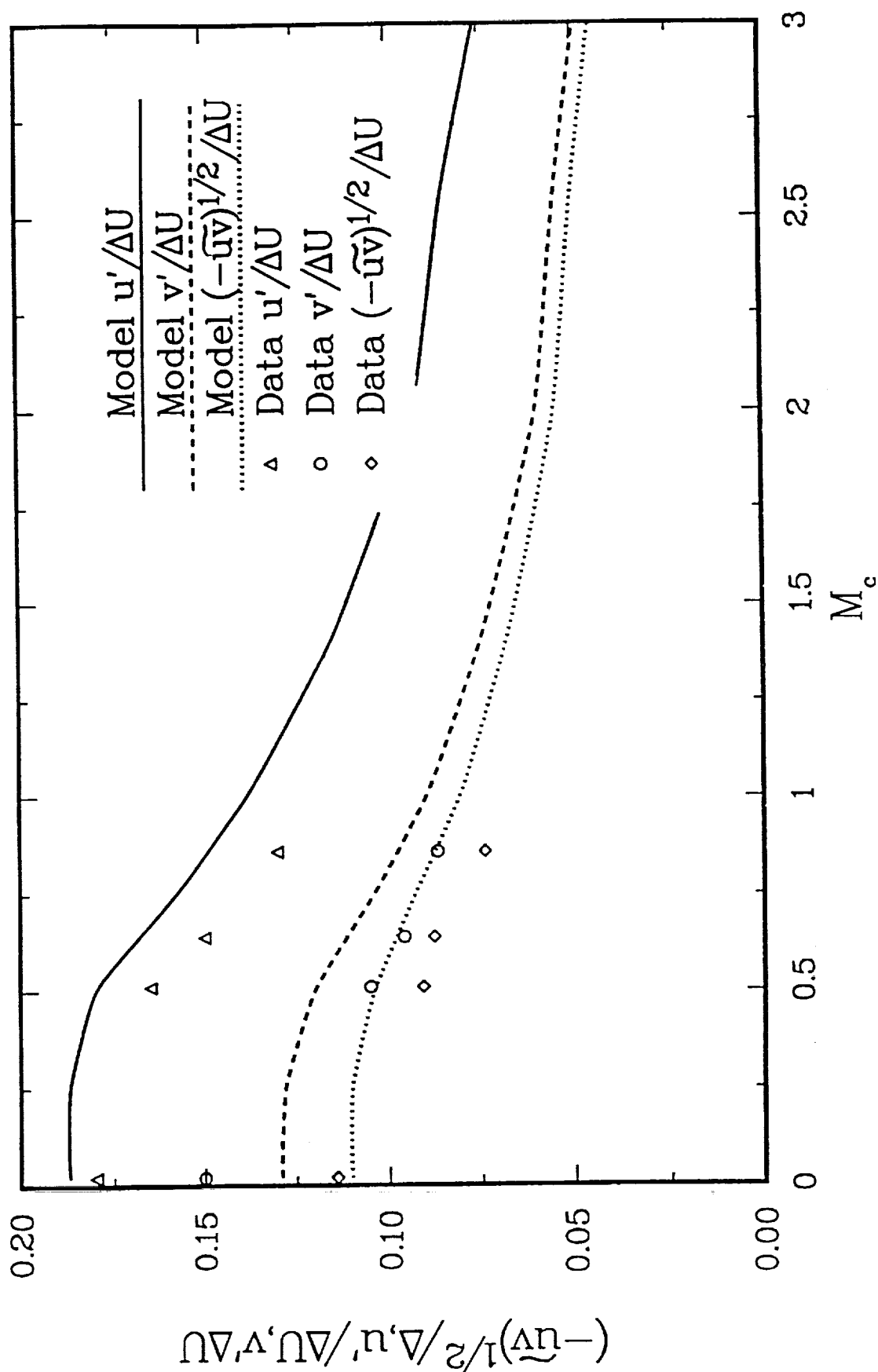
Total dissipation (for given kurtosis  $K$  of  $m$ )

$$\epsilon_{tot} = \epsilon_s \{1 + c_d F_k(M_t)\}$$

# Normalized Mixing Layer Growth Rate



# Centerline Turbulence Intensities in Mixing Layer





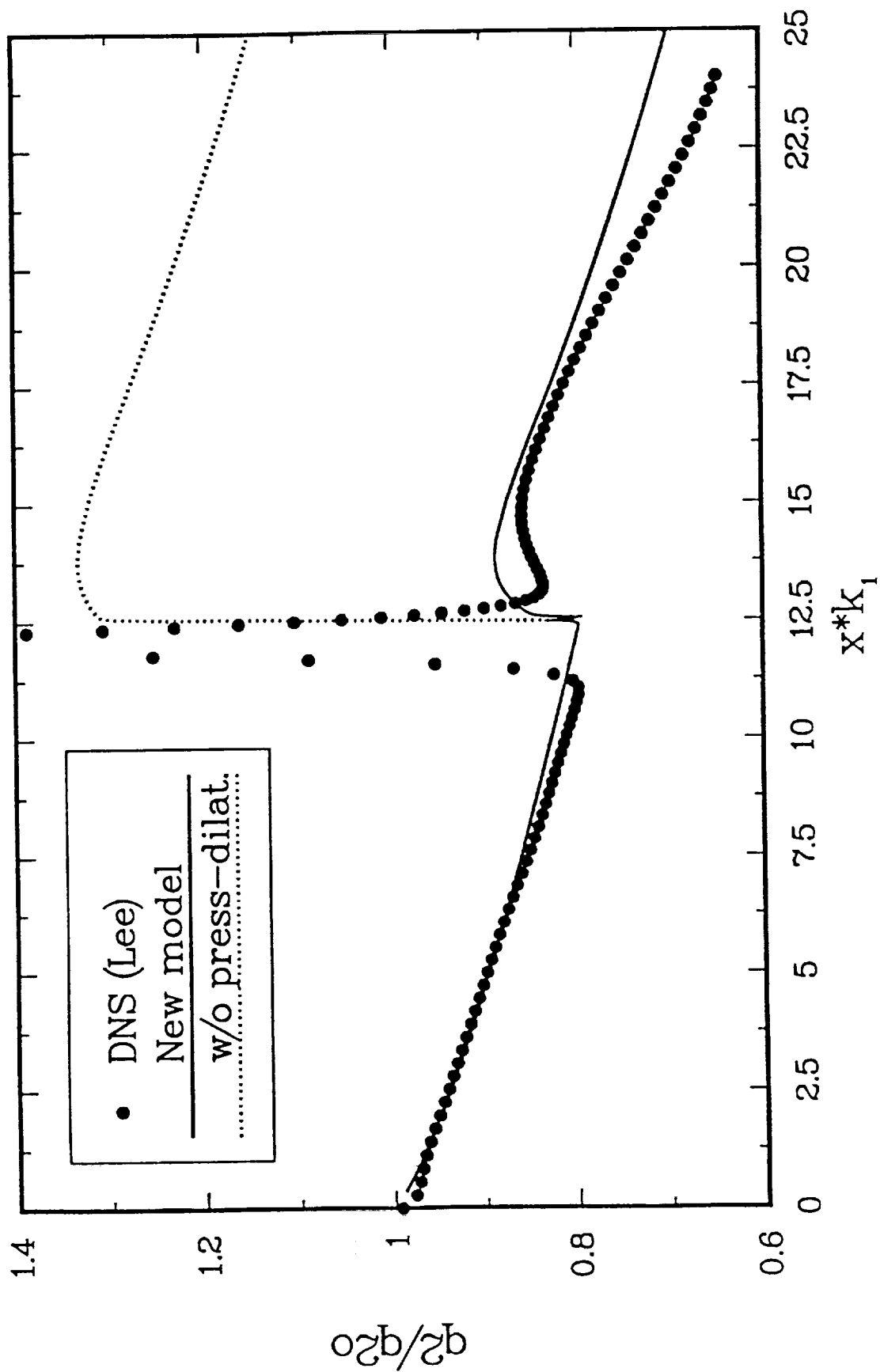


Fig. 3 Response of turbulent energy to the normal shock

# **CONCLUSIONS**

## **DIRECT SIMULATION**

**Useful for studying detailed physics**

**Understanding leads to new/improved models**

**Wide range of examples**

**Building block flows**

**Further examples:**

**Compressed turbulence**

**Strained turbulence**

**Reacting flows–flamelet models**

**Stratified flows**

**Atmosphere**

# **FUTURE**

## **DIRECT SIMULATION**

**More complex flows**

**Wider range of applications**

**Wide range of examples**

**Building block flows**

## **LARGE EDDY SIMULATION**

**Complex flows**

**Interaction between LES/RANS**

**Possible engineering applications**

# **FLOWS SIMULATED—CONTINUED**

**Free shear flows**

**Mixing layers**

**Wakes, jets**

**Wall bounded flows**

**Channel**

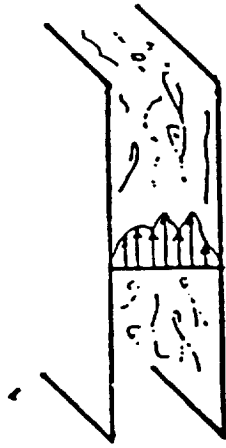
**Boundary layer**

**Couette flow**

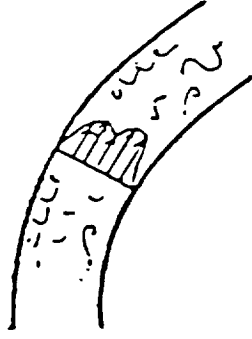
# Inhomogeneous Incompressible Turbulence

## Wall-bounded flows

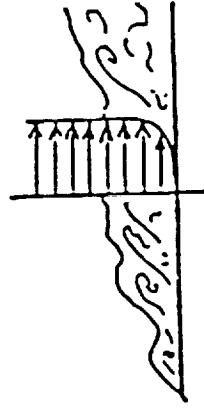
Channel



Curved channel

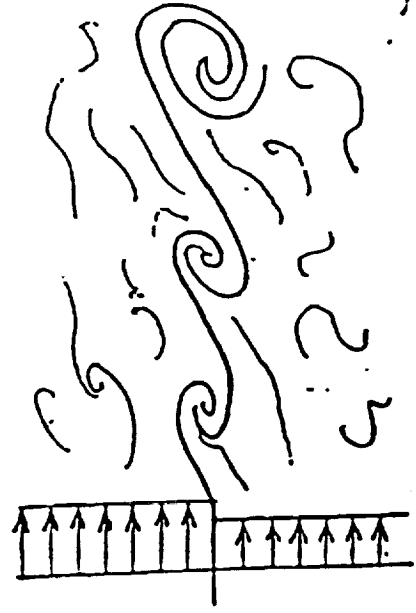


Boundary Layer

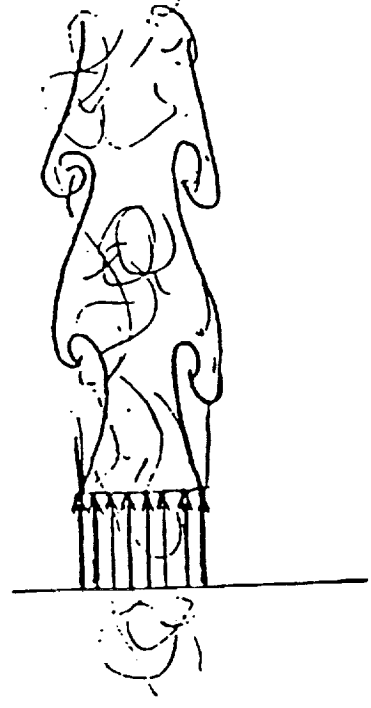


## Free shear flows

Mixing Layer



Plane Jets



# **USES OF SIMULATIONS**

**Study physics of turbulence**

**Test validity of models**

**Gain insights for new models**

# **EXAMPLES--MODEL TESTING/VALIDATION**

**Boundary layers**

**Mixing layers**

**Atmospheric models**

**Stratified turbulence**

# **EXAMPLES-MODEL DEVELOPMENT**

**Rotating turbulence**

**Compressed turbulence**

**Compressible turbulence**

**Wall layers**

**Flamelet models**



# **ROTATING TURBULENCE**

**Isotropic turbulence subject to rotation**

**Important building block flow**

**History:**

**Several experiments—disagree**

**Simulations:**

**Bardina & Ferziger**

**Aupoix & Cousteix**

**Mansour & Coleman**

# **RESULTS--ROTATING TURBULENCE**

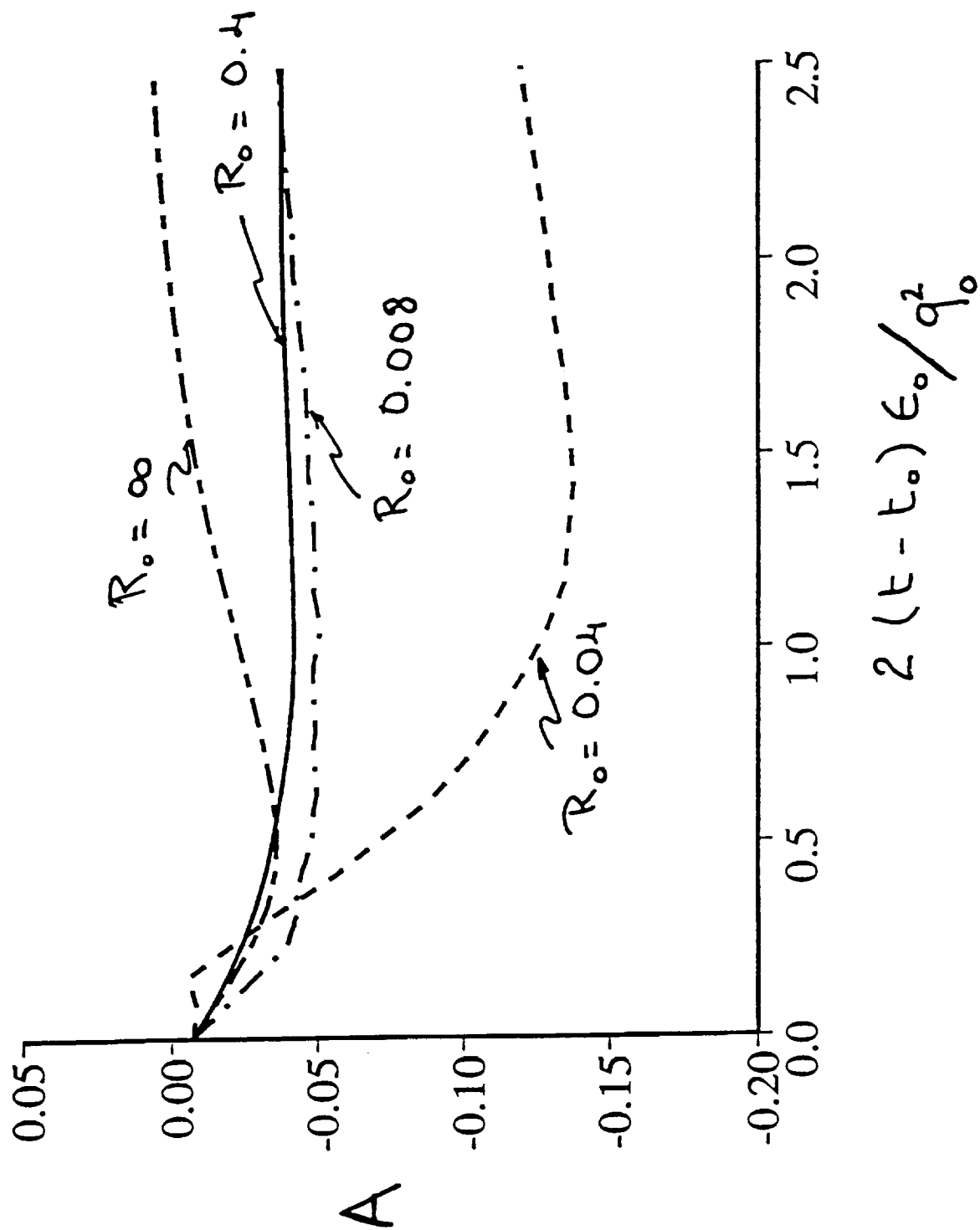
**Turbulence decay reduced**

**Reduced transfer to small scales**

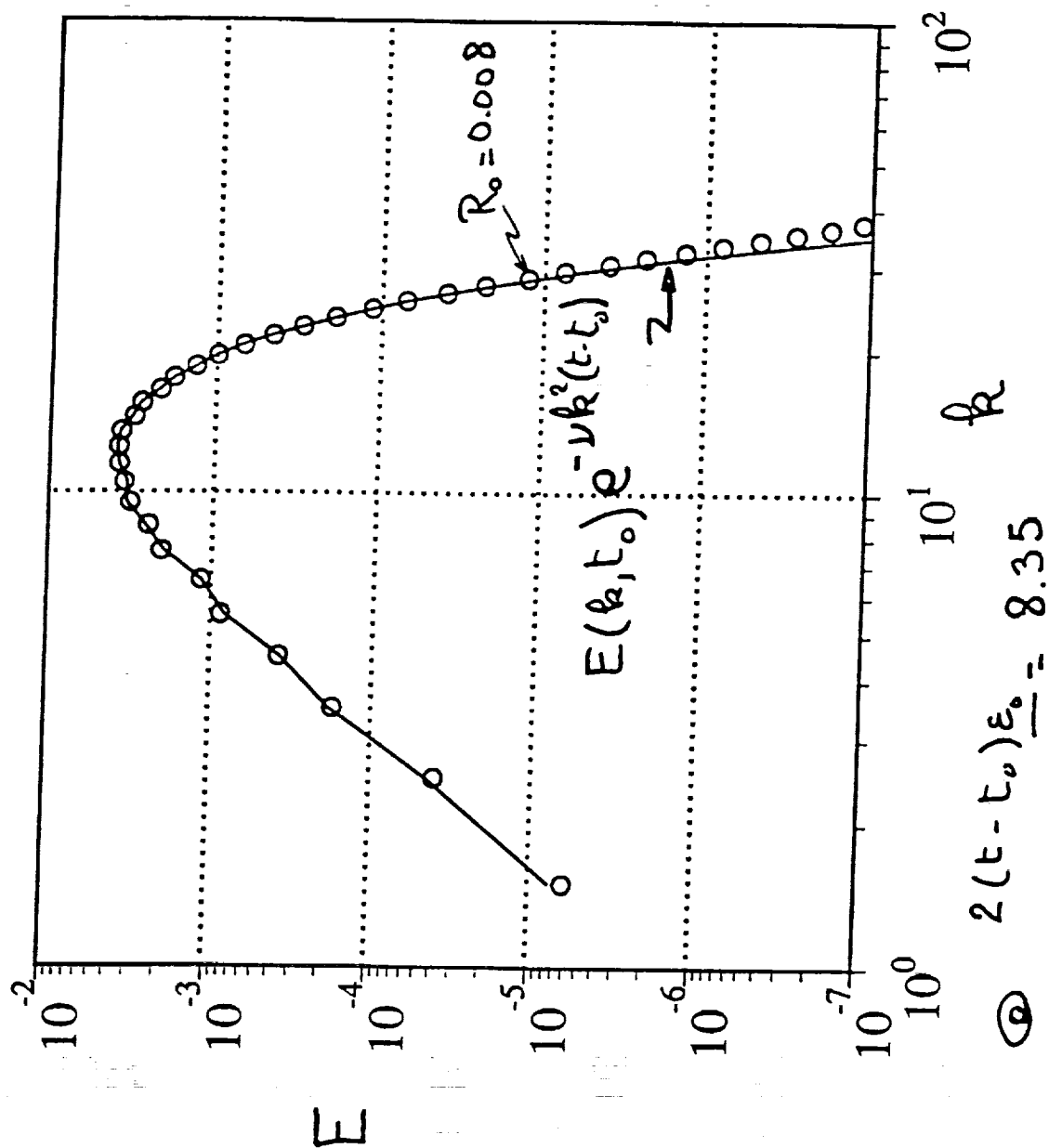
**Anisotropy only at moderate  $Ro$**

**Several models developed**

# ANISOTROPY MEASURE



# ENERGY SPECTRUM



# **WALL BOUNDED FLOWS**

**Near-wall turbulence difficult**

**Existing models not totally satisfactory**

**Mansour & coworkers:**

**Tested approximations to pressure-strain**

**Computed dissipation equation budget**

**Found scaling laws for various terms**

**Durbin:**

**New model based on equation for  $v^2$**

**No need for special low-Re model**

**Good predictions of boundary layers**

## Summary

Have modelled blocking effect with new  $k - \varepsilon - v$  model.

Elliptic relaxation model for  $k_{22}$  accounts for strong non-homogeneity and introduces homogeneous solutions in the  $\overline{v^2}$  part of the model.

This is able to describe near-wall transport in channel flow; holds promise for non-equilibrium flow—which is where models are most needed.

## DISCUSSION

B.E. Launder (to S. Orszag)

You showed us values of  $C_{\epsilon 1}$  and  $C_{\epsilon 2}$  which would give too high a decay rate of grid turbulence.

S. Orszag (reply)

There is some question about what the decay rate really is.

B.E. Launder (to S. Orszag)

You talked about  $C_{\epsilon 2}$  but you didn't say how  $C_{\epsilon 1}$  emerged.

S. Orszag (reply)

That's the same calculation.

S. Pope (to S. Orszag)

Calculations you did with backward facing step, what boundary conditions did you use?

S. Orszag (reply)

That was not a full RNG calculation. It should have been done using interpolation formulation for the various constants all the way to the wall. Instead it was done using the a fit right to the log layer.

T. Gatski (to W.K. George)

Two things modelers are looking for validation and calibration. Bill referred to the kind of experiments we use for validation. The kind of work you (J.H. Ferziger) do with DNS has been building block for calibration. Do you want to design the experiments to validate our model or calibrate?

W.K. George (reply)

I would like to design experiments which would invalidate your models.

T. Gatski (to W.K. George)

But that would be destructive for both of us.

P. Spalart (to J.H. Ferziger)

I differ with your description of DNS as exact solution. I would like to say that my solution are not exact. I spend quite a lot of time thinking how I can keep the error small. If I can double the number of the grid points in each direction I would sleep better but it would take fifty years instead of two years to finish the simulation.

J.H. Ferziger (reply)

I should have said that in my talk that any numerical calculation is approximate and I hope we work hard to keep them small. There are errors due to numerical methods and errors due to the fact that we have limited computer time.



P. Spalart (to W.K. George)

A comment on Bill's theory that it's a power law instead of log law. I think in your original APS abstract you make it sound like it's just a matter of taste if you use defect law or your theory.

W.K. George (reply)

In the original APS abstract I wasn't clear why the theory comes apart asymptotically and I feel much confident now that the existing theory is wrong.

P. Spalart (to W.K. George)

There is very different Galilean invariance to those two theories and its not a matter of test. Defect law says we are coming from the free stream and we don't know how fast the wall is moving; it may be a moving belt. Your theory doesn't do that.

W.K. George (reply)

That's right. There is no question that there is a lot of sorting out to be done.

B.E. Launder (to W.K. George)

It's very interesting that he (or it) brings out into question the universality. If you got flows to decrease rapidly with distance from the wall as you do in low  $Re$  channel flows there is data going back to fifties that your log-log constant goes up. So logically you would expect log-log constant would go down in adverse pressure gradients. The implicit faith shown in sectors of fluid community in the universality of the log law I think is misplaced.

There is now emerged which I think an excellent paper by Nagano (at the upcoming SFC in Munich) showing what seems to me a clear dependence of log-log constant on shear stress gradient. In adverse pressure gradients lower log-log constants than you find in zero pressure gradients. Bill gets unhappy at unacknowledged at his private discoveries as all of us do, I suggested this in a paper about eight years ago.

W.K. George (reply)

I presented this in 1978.

P. Spalart (to B.E. Launder)

I have results that show that in moderate pressure gradients log-log going down. At  $y^+ = 50$  it goes down by almost one wall unit at  $\beta = 2$  which is not very strong at all.

S. Pope (to P. Spalart)

You very quickly mentioned that you use DNS data for guidance and not calibration. Could you expand on that?

P. Spalart (reply)

If I calibrate turbulence model for boundary layer based on flat plate results I'll get too high  $Re$ , so I don't have DNS results which I'll trust within 10% to extrapolate to high  $Re$ .

G. Hwang (to J.H. Ferziger)

You talked about P. Durbin model that uses  $\overline{v^2}$  as damping function. But

problem is we want to use this model in multi-dimensions. Then you are talking about  $k - \overline{v^2}$  and  $\epsilon$  equation which is as complicated as Reynolds stress models. I am not disputing the model there are some good points to it.

A. Yoshizawa (to J.H. Ferziger)

You pointed out the possibility of LES as engineering tool but our experience shows we can not perform LES with Smorginsky constant fixed. LES critically depends on Smorginsky constant e.g. we perform LES of channel flow using  $C_s = 3.1$  but using this constant we can not simulate e.g. backward facing step. My opinion is without overcoming this difficulty LES can not become engineering tool.

J.H. Ferziger (reply)

I agree with you. There is a new model which does overcome some of these difficulties. I don't want to say that all the difficulties are overcome but we have hope.

J. Bardina (to A. Yoshizawa)

We have investigated DIA, RDT and LES. LES is much simpler than DIA. You have difficulty with Smorginsky constant but we know that this constant is not right. It's not universal for all flows because only thing it's doing more is dissipating more energy; that's all. At high  $Re$  you are putting more energy at small scales and there are many other effects which you have to put there.

E. Reshotko

Bill brought up many of the problems in looking at experiments. There are many more which try to fulfill some of the desires and expectations that have been brought up here at the workshop. First any self-respecting

experimentalist is not there to design experiments to validate or calibrate a theory. First of all he is there to discover new physics. May be a theoretician wants to see if the physics is reproduced by the model.

But when it comes to doing that particularly in measuring turbulence we come up with the problem of how to measure at a point. We have all these wonderful things at a point when we have probes which are not a point. Recently we had experience with multiple wire probe that showed our probe was not measuring at a point although our probe was less than 0.1mm in overall size. And this problem becomes worse if one goes to high speeds. I understood just a few years ago why all good turbulence measurers were working in large facilities and in low speeds because only in that way you can feel reasonably secure that in terms of wall units you are operating at a point. We tried running some experiments at 100 ft/sec and found that our probe was 100 wall units which typically a spanwise streak size. In compressible flows, aside from increased speeds and increased probe dimensions in wall units, we also have the problem of calibration in transonic regimes. It's not that we don't know how a hot-wire works in transonic regimes. It's so sensitive to Mach number in transonic regime that there is not a way of saying it's reliable. I am worried about the double and triple correlations in boundary layers with the present probes. One of the things H. Nagib is doing is looking at probe miniaturization and I encourage this but until then I think prospects of getting detailed compressible flow measurements are dim.

J.H. Ferziger (to all participants)

I thought it would be interesting to throw out at the modelers that what do they think is missing in the experiments, simulations and theory?

B.E. Launder

I ponder from time to time about these nice homogeneous flows which people use to come up with constants in dissipation equation as Steve Orszag was talking about. Question comes to my mind that its the variation in inhomogeneity which we are interested in looking at. The variation in spatial length scales ought to enter in our closures in ways other than the diffusive like terms. That is to say perhaps if we are thinking of dissipation equation having adjacent to one another layers of different length scales are going to be promoting spectral transport of energy removed more readily than you'll find in homogeneous flows. So I ask myself if DNS can help clarify this.

J.H. Ferziger (to B.E. Launder)

Are those relatively simple inhomogeneous flows in that regard.

B.E. Launder (reply)

I think simpler inhomogeneous flow you are talking about is channel flow where everything is so dominated by the fascinating structure of the near wall sublayer. If you could do simulation away from wall where low  $Re$  dissipation issues are dominated.

J.H. Ferziger (to B.E. Launder)

Then there are mixing layers.

B.E. Launder (reply)

Maybe mixing layer results will be valuable. I don't think they ought to belong to this question. That is your dissipation equation needs to be different in inhomogeneous from that in homogeneous.

J.H. Ferziger (to B.E. Launder)

We tried doing flow simulation of experiments of Warhaft. May be we can collaborate.

B.E. Launder (reply)

Maybe those experiments themselves will answer. There are no mean velocity gradients in that experiment John (to J.L. Lumley).

J.L. Lumley (to B.E. Launder)

No mean velocity gradients - just a gradient of scale. I was just going to draw attention to that.

J. Bardina (to B.E. Launder)

Brian are you suggesting that homogeneous flows are not a valid test for  $k-\epsilon$  model since it would not account for inhomogeneous part in shear flows.

B.E. Launder (reply)

We should look at homogeneous flows as building blocks and may be we shouldn't say going from homogeneous to inhomogeneous flows just adds a diffusion like transport term but may be adds other as well.

J. Bardina (to B.E. Launder)

The  $k - \epsilon$  model worked good for homogeneous flows we tested and it wasn't tuned for these. We looked at homogeneous shear, and plane strain flows and it did very well. It didn't do well for rotational flows because effect of rotation isn't accounted for.

W.K. George (to B.E. Launder)

I think you are right Brian. If you look at Antonio's dissipation results all but one derivative is way out of line. I have come to believe that it associated with inhomogeniety. And if you look at each term in the equation, if the flow is truly locally homogeneous you can not produce any of those. This question is best resolved by DNS of inhomogeneous situation at low  $Re$ .

J. Weinstock (to B.E. Launder)

I wouldn't have any doubt that inhomogeniety would cause changes other then diffusive transport. Nature of the change is such that we can not tell until we do it. I did a calculation where I accounted for strong time variation where the turbulence energy is changing at times of order of eddy circulation time and a simple result came out of that. It wasn't diffusive transport. Eddy circulation becomes function of turbulence time scale. So the coefficients involving damping become function of rate of change of turbulence. I think if we put spatial inhomogeniety we would come up with something related to that. It's obviously not diffusive.

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